The formation of giant planets: Constraints from interior models

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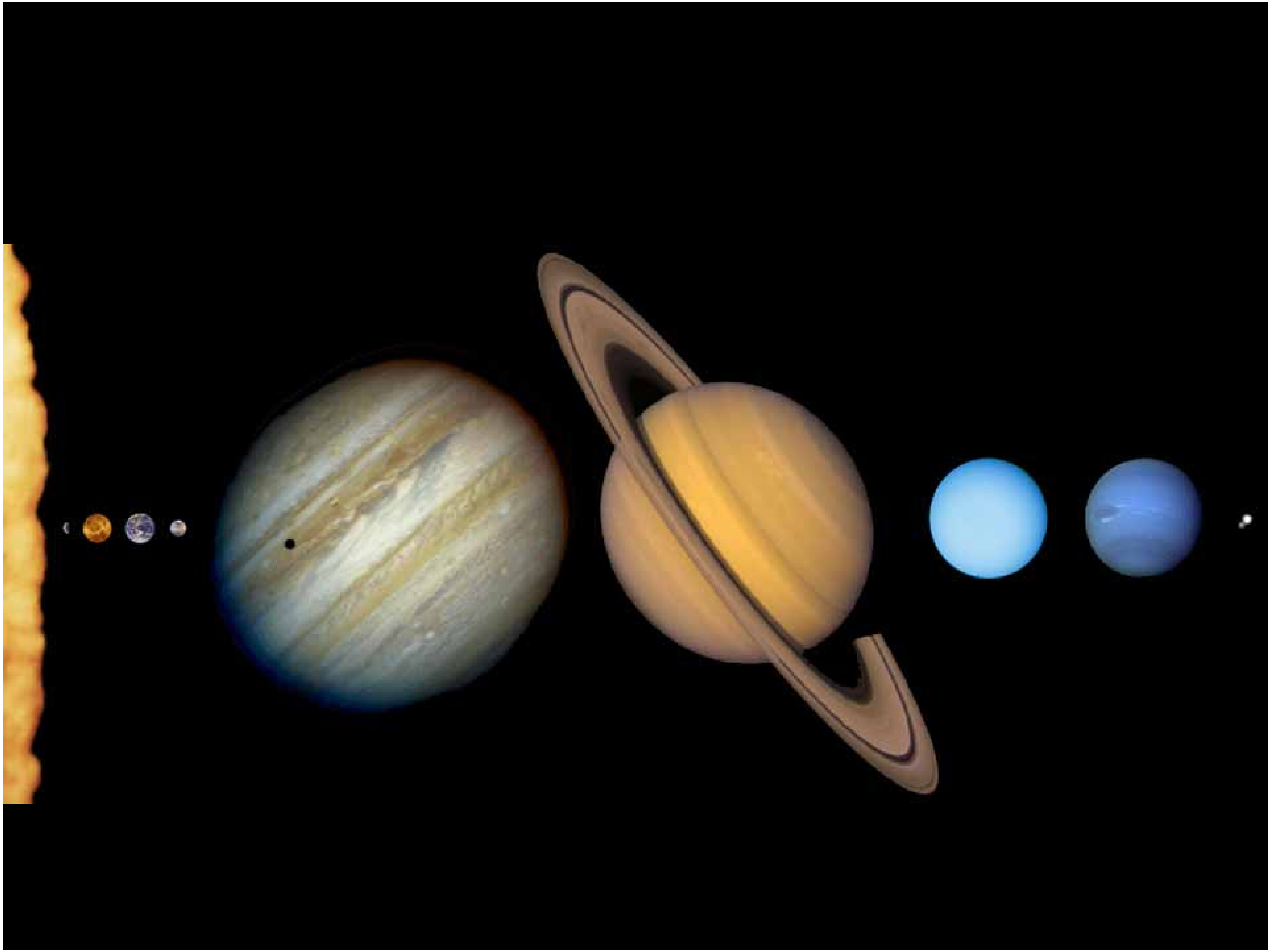
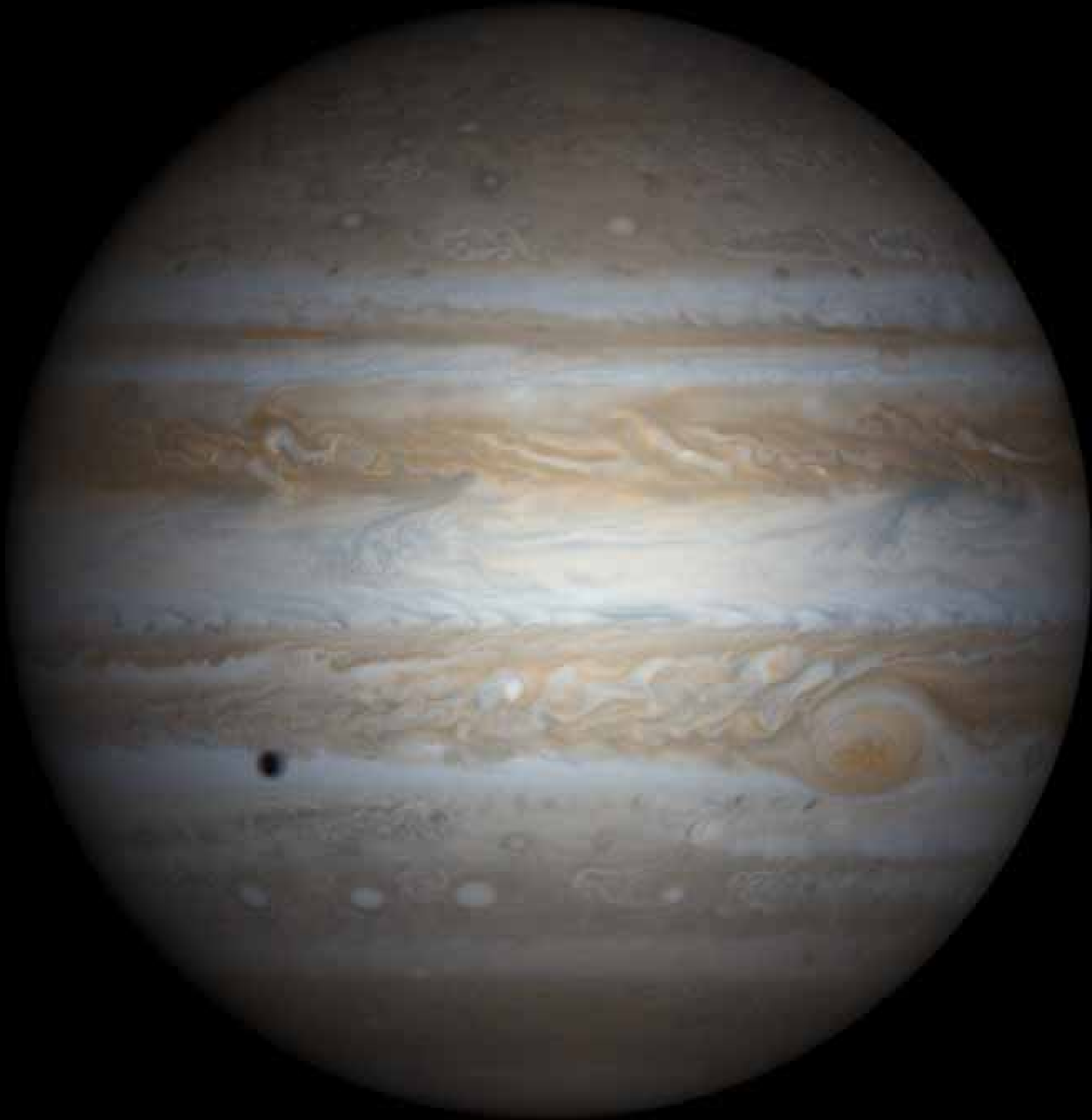
(Guillot, Ann. Rev. Earth & Plan. Sci. 2005

& Saas-Fee course 2001, in press)



Jupiter



Jupiter: clouds & vortexes

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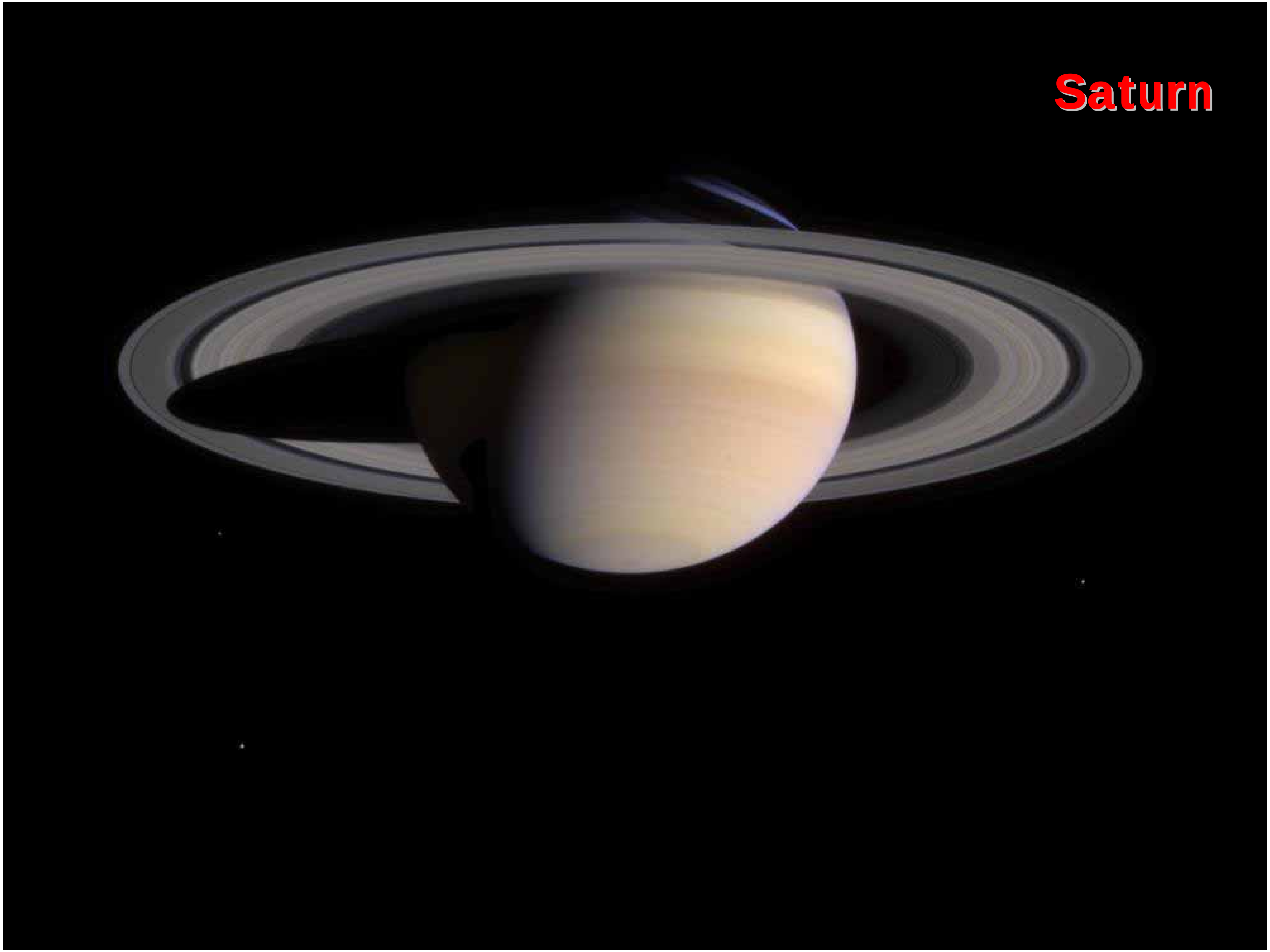
Saturn

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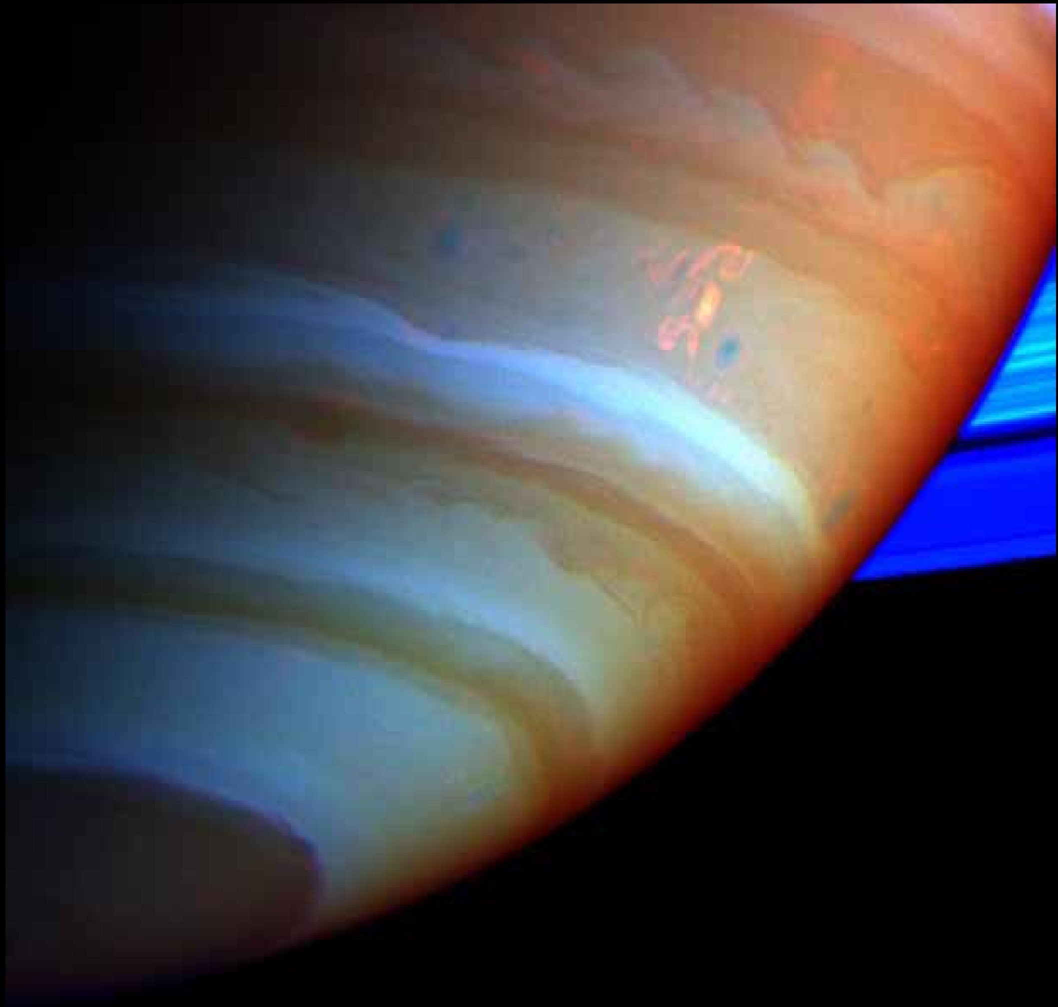
Saturn: the “dragon” storm



Uranus

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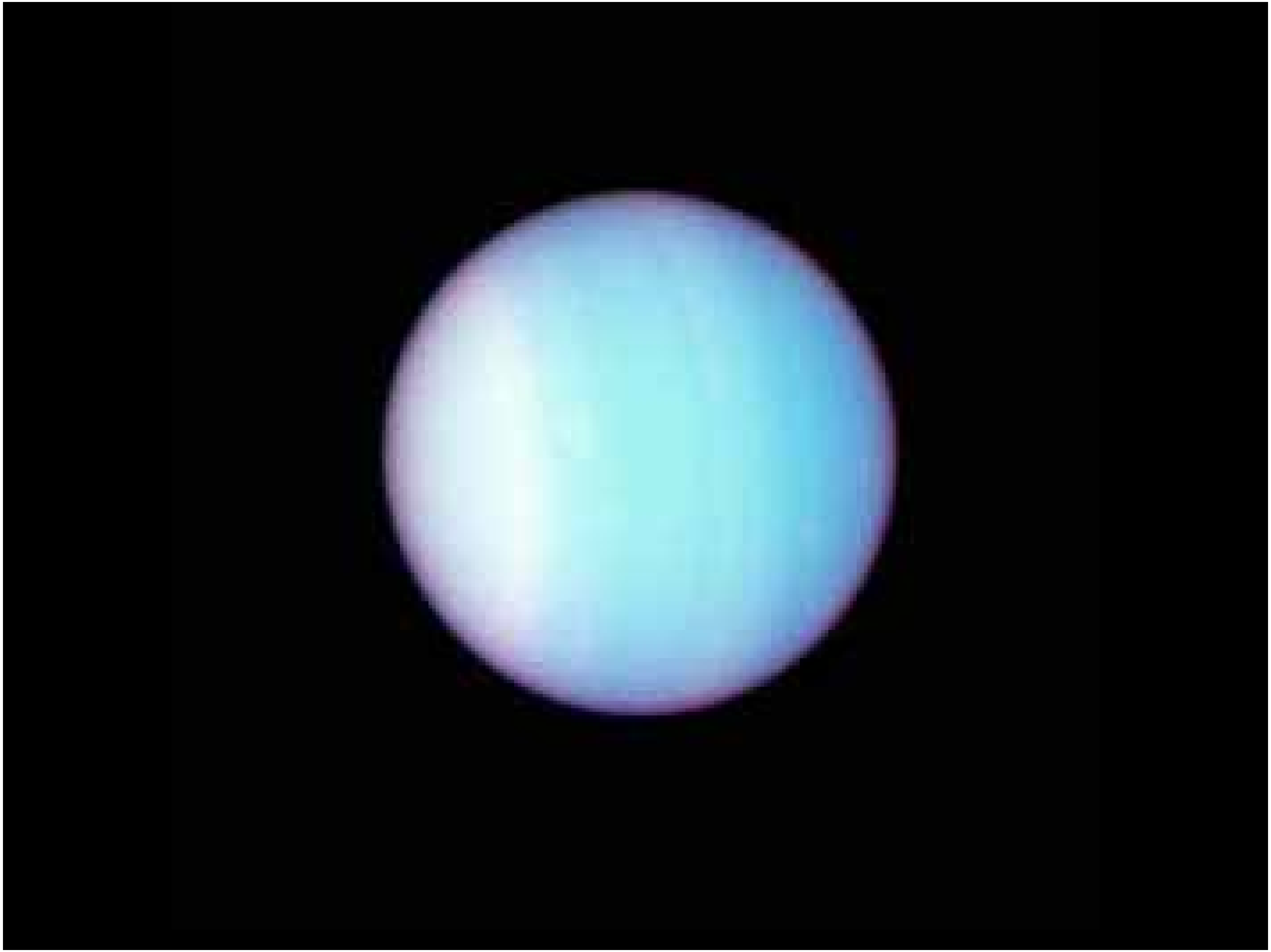

Neptune

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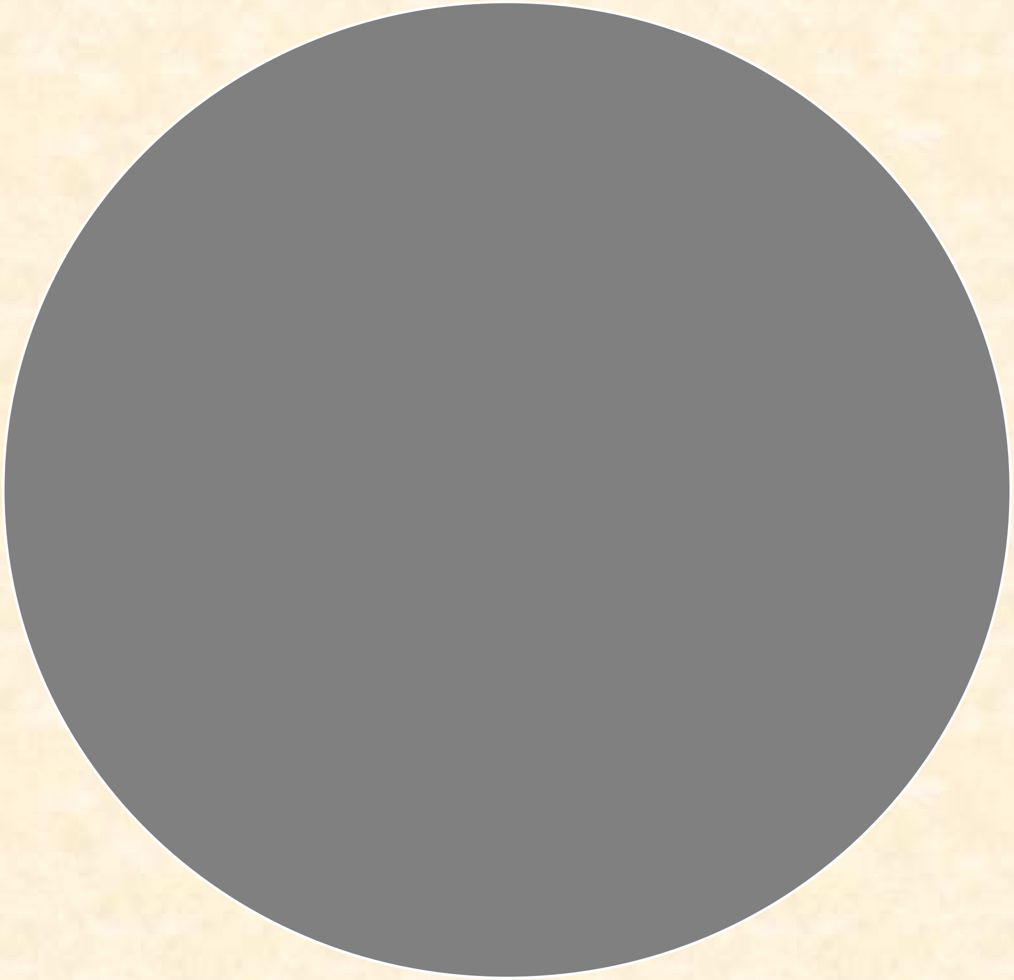
Neptune


Both Hemispheres of Neptune
ST ScI OPO · February 1995 · D. Crisp (JPL), WFPC2 Science Team, NASA
HST · WFPC2
2/3/95 zgl

Models now!

Giant planets: theoretical models



Mass

Radius

Luminosity

Atmospheric T-P profile

Atmospheric composition

Rotation rate, gravity field

Basic data

TABLE 1 Characteristics of the gravity fields and radii

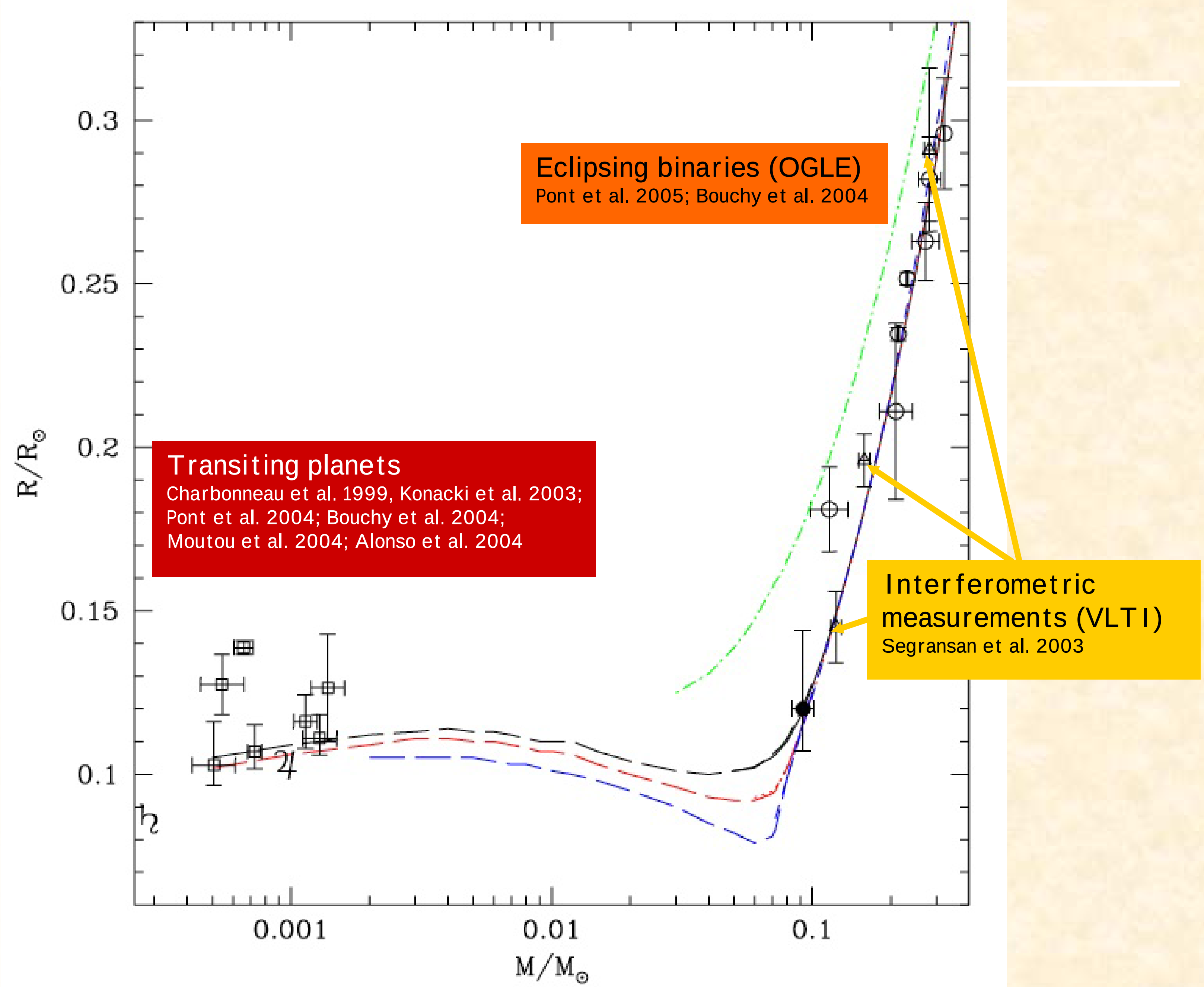
	Jupiter	Saturn	Uranus	Neptune
$M \times 10^{-29}$ [g]	18.986112(15)[a]	5.684640(30)[b]	0.8683205(34)[c]	1.0243542(31)[d]
$R_{eq} \times 10^{-9}$ [cm]	7.1492(4)[e]	6.0268(4)[f]	2.5559(4)[g]	2.4766(15)[g]
$R_{pol} \times 10^{-9}$ [cm]	6.6854(10)[e]	5.4364(10)[f]	2.4973(20)[g]	2.4342(30)[g]
$\bar{R} \times 10^{-9}$ [cm]	6.9894(6)[h]	5.8210(6)[h]	2.5364(10)[i]	2.4625(20)[i]
$\bar{\rho}$ [g cm^{-3}]	1.3275(4)	0.6880(2)	1.2704(15)	1.6377(40)
$J_2 \times 10^2$	1.4697(1)[a]	1.6332(10)[b]	0.35160(32)[c]	0.3539(10)[d]
$J_4 \times 10^4$	−5.84(5)[a]	−9.19(40)[b]	−0.354(41)[c]	−0.28(22)[d]
$J_6 \times 10^4$	0.31(20)[a]	1.04(50)[b]	...	...
$P_\omega \times 10^{-4}$ [s]	3.57297(41)[j]	3.83577(47)[j]	6.206(4)[k]	5.800(20)[l]
q	0.08923(5)	0.15491(10)	0.02951(5)	0.02609(23)
C/MR_{eq}^2	0.258	0.220	0.230	0.241

Basic data

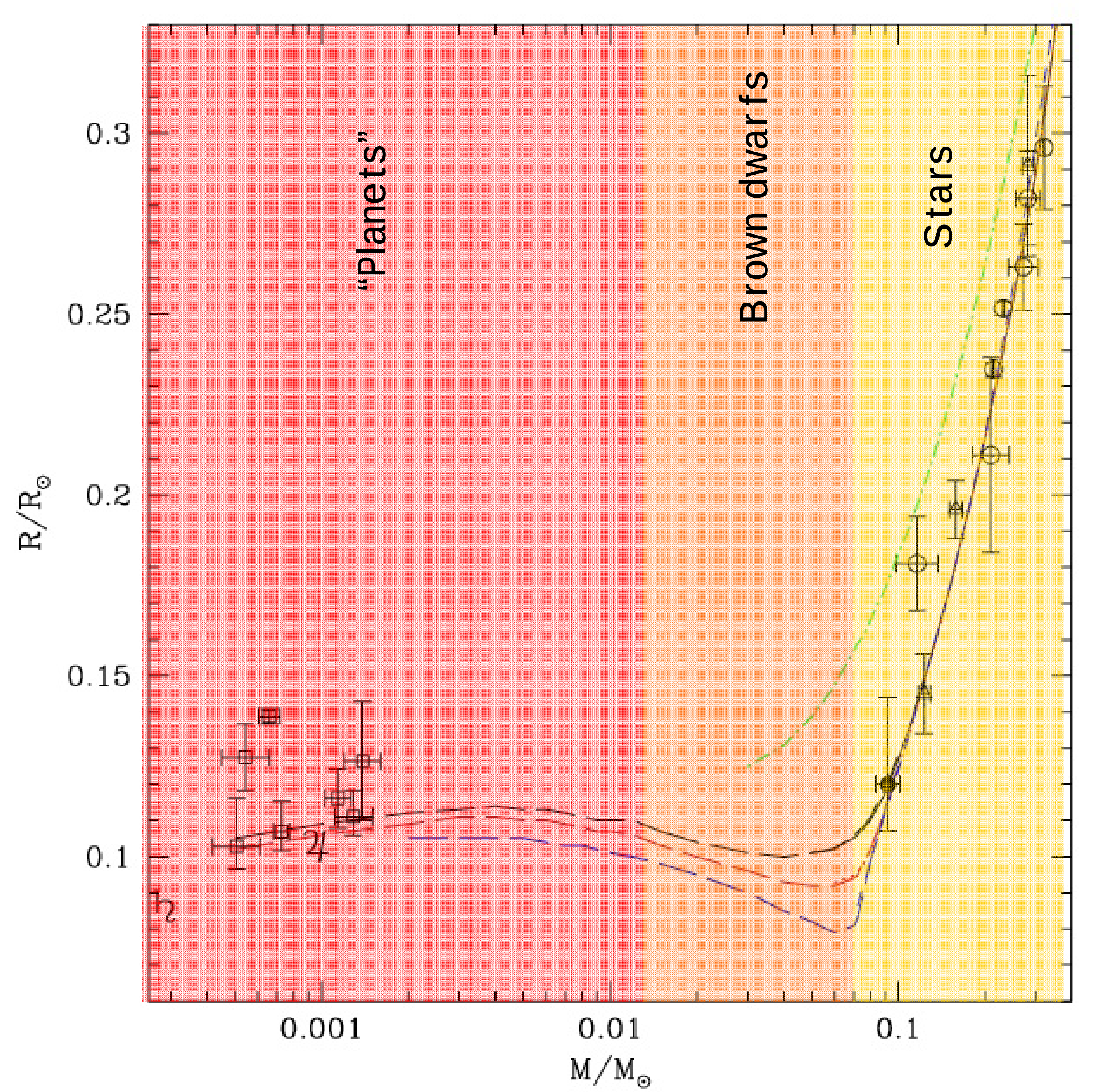
TABLE 2 Energy balance as determined from Voyager IRIS data[a]

	Jupiter	Saturn	Uranus	Neptune
Absorbed power [10^{23} erg · s^{-1}]	50.14(248)	11.14(50)	0.526(37)	0.204(19)
Emitted power [10^{23} erg · s^{-1}]	83.65(84)	19.77(32)	0.560(11)	0.534(29)
Intrinsic power [10^{23} erg · s^{-1}]	33.5(26)	8.63(60)	0.034(38)	0.330(35)
Intrinsic flux [erg · s^{-1} · cm^{-2}]	5440.(430)	2010.(140)	42.(47)	433.(46)
Bond albedo	0.343(32)	0.342(30)	0.300(49)	0.290(67)
Effective temperature [K]	124.4(3)	95.0(4)	59.1(3)	59.3(8)
1-bar temperature[b] [K]	165.(5)	135.(5)	76.(2)	72.(2)

[a] After Pearl & Conrath (1991).
[b] Lindal (1992).



Eclipsing binaries (OGLE)
Pont et al. 2005; Bouchy et al. 2004
Transiting planets
Charbonneau et al. 1999, Konacki et al. 2003;
Pont et al. 2004; Bouchy et al. 2004;
Moutou et al. 2004; Alonso et al. 2004
Interferometric measurements (VLTI)
Segransan et al. 2003
R/R⊙
M/M⊙
0.3
0.25
0.2
0.15
0.1
0.001
0.01
0.1


"Planets"
Brown dwarfs
Stars
0.3
0.25
0.2
0.15
0.1
R/R⊙
0.001
0.01
0.1
M/M⊙

The equations of (sub)stellar structure

$$\begin{aligned}
\frac{\partial P}{\partial r} &= -\rho g \\
\frac{\partial T}{\partial r} &= \frac{\partial P}{\partial r}\frac{T}{P}\nabla_T. \\
\frac{\partial m}{\partial r} &= 4\pi r^2 \rho. \\
\frac{\partial L}{\partial r} &= 4\pi r^2 \rho \left(\dot{\epsilon} - T\frac{\partial S}{\partial t}\right)
\end{aligned}$$

$$\left\{\begin{aligned}
\frac{\partial P}{\partial m} &= -\frac{Gm}{4\pi r^4} \\
\frac{\partial T}{\partial m} &= \left(\frac{\partial P}{\partial m}\right)\frac{T}{P}\nabla_T, \\
\frac{\partial r}{\partial m} &= \frac{1}{4\pi r^2 \rho}, \\
\frac{\partial L}{\partial m} &= \dot{\epsilon} - T\frac{\partial S}{\partial t},
\end{aligned}\right.$$

$$\rho = \rho(P, T, \{X_i\}); \qquad S = S(P, T, \{X_i\})$$

The equations of (sub)stellar structure

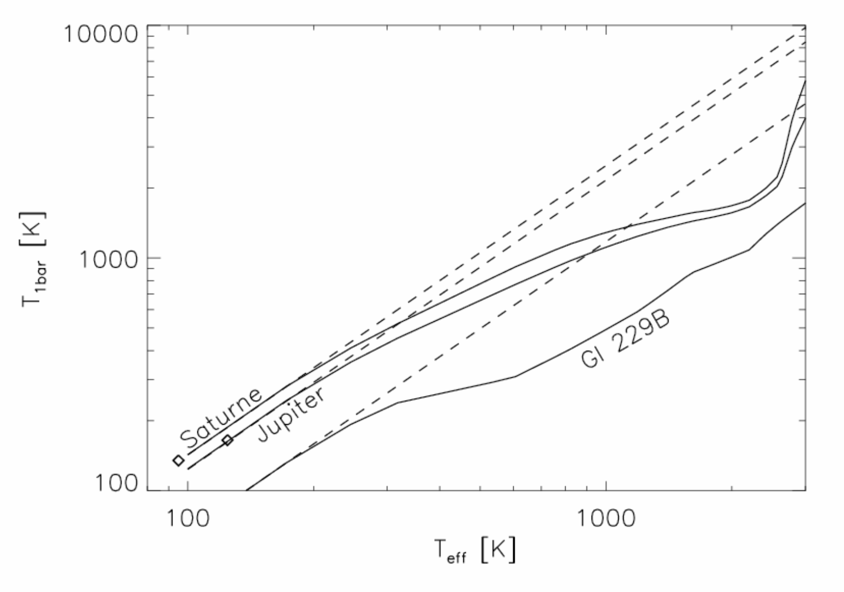
Boundary conditions

$$m = 0 \longrightarrow r = L = 0$$
$$m = M \longrightarrow P = P_{\text{phot}}(g, L)$$
$$T = T_{\text{phot}}(g, L)$$

Example: Eddington approximation

$$T = T_{\text{eff}}$$
$$P = \frac{2}{3}\frac{g}{\kappa}$$

Atmospheric model:



A note on the thermal equation

We wrote:

$$\frac{\partial T}{\partial m} = \frac{T}{P}\frac{\partial P}{\partial m}\nabla_T$$

$$\nabla_T \equiv \frac{d\ln T}{d\ln P}$$

In a radiative environment:

$$F = -K\frac{dT}{dr}$$

$$\frac{dT}{dr} = -\frac{1}{K}\frac{L}{4\pi r^2}$$

$$\nabla_{\rm rad} = \frac{3}{64\pi\sigma G}\frac{\kappa P L}{m T^4},$$

Schwarzschild's criterion for convection:

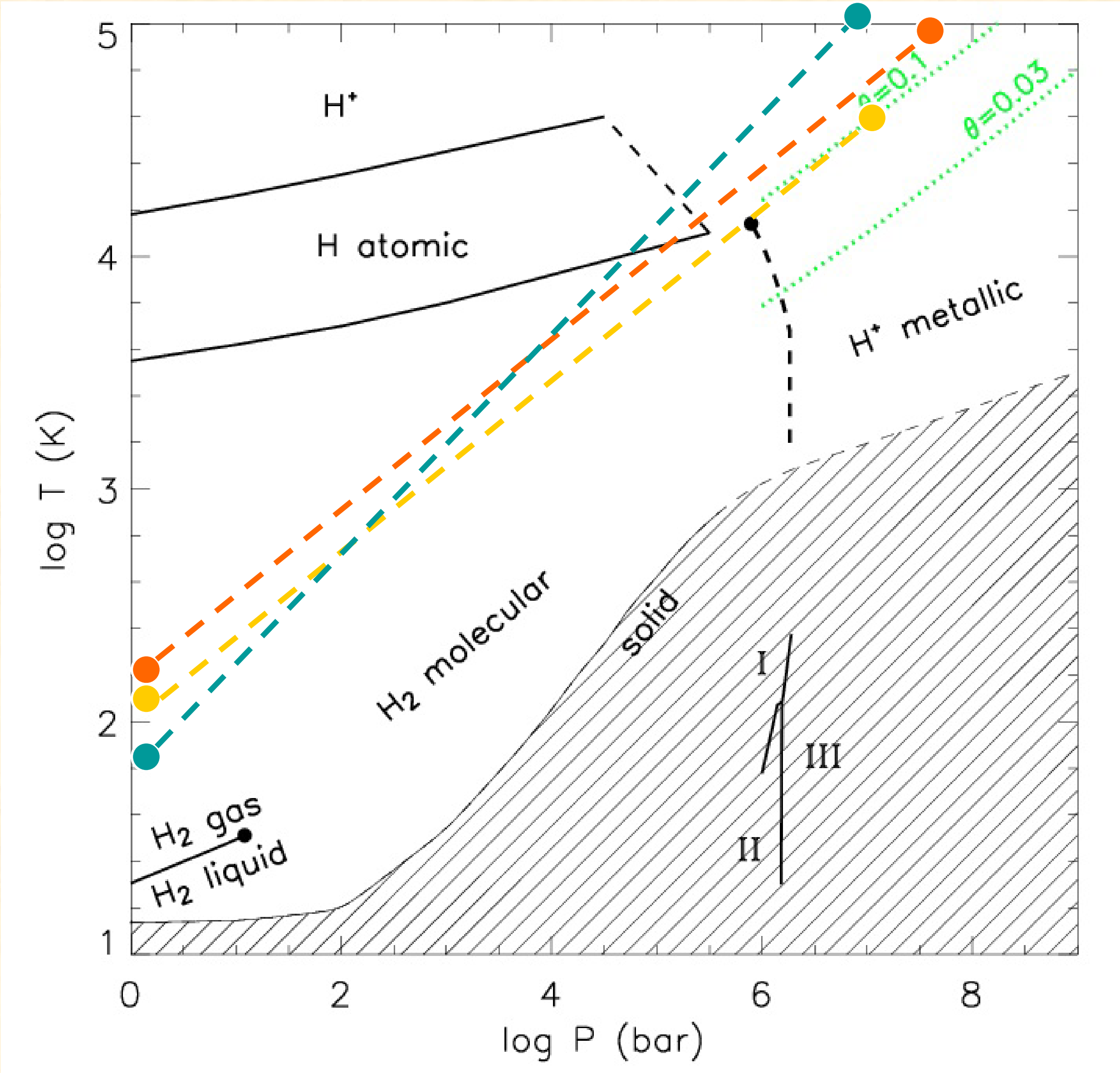
$$\nabla_{\rm rad} > \nabla_{\rm ad}$$

In a convective environment (MLT):

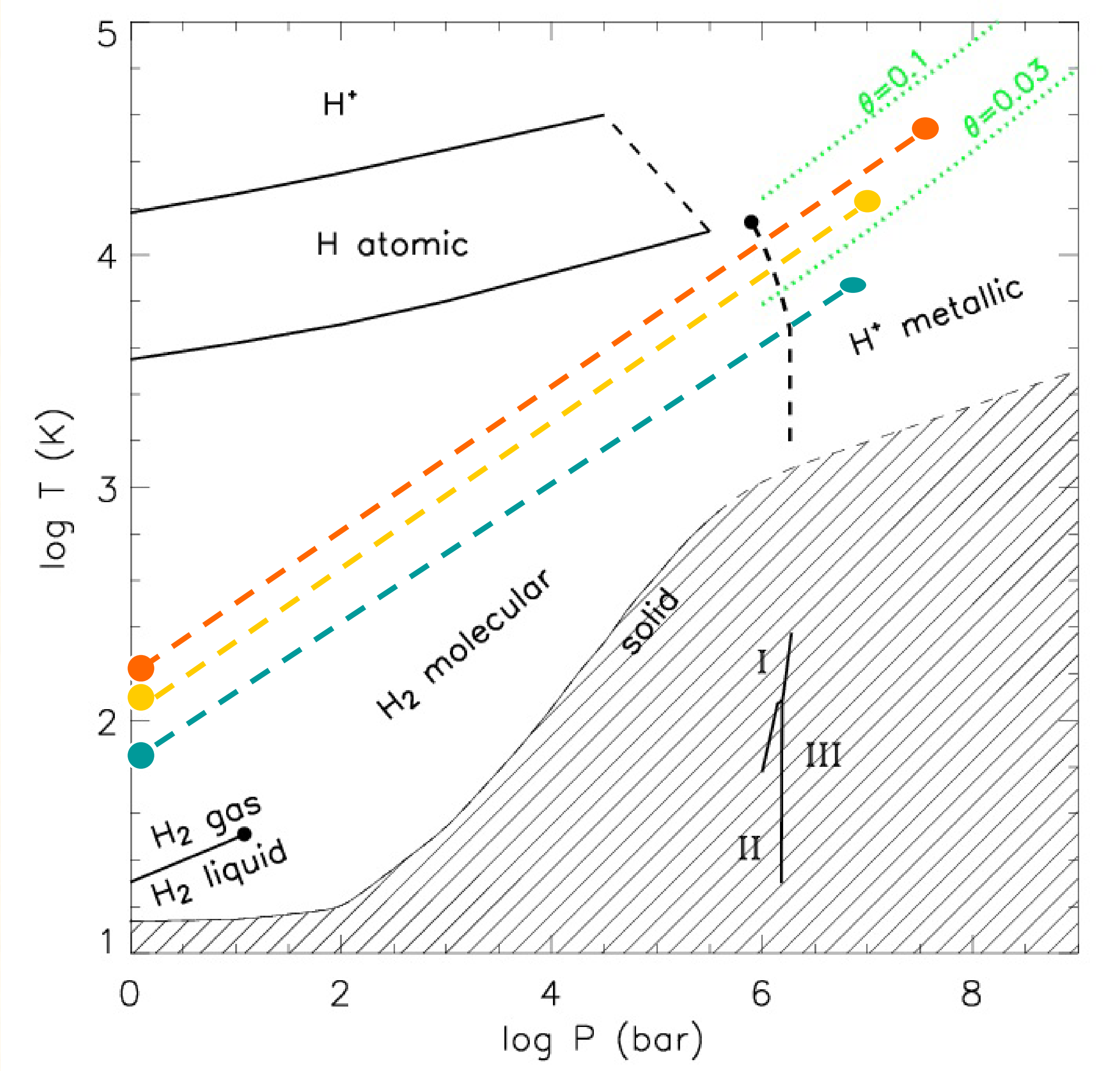
$$\nabla_T - \nabla_{\rm ad} \sim \left[\frac{4\sqrt{2}}{\alpha^2\delta^{1/2}}\frac{F_{\rm conv}}{c_P T(\rho P)^{1/2}}\right]^{2/3},$$

$$v \sim \left[\frac{\alpha\delta}{4}\frac{P}{\rho c_P T}\frac{F_{\rm conv}}{\rho}\right]^{1/3}.$$

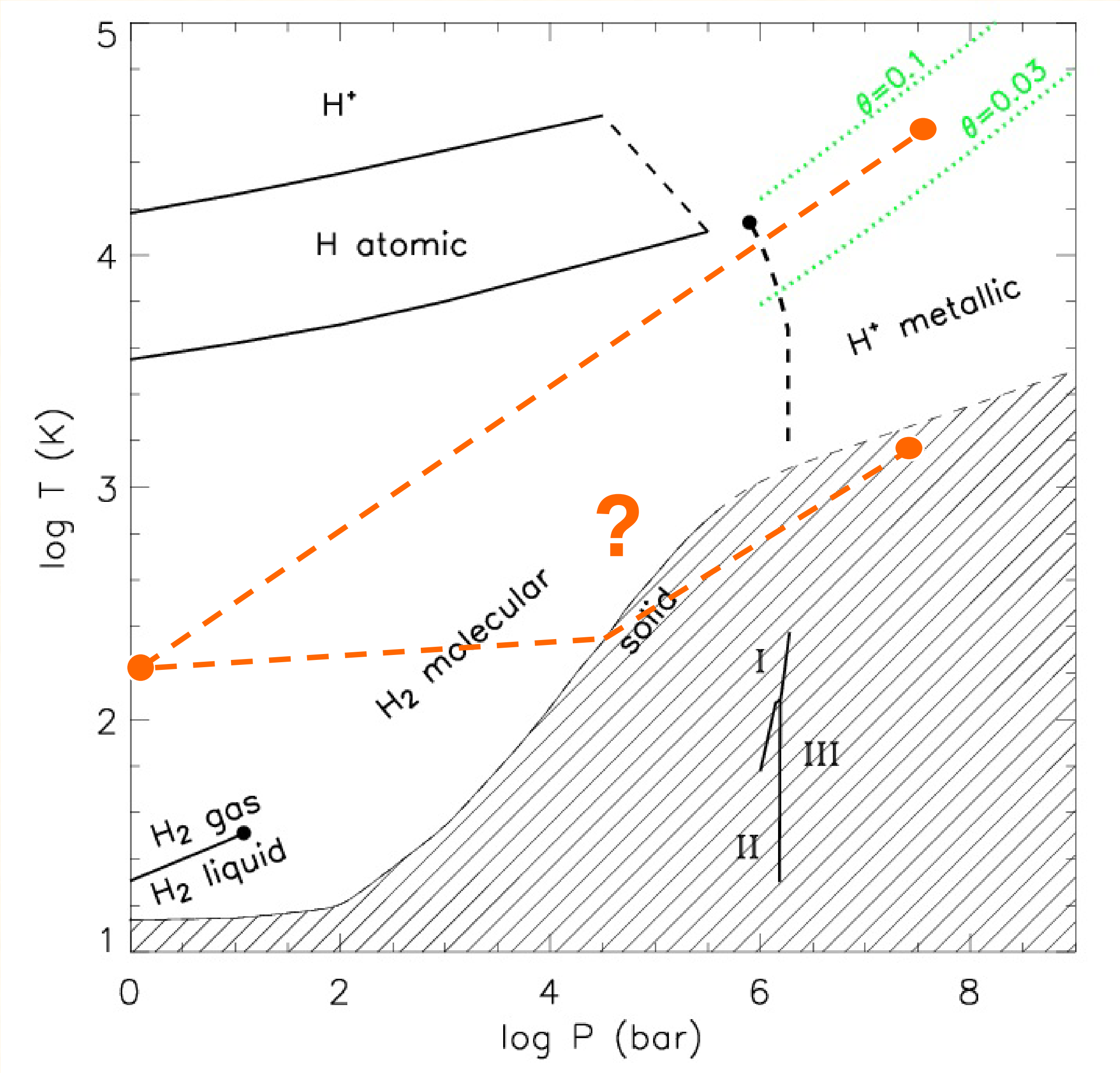
The hydrogen phase diagram



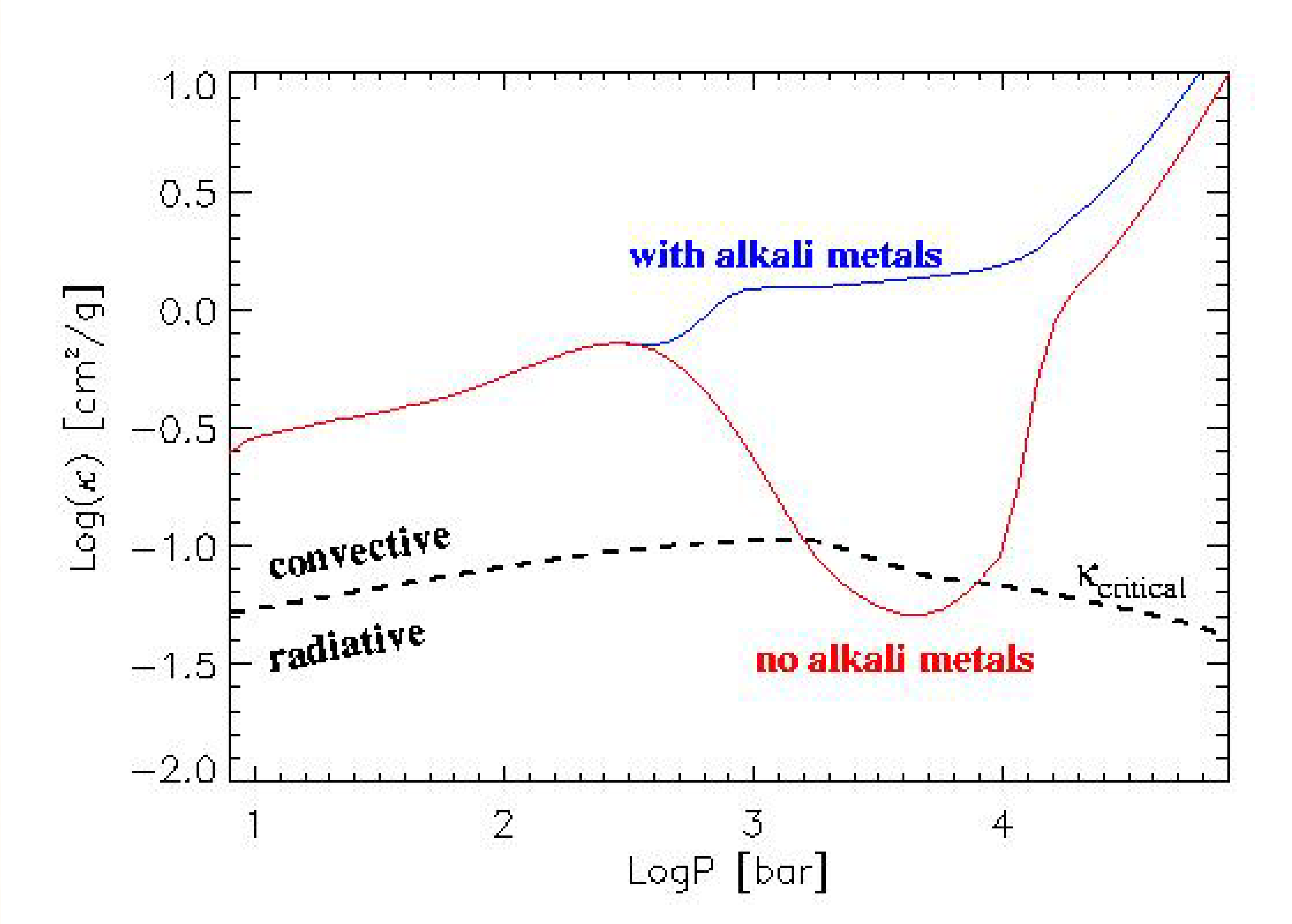
The hydrogen phase diagram



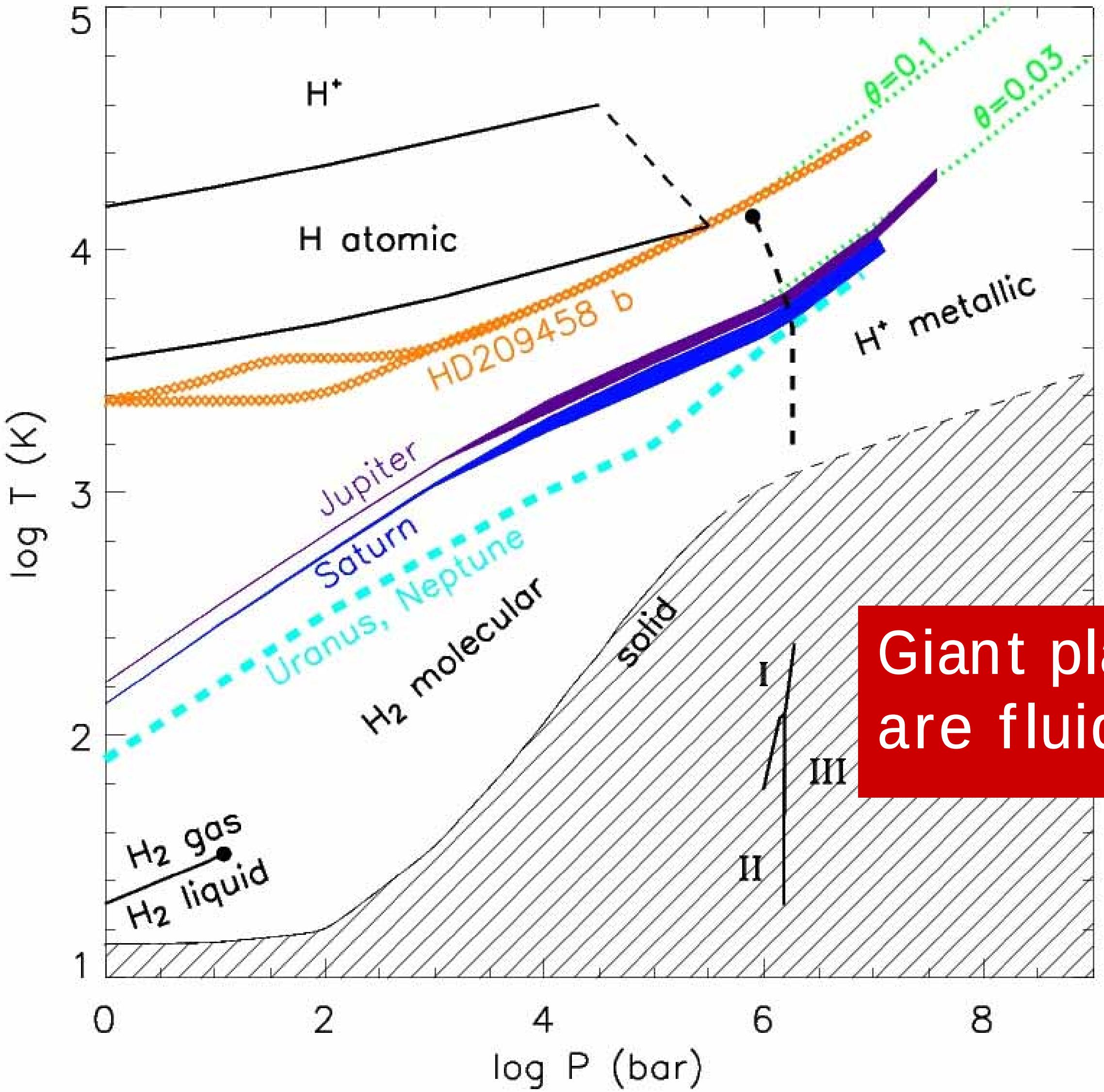
The hydrogen phase diagram



Opacities…


1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Log(κ) [cm²/g]
1
2
3
4
LogP [bar]
with alkali metals
no alkali metals
convective
radiative
κcritical

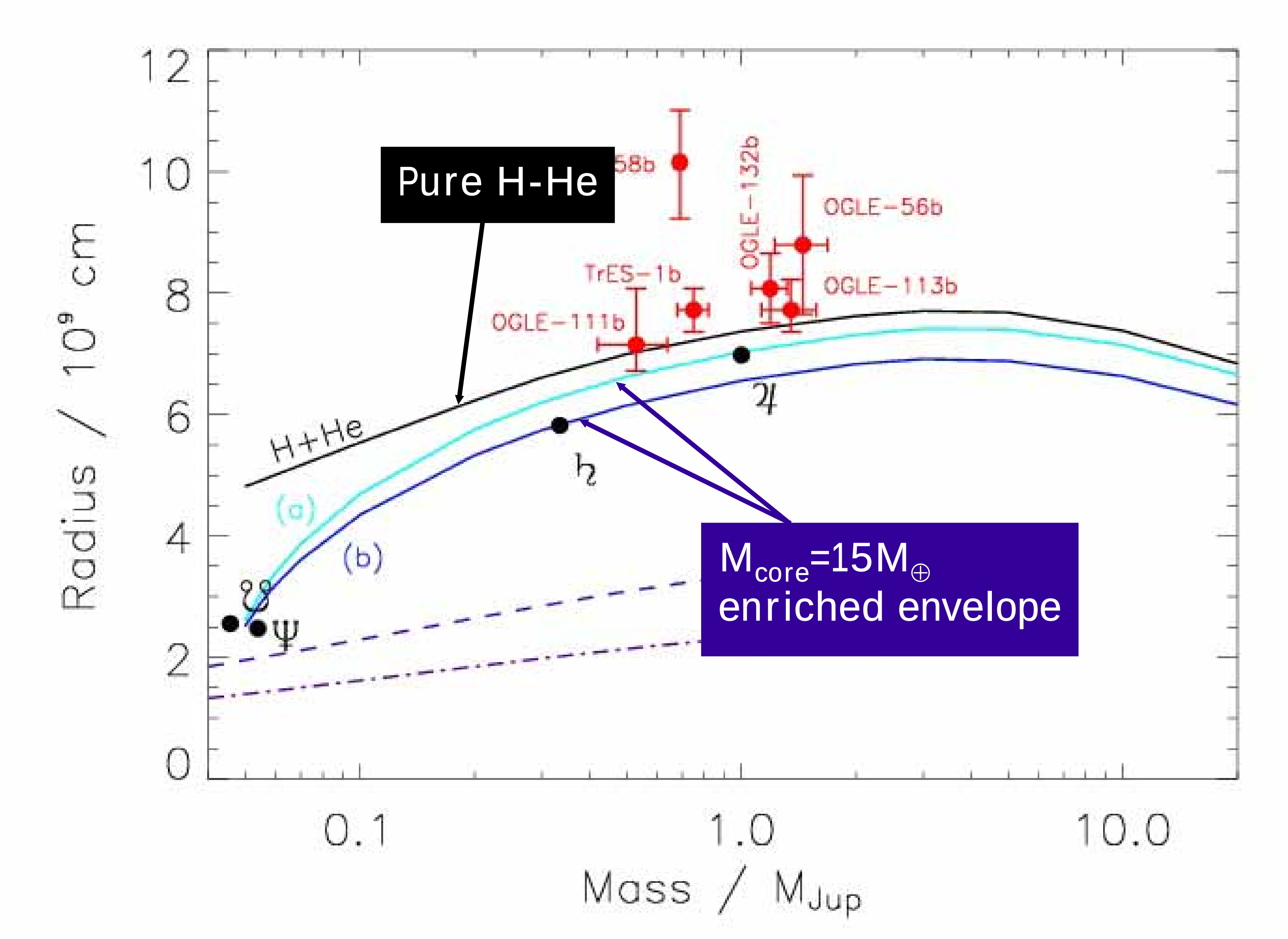
The hydrogen phase diagram



Giant planets are fluid

Let’s work on these mass-radius relations

Mass-radius relation (~isolated objects)



Polytropic solutions

$$P = K\rho^{\gamma} \equiv K\rho^{1+1/n}$$

$$\begin{cases} \dfrac{dP}{dr} = -\dfrac{d\Phi}{dr}\rho \\ \dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2\dfrac{d\Phi}{dr}\right) = 4\pi G\rho \end{cases}$$

$$z = Ar, \qquad A^2 = \frac{4\pi G}{(n+1)K}\rho_c^{\frac{n-1}{n}}$$

$$w = \frac{\Phi}{\Phi_c} = \frac{\rho}{\rho_c}$$

$$\frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{dw}{dz}\right) + w^n = 0$$

Polytropic solutions

$$P = K\rho^{\gamma} \equiv K\rho^{1+1/n}$$

$$z = Ar, \qquad A^2 = \frac{4\pi G}{(n+1)K}\rho_c^{\frac{n-1}{n}}$$

$$w = \frac{\Phi}{\Phi_c} = \frac{\rho}{\rho_c}$$

$$\left(\frac{r}{z}\right)^2 = \frac{1}{4\pi G}(n+1)K\rho_c^{\frac{1-n}{n}}$$

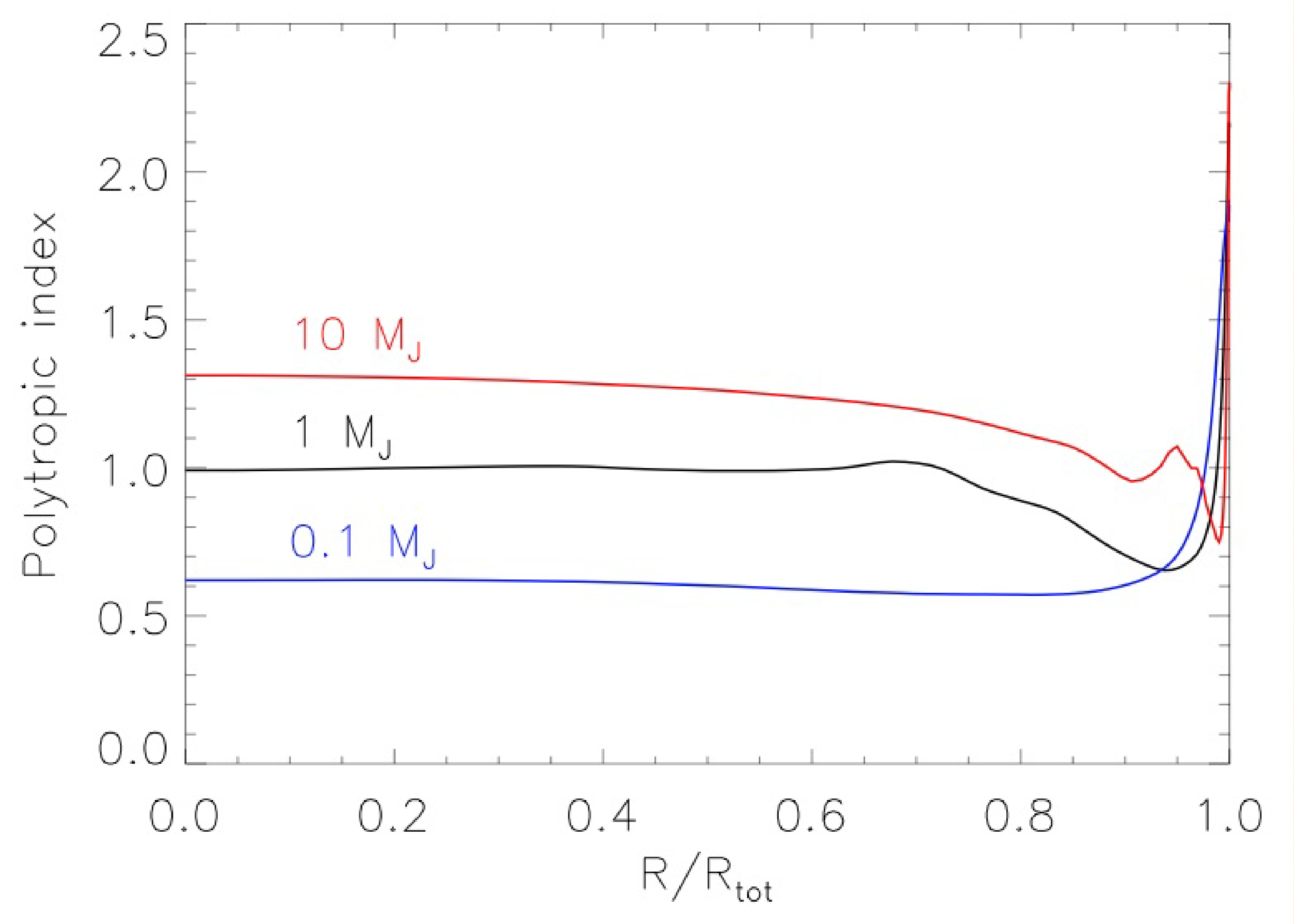
$$\begin{aligned} m(r) &= \int_0^r 4\pi r^2 \rho dr \\ &= 4\pi\rho_c \frac{r^3}{z^3}\int_0^z w^n z^2 dz \\ &= 4\pi\rho_c r^3 \left(-\frac{1}{z}\frac{dw}{dz}\right)_{z=z_n} \end{aligned}$$

$$M = 4\pi\rho_c R^3 \left(-\frac{1}{z}\frac{dw}{dz}\right)_{z=z_n},$$

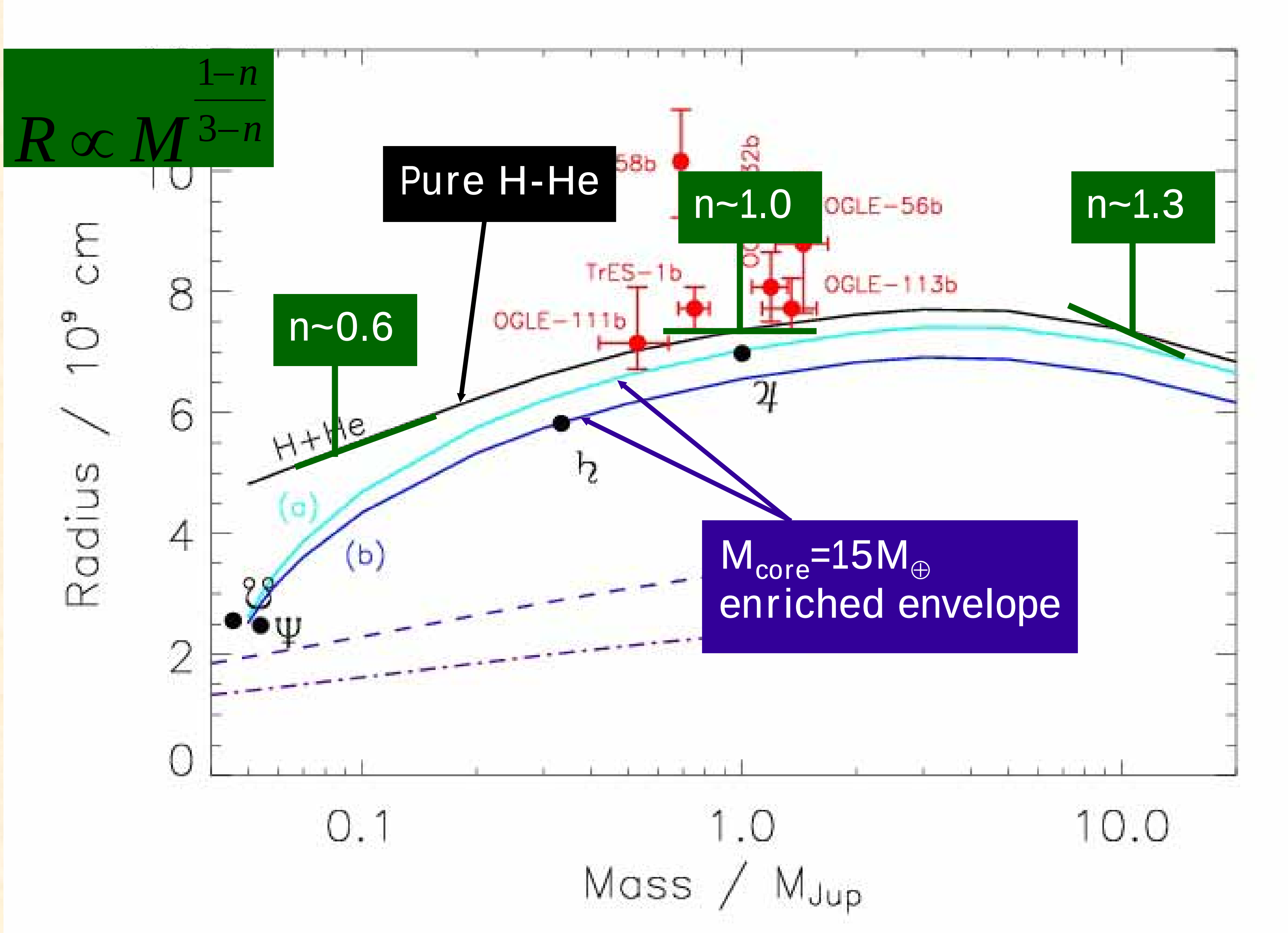
$$R = z_n \left[\frac{1}{4\pi G}(n+1)K\right]^{1/2} \rho_c^{\frac{1-n}{2n}}$$

$$R \propto K^{\frac{n}{3-n}} M^{\frac{1-n}{3-n}}$$

Polytropic index



Mass-radius relation (~isolated objects)

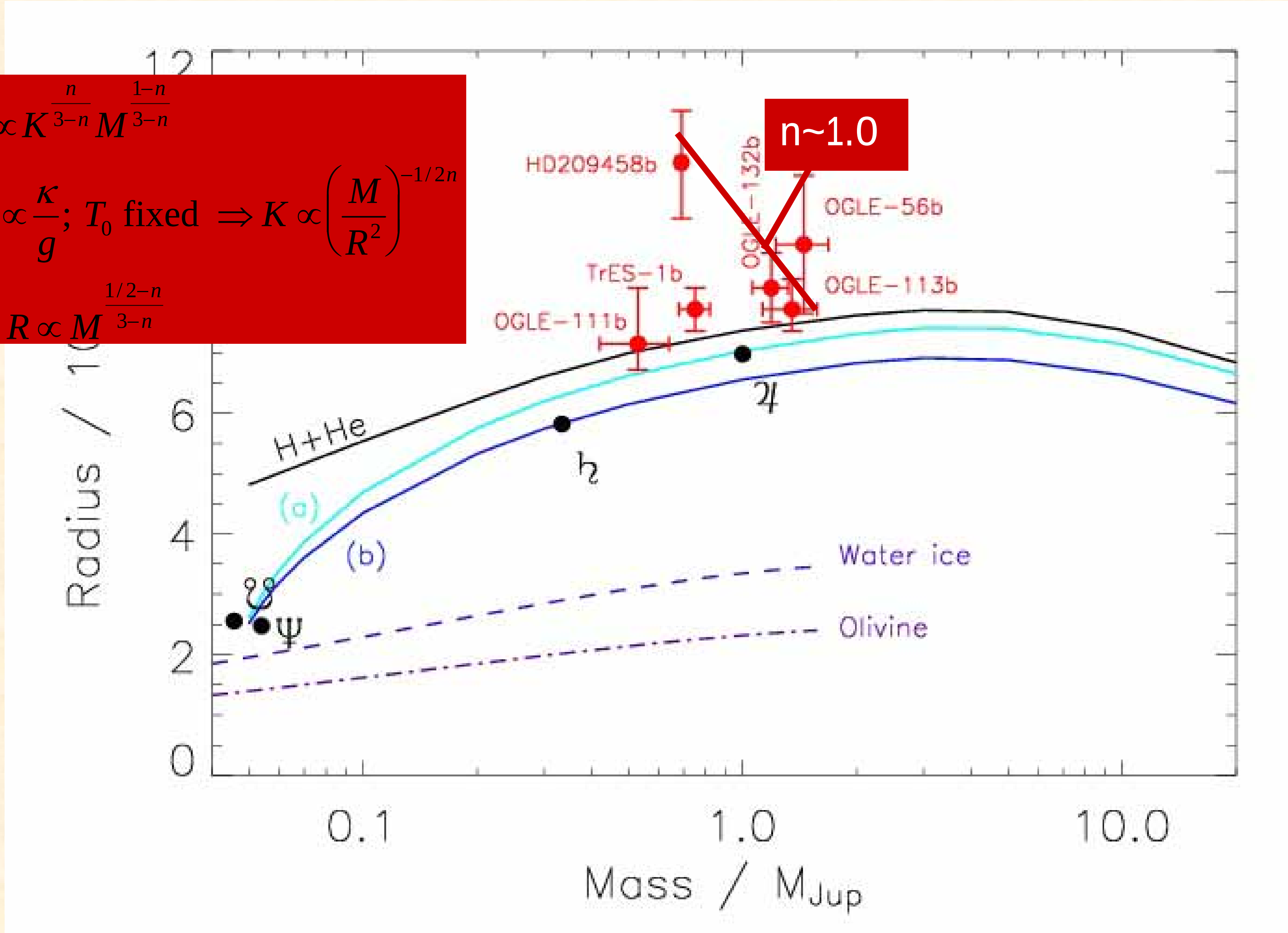


Mass-radius relation (irradiated objects)

$$R \propto K^{\frac{n}{3-n}} M^{\frac{1-n}{3-n}}$$

$$P_0 \propto \frac{\kappa}{g};\ T_0 \text{ fixed } \Rightarrow K \propto \left(\frac{M}{R^2}\right)^{-1/2n}$$

$$\Rightarrow R \propto M^{\frac{1/2-n}{3-n}}$$



How do the giant planets evolve?

The cooling of giant planets

The virial theorem:

$$\xi \equiv 3P/u\rho$$

$$\xi E_{\rm i} + E_{\rm g} = 0,$$

$$W = E_{\rm i} + E_{\rm g}$$

$$\frac{dW}{dt} + L = 0$$

$$L = (\xi - 1)\frac{dE_{\rm i}}{dt} = -\frac{\xi - 1}{\xi}\frac{dE_{\rm g}}{dt}.$$

monoatomic perfect gas => ξ=2

degenerate electron gas => ξ=2

For giant planets: $E_i \approx E_{el} + E_{ion} \approx E_{el}$ (θ is large)

The cooling of giant planets

For giant planets: $E_i \approx E_{el} + E_{ion}$ and $E_{el} \gg E_{ion}$ (θ is large)

$$E_{\rm el} \propto \rho^{2/3}$$

$$E_{\rm g} \propto 1/R \propto \rho^{1/3}$$

$$\dot{E}_{\rm el} \approx 2(E_{\rm e}/E_{\rm g})\dot{E}_{\rm g}$$

degenerate electron gas => $\xi=2$

$$L = (\xi - 1)\frac{dE_{\rm i}}{dt} = -\frac{\xi - 1}{\xi}\frac{dE_{\rm g}}{dt}.$$

$$\dot{E}_{\rm e} \approx -\dot{E}_{\rm g} \approx 2L$$

$$L \approx -\dot{E}_{\rm ion} \propto -\dot{T}$$

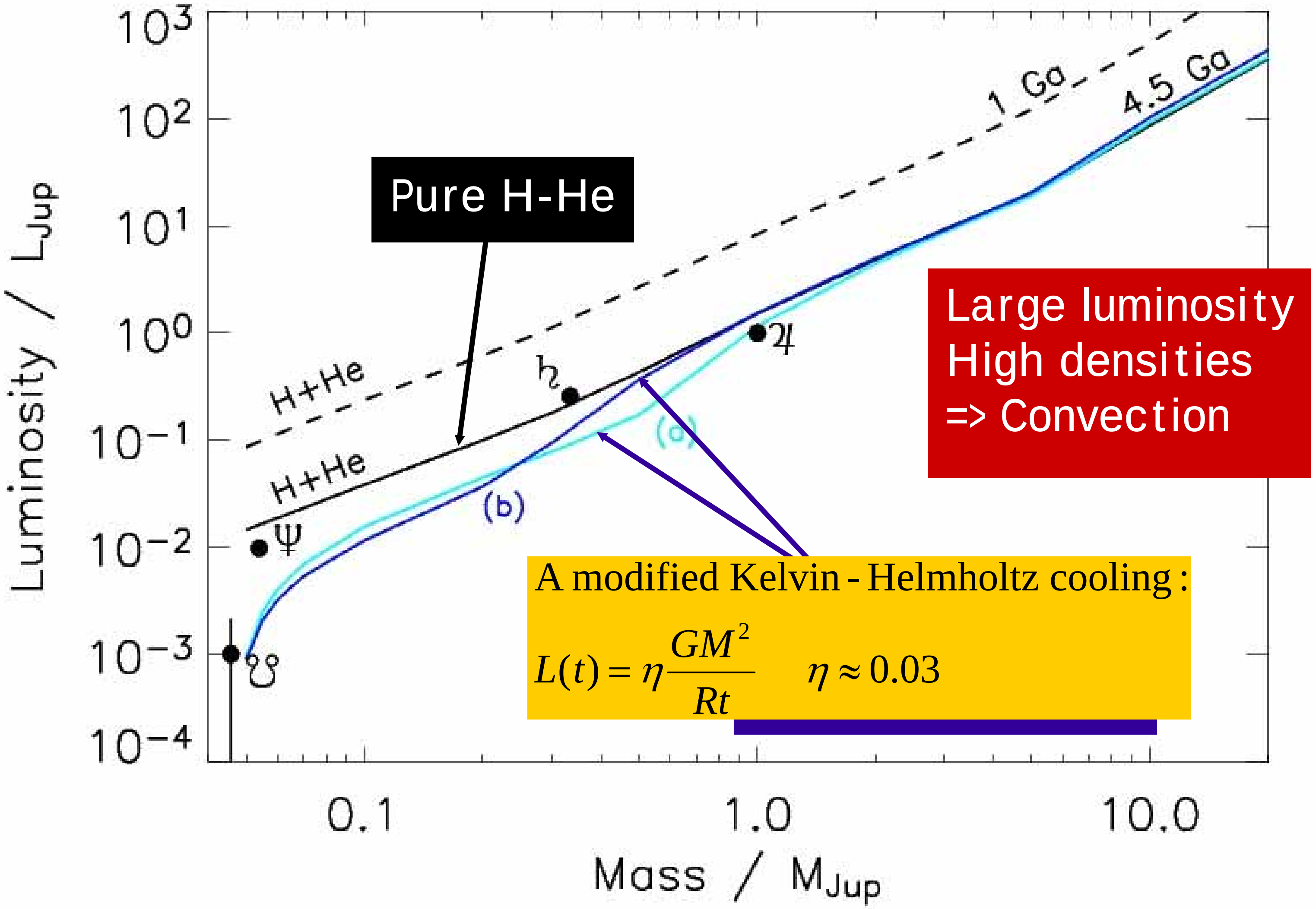
A modified Kelvin-Helmoltz contraction

$$L \approx \eta \frac{GM^2}{R\tau}$$

At the beginning of contraction (perfect gas) => $\eta \approx 1/2$:
a significant fraction of the gravitational energy is radiated away; The remaining fraction heats up the interior

When degeneracy sets in => $\eta \approx \theta \approx 0.03$
The gravitational energy is almost entirely used up in the (non-thermal) increase of the electronic pressure.
The luminosity is due to the *cooling* of the ions.

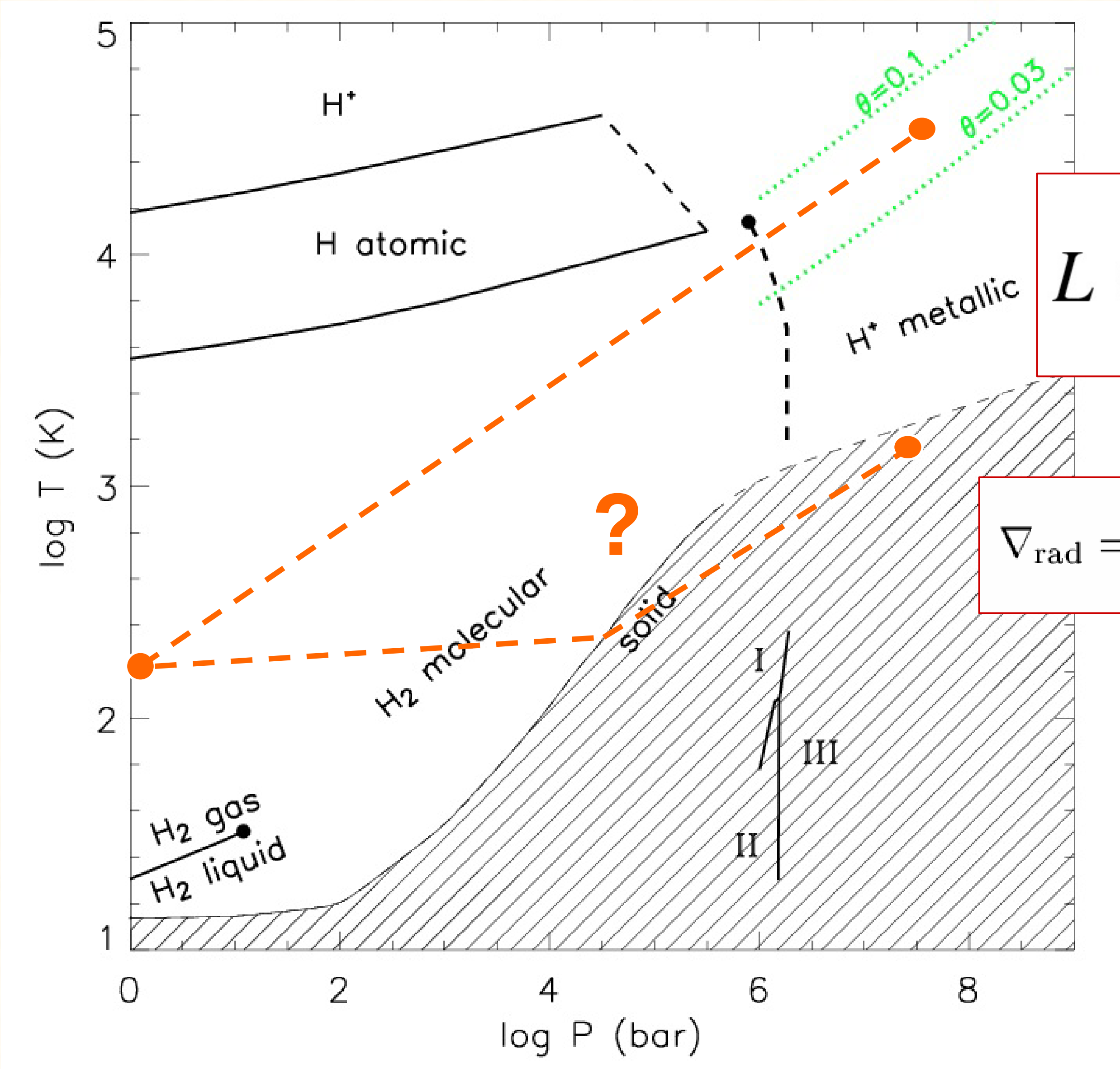
Mass-luminosity relation



A modified Kelvin-Helmholtz cooling:

$$L(t) = \eta \frac{GM^2}{Rt} \qquad \eta \approx 0.03$$

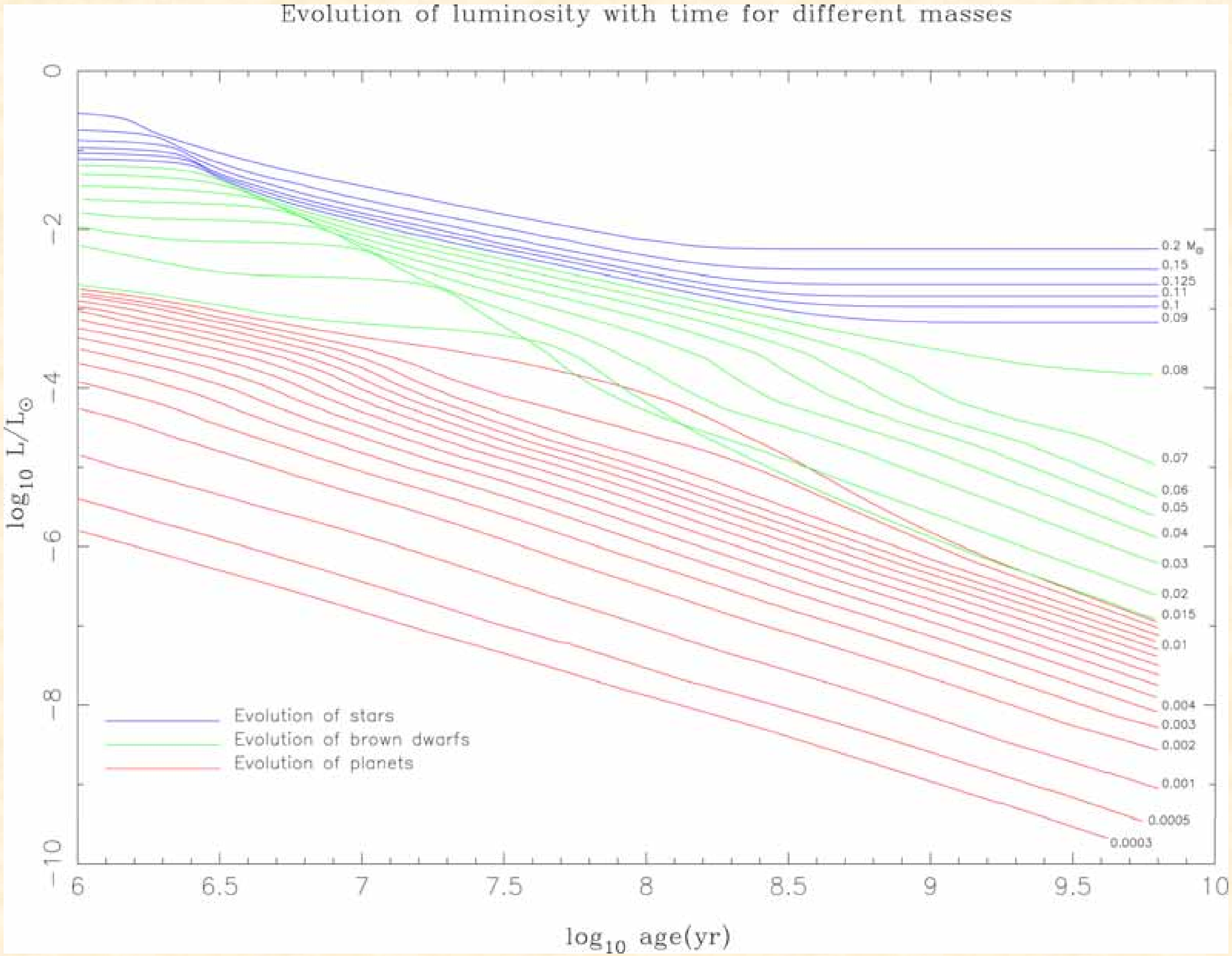
Problem: how long before Jupiter solidifies?



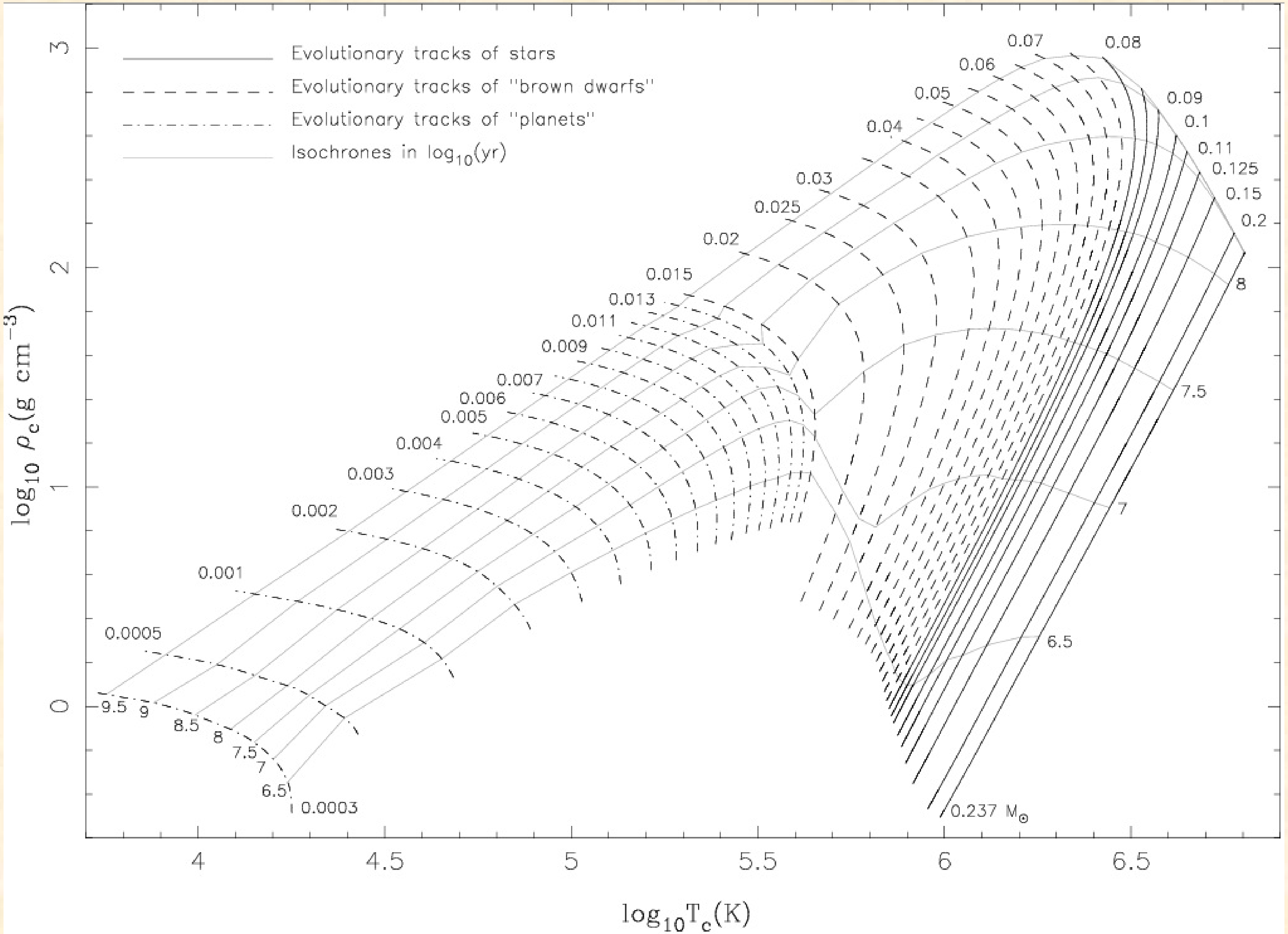
$$L \approx \eta \frac{GM^2}{R\tau}$$

$$\nabla_{\rm rad} = \frac{3}{64\pi\sigma G}\frac{\kappa P L}{m T^4},$$

Evolution tracks



Evolution tracks



Is neglecting the stellar irradiation ok?

Irradiated planets

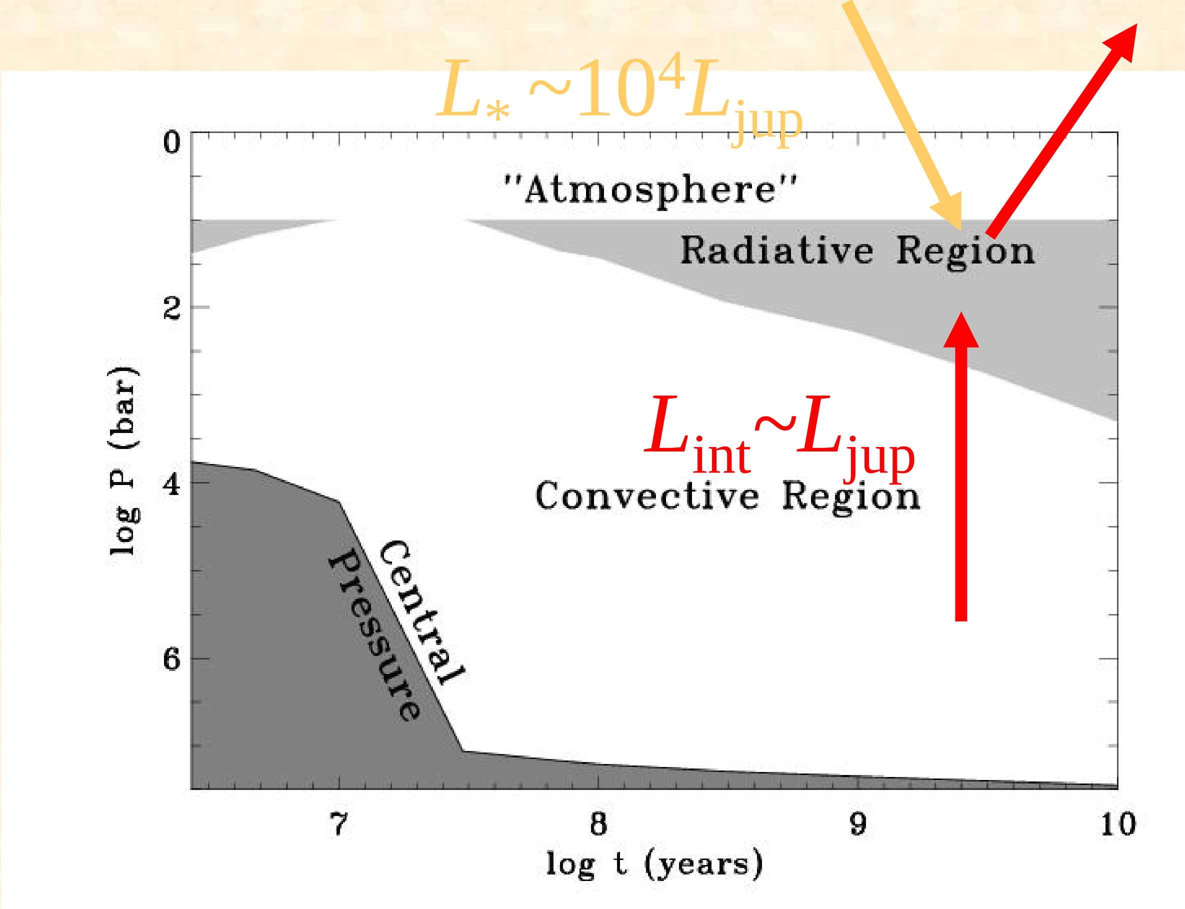
QuickTime™ et un
décompresseur GIF
sont requis pour visionner cette image.

- Proximity to the star -> significant contribution due to the stellar irradiation
- The atmospheric temperature is close to the « equilibrium » temperature:

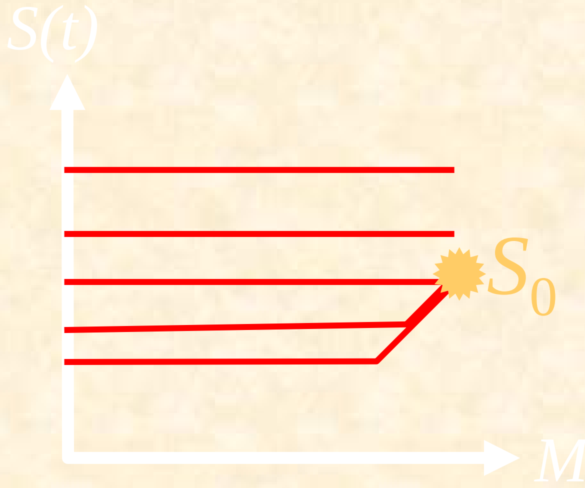
$$T_{eq} = f\, T_* \sqrt{\frac{R_*}{2a}}$$

- Teq=1400 to 2000K (HD209458b to OGLE planets)
- T1bar=2000 to 3000K (rough estimate)

Importance of stellar irradiation

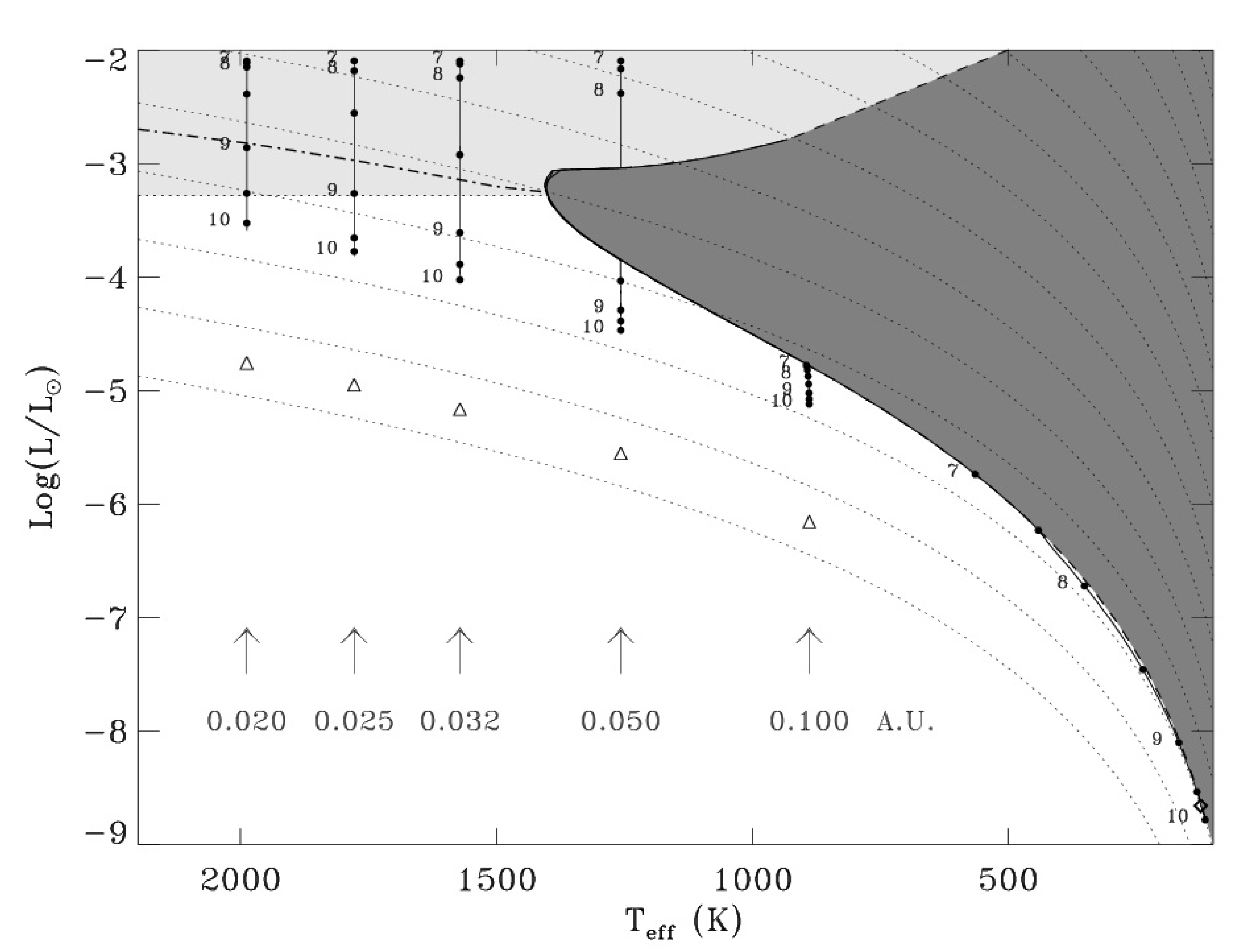


$T_0(L_{int} \sim 0) = cte$
$P_0(L_{int} \sim 0) = cte$



The evolution of Pegasi planets can be approximated by using a simplified boundary condition $T_{1bar} = T_{eq}$

An HR diagram for giant planets



Application to real data!

Discovered transiting planets

Table 3: Systems with transiting Pegasi planets discovered so far

	Age [Ga]	[Fe/H]	a [AU]	$T^{\star}_{eq}$ [K]	M_p/M_J	$R_p/10^{10}$ cm
HD209458[a]	$4-7$	0.00(2)	0.0462(20)	1460(120)	0.69(2)	1.02(9)
OGLE-56[b]	$2-4$	0.0(3)	0.0225(4)	1990(140)	1.45(23)	0.88(11)
OGLE-113[c]	?	0.14(14)	0.0228(6)	1330(80)	0.765(25)	$0.77(^{+5}_{-4})$
OGLE-132[d]	$0-1.4$	0.43(18)	0.0307(5)	2110(150)	1.19(13)	0.81(6)
OGLE-111[e]	?	0.12(28)	0.0470(10)	1040(160)	0.53(11)	$0.71(^{+9}_{-4})$
TrES-1[f]	?	0.00(4)	0.0393(11)	1180(140)	0.75(7)	0.77(4)

* Equilibrium temperature calculated on the basis of a zero planetary albedo

[a]Cody & Sasselov (2002), Brown et al. (2001)

[b]Torres et al. (2004), Sasselov (2003), Konacki et al. (2003)

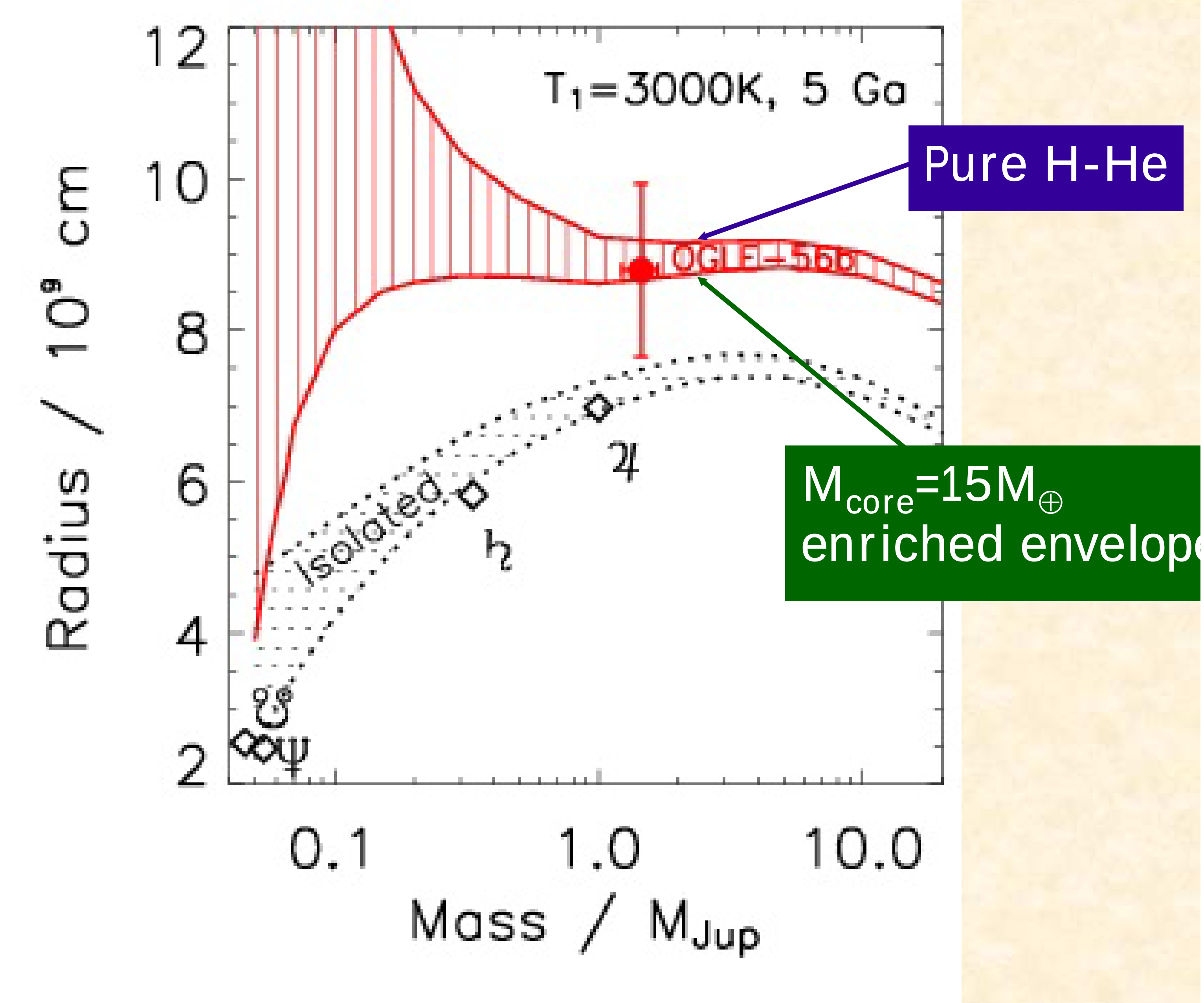
[c]Bouchy et al. (2004), Konacki et al. (2004)

[d]Moutou et al. (2004)

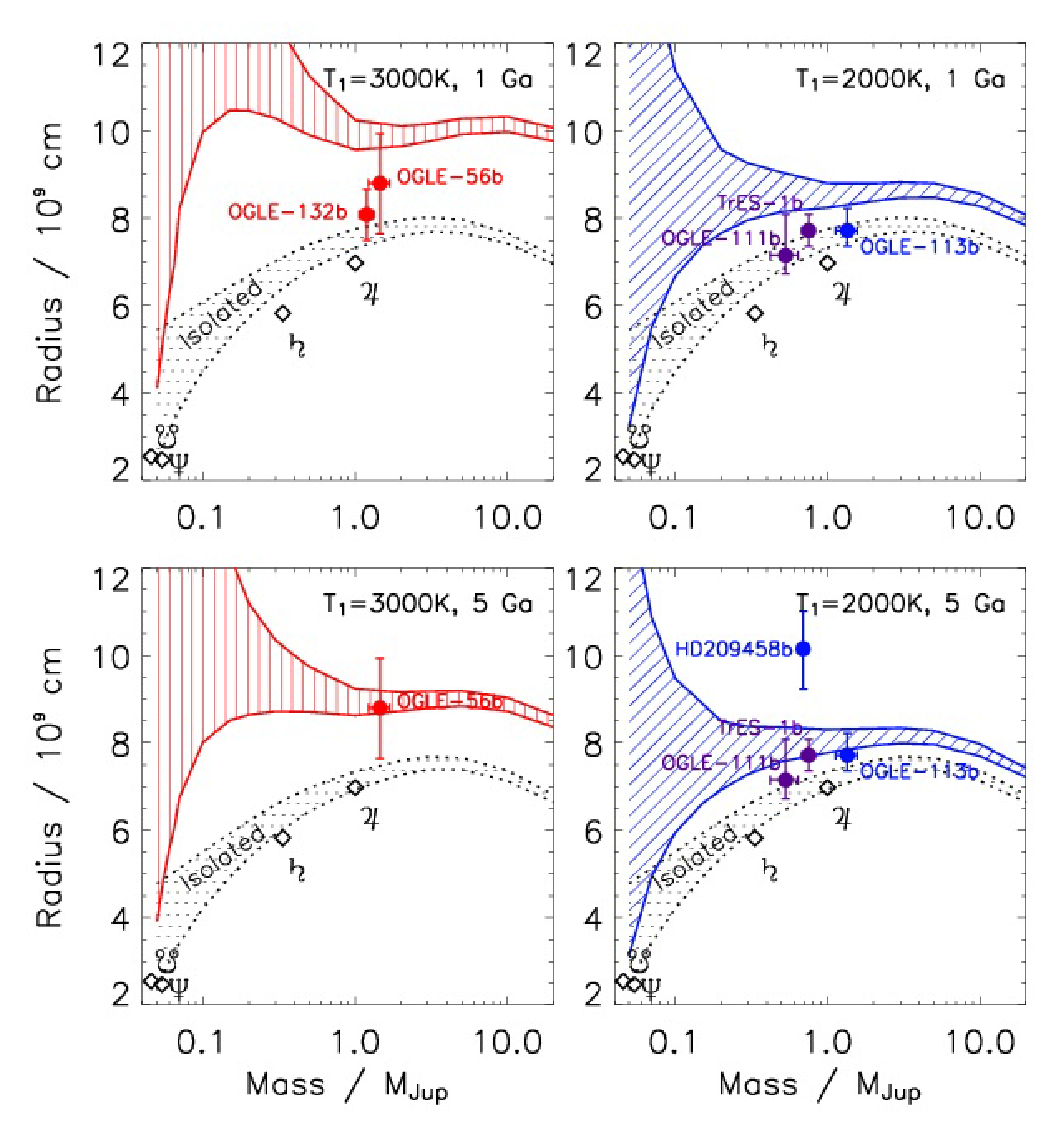
[e]Pont et al. (2004)

[f]Laughlin et al. (2004), Sozzetti et al. (2004), Alonso et al. (2004)

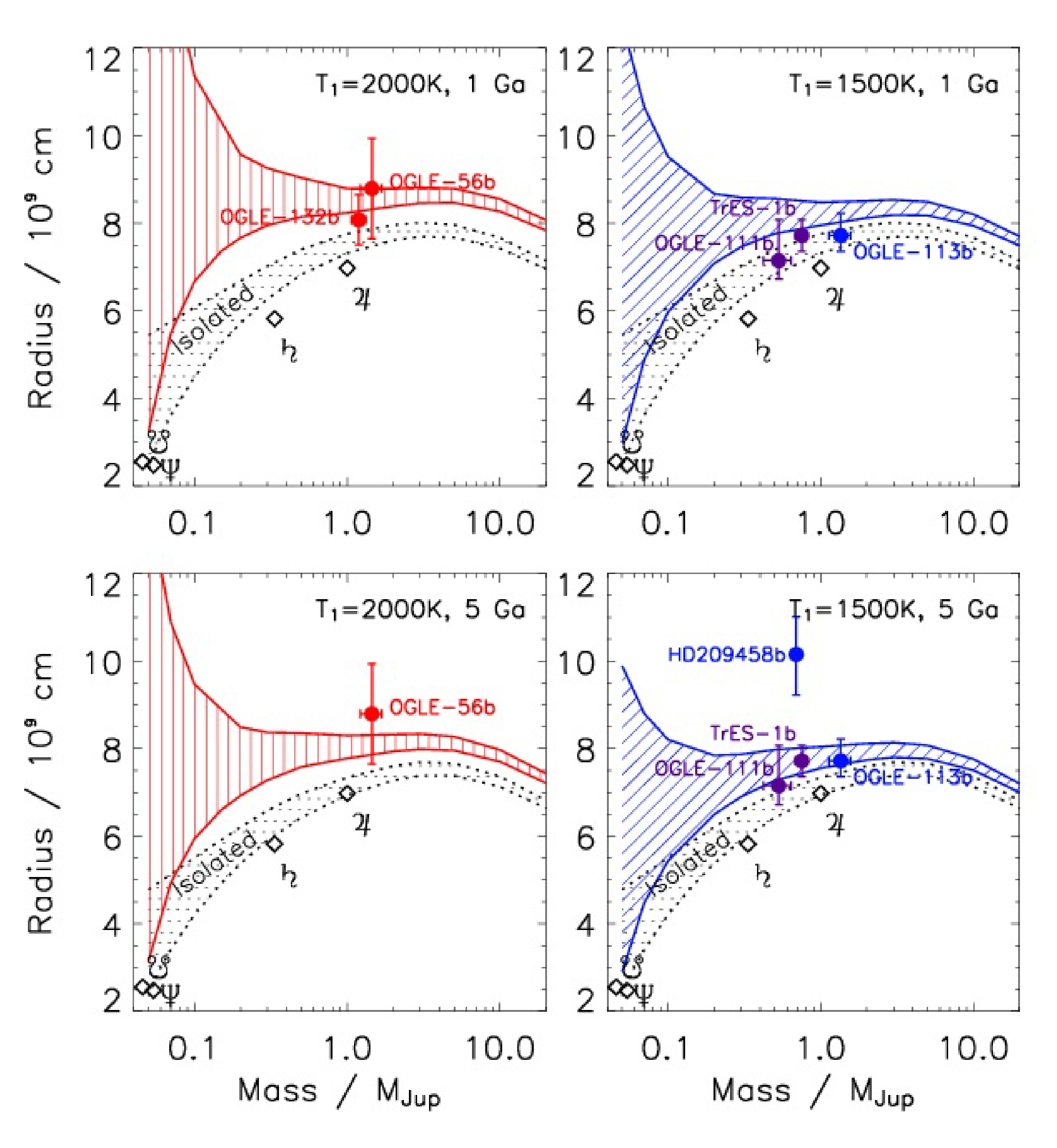
Mass-radius relation (irradiated objects)



Mass-radius relation (irradiated objects)



Mass-radius relation (irradiated objects)

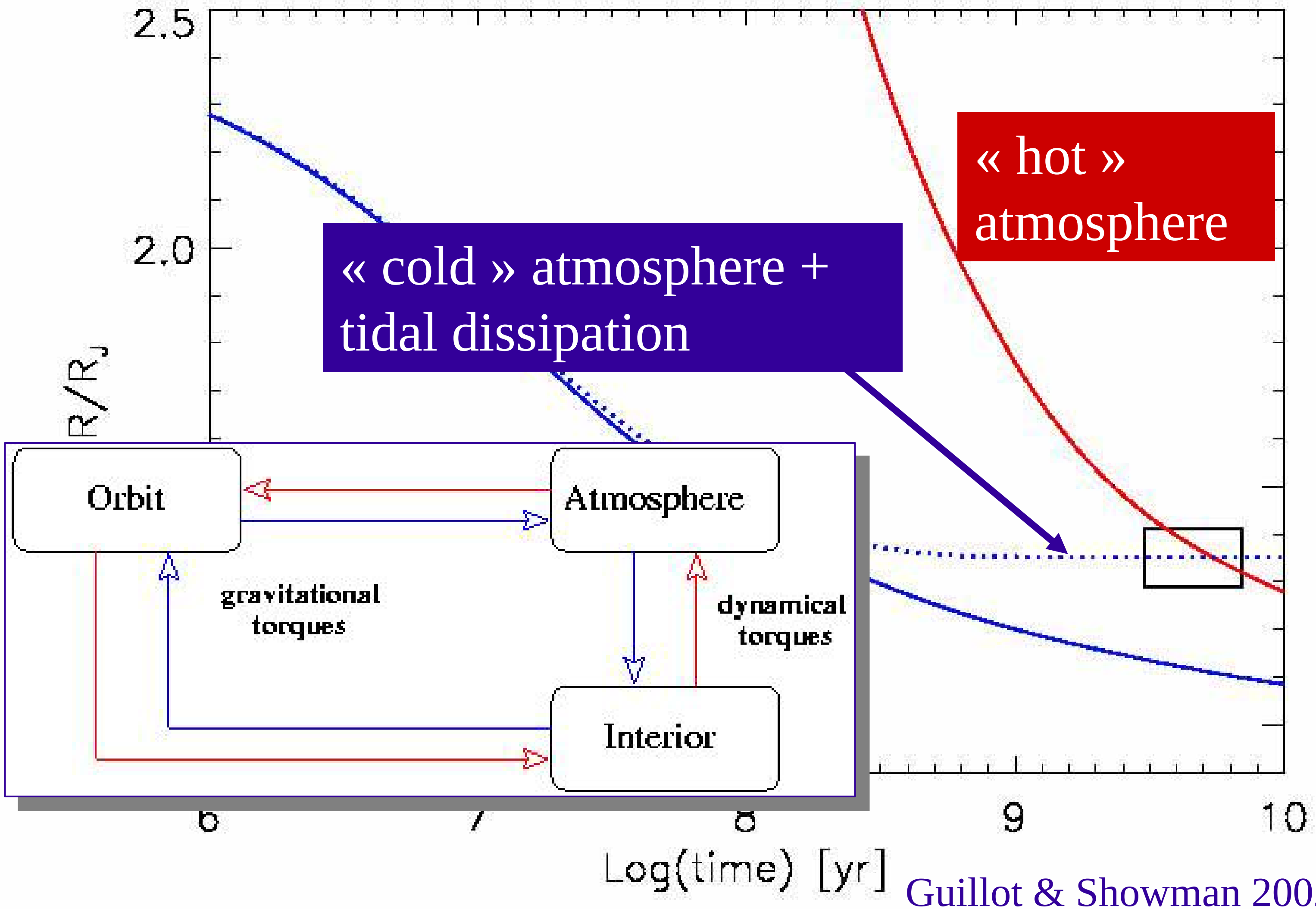


Tides

The significance of tides

- Based on HD209458b parameters:
- Gravitational energy required to change the radius by 10%
  - 2×10^{42} erg
- Energy available from the circularization of the orbit
  - 4×10^{42} erg (for e=0.1)
- Energy available from the synchronization of the planet's spin
  - 0.2×10^{42} erg
- => Tides may play a role but require either
  - A forced eccentricity (Bodenheimer et al.)
  - Continuous generation of K.E. in the interior (Showman & Guillot)

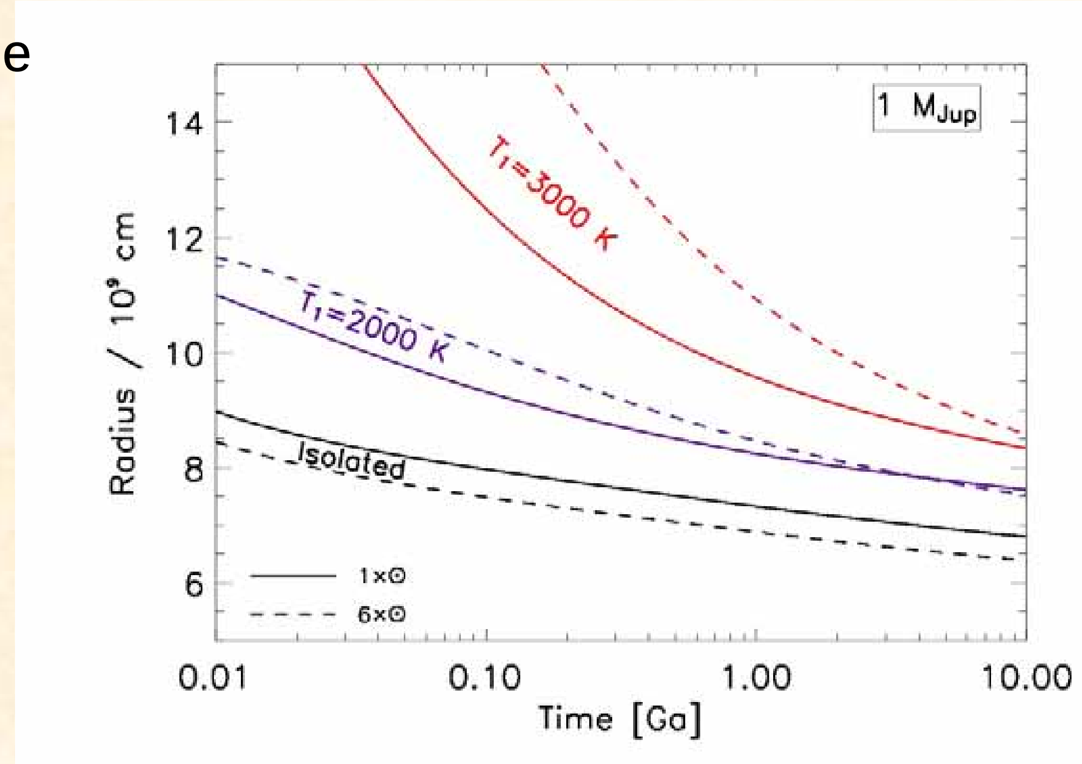
The contraction of HD209458b



Guillot & Showman 2002

Heavy elements & the evolution of giant planets

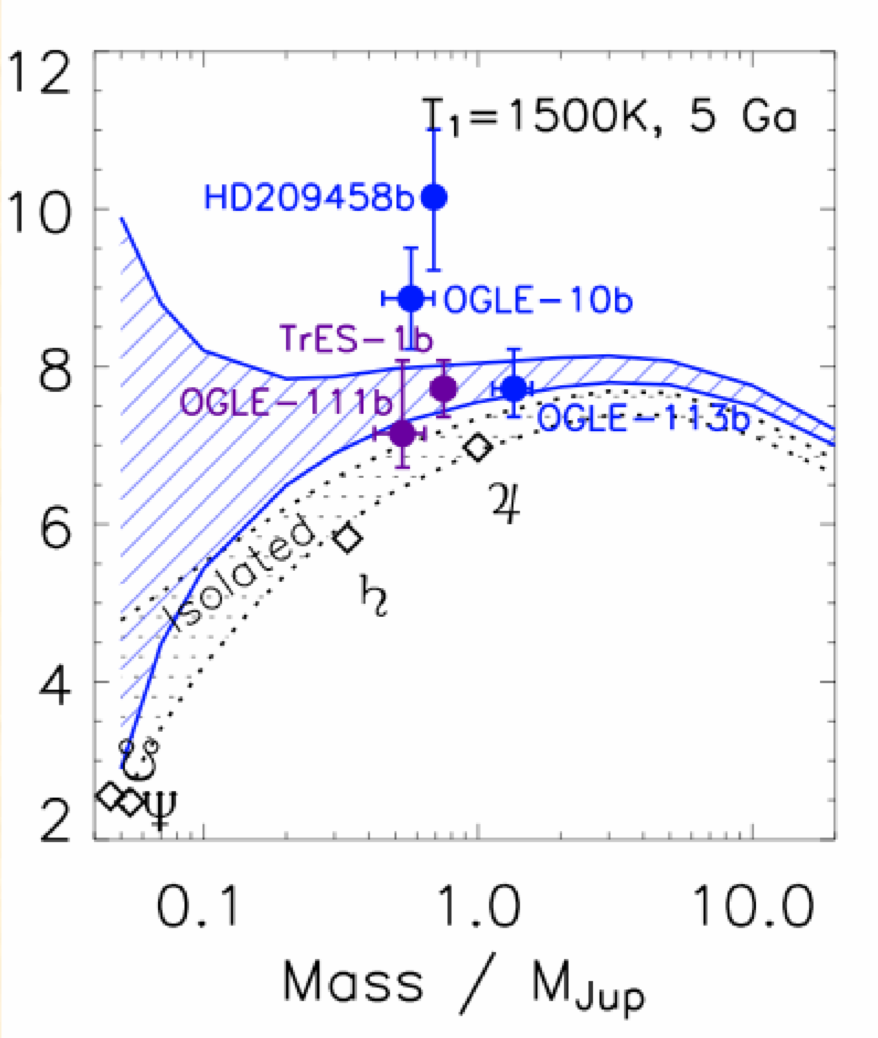
- Heavy elements in the core
  - ⇒ Smaller radii
  - ⇒ ΔR/R proportional to Mc/Mtot
- Heavy elements in the envelope
  - ⇒ Larger mean molecular weight
  - ⇒ Higher opacities (slower cooling)



Measured radii: Indicate that some planets *may* have cores ~20 Earth masses.

The problem of the "inflated" planets

- HD209458b and OGLE-10b have radii that are too large to be reproduced by standard evolution models
  - ⇒ A forced eccentricity (Bodenheimer et al. 2001, 2003)
  - ⇒ Kinetic energy generation and dissipation by tides (Showman & Guillot 2002)
  - ⇒ Inaccurate stellar radii? (Burrows et al. 2003)
  - ⇒ Planets caught in a runaway evaporation phase? (Baraffe et al. 2003)

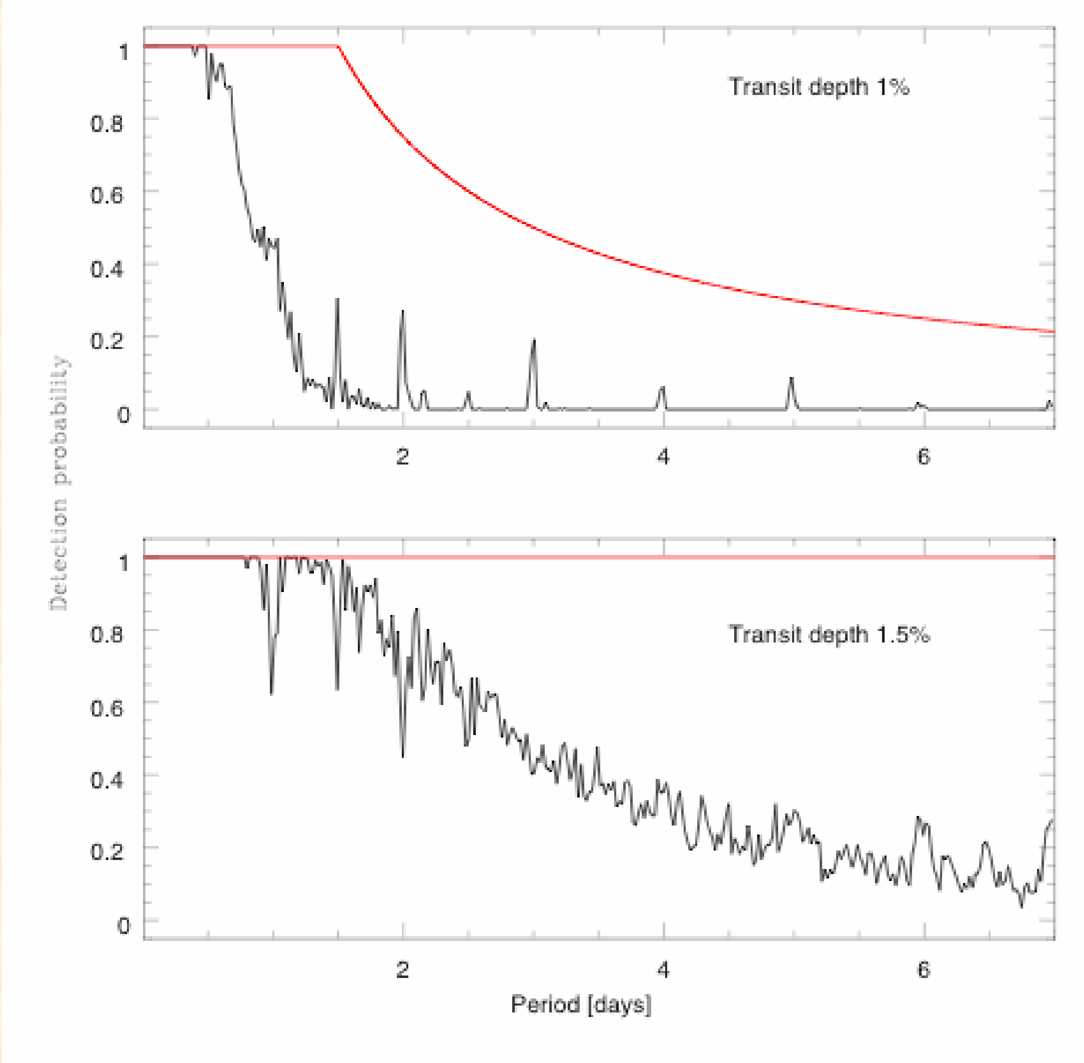


A possible explanation

- Conjecture:
  - All Pegasides are subject to the “inflation effect” but the core masses/mass of heavy elements are very variable from one planet to another
- Advantages:
  - Explains HD209458b and OGLE-110b at the same time as all other planets
  - “Small” planets like OGLE-111b, -113b and -132b also have parent stars with the highest [Fe/H]
  - Contrary to Jupiter, Pegasides can’t eject planetesimals from their system: $(GM/a)^{1/2} \gg (2GM/R)^{1/2}$; Disk properties directly impact planet composition
- Prediction:
  - Planets with low-irradiation and/or further from their star should be smaller <u>on average</u>.

Transit detection: a bias towards larger planets

- Transit surveys (e.g. OGLE) are strongly biased towards the detection of large planets (figure: Pont & Bouchy 2004)
- Suggests that we may have missed planets with radii <1 Rj.



Transiting planets: some conclusions (as of 02/05)

- A simple model with a H-He envelope and a dense core of mass (0 to 15 $M_\oplus$) reproduces the luminosity and radius of most known giant planets
  - Jupiter, Saturn
  - ~5 transiting extrasolar planets
- However, HD209458b is anomalously large!
  - A non-zero forced eccentricity? (Bodenheimer et al. 2001, 2003)
  - Atmospheric kinetic energy dissipated in the interior? (Guillot & Showman 2002)
  - A problematic determination of the stellar radius? (Burrows et al. 2003)
  - A chance effect due to runaway evaporation? (Baraffe et al. 2004)
- We should expect variations of the compositions
  - Variations in [Fe/H]
    - OGLE-TR-132: may be too small => core?
  - Important difference with Jupiter: Pegasi planets can't eject planetesimals (Guillot & Gladman 2000; Guillot 2005)
  - History matters (Burrows et al. 2000)
  - Evaporation may affect the closest planets (Lammer et al. 2003; Lecavelier des Etangs et al. 2004)

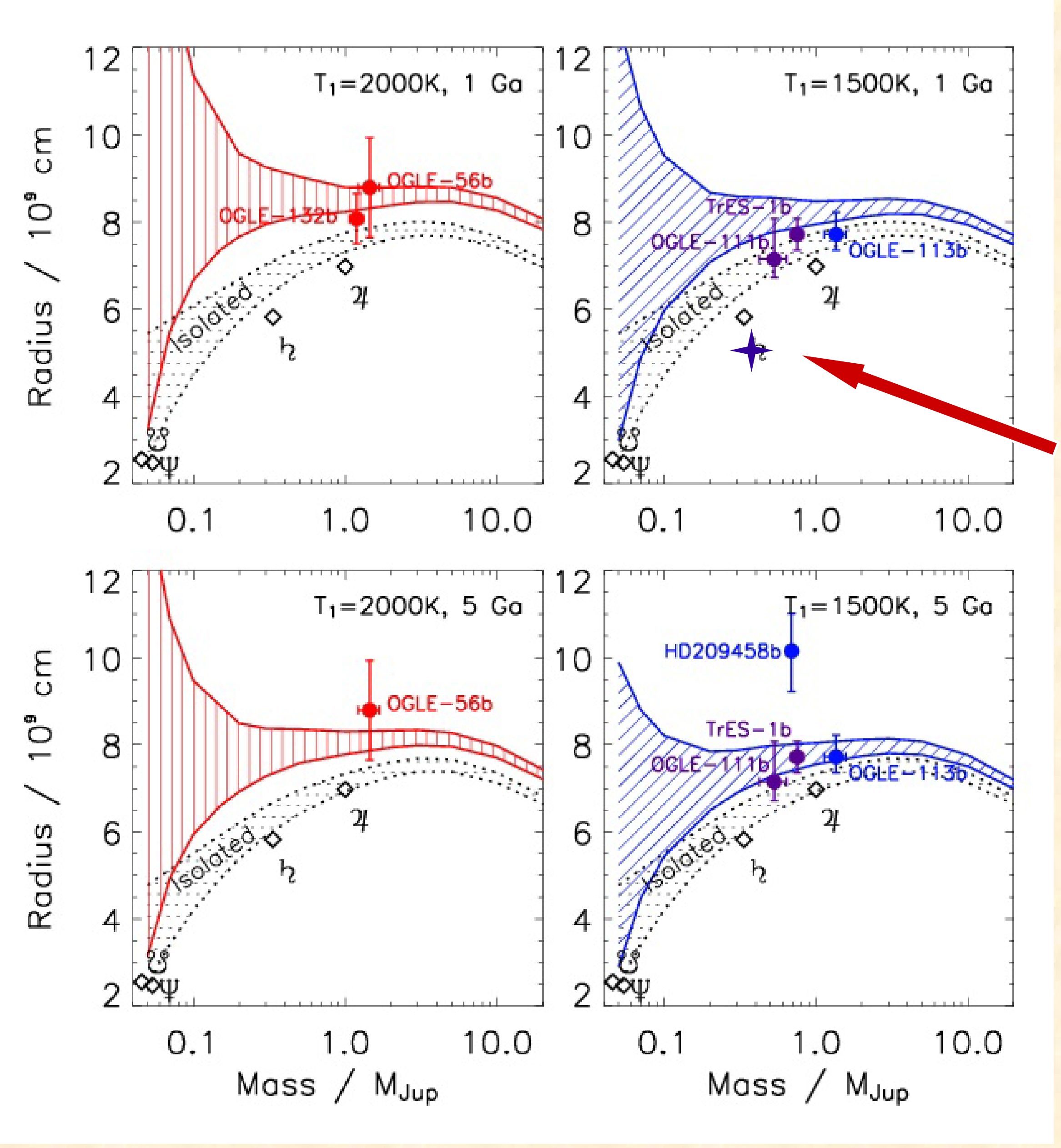
What about a very dense planet?

HD1492026b!

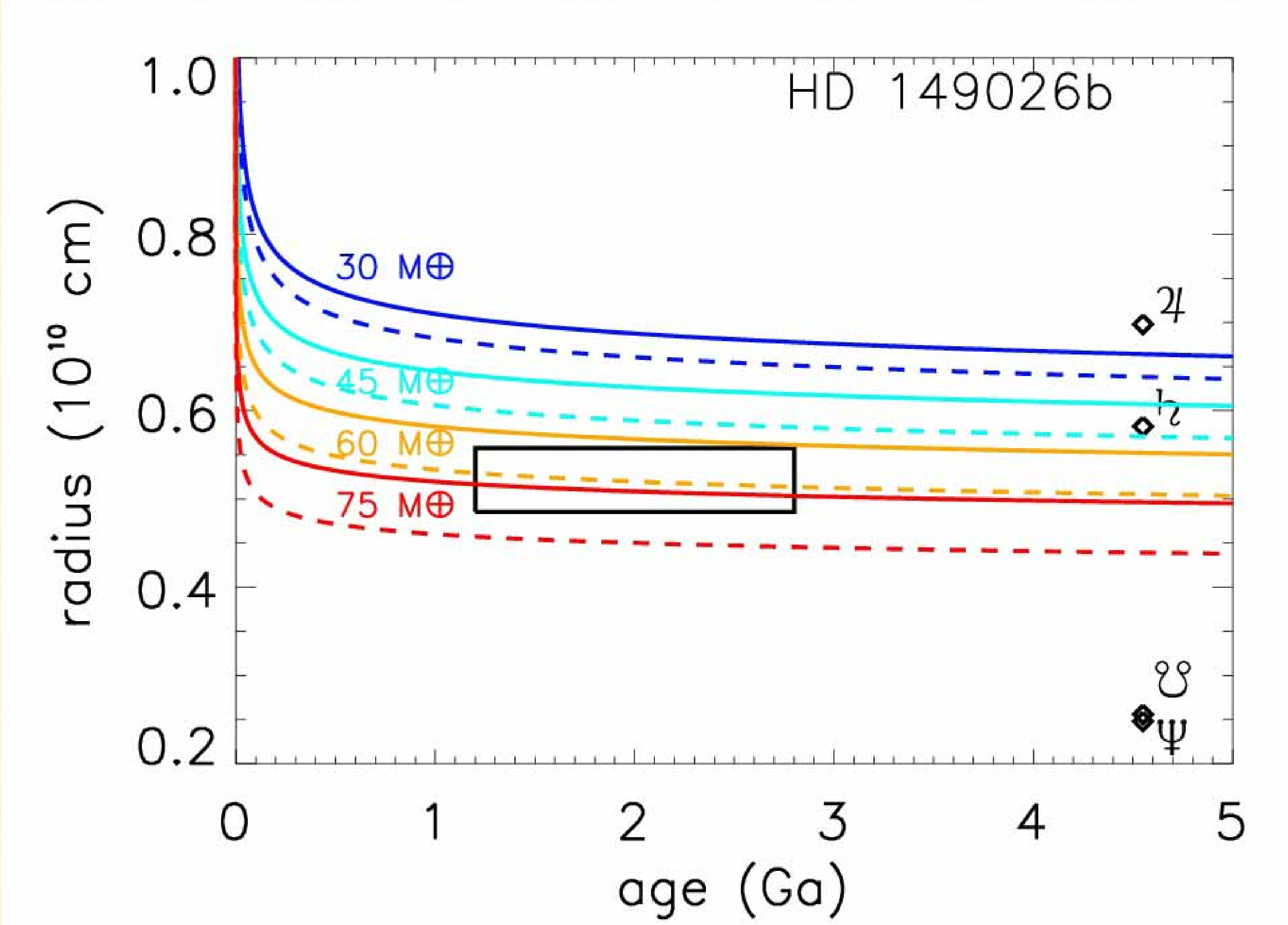
Table 3: Systems with transiting Pegasi planets discovered so far

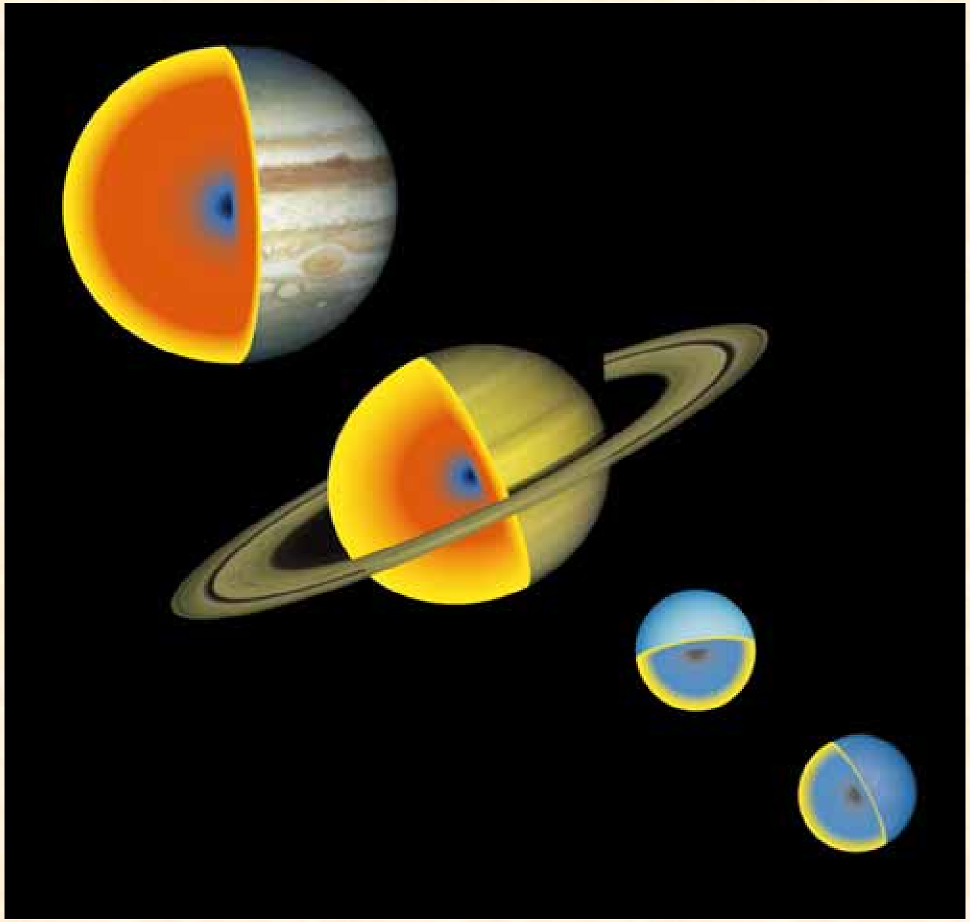
	Age [Ga]	[Fe/H]	a [AU]	$T^{\star}_{eq}$ [K]	M_p/M_J	$R_p/10^{10}$ cm
HD209458[a]	4 – 7	0.00(2)	0.0462(20)	1460(120)	0.69(2)	1.02(9)
OGLE-56[b]	2 – 4	0.0(3)	0.0225(4)	1990(140)	1.45(23)	0.88(11)
OGLE-113[c]	?	0.14(14)	0.0228(6)	1330(80)	0.765(25)	$0.77(^{+5}_{-4})$
OGLE-132[d]	0 – 1.4	0.43(18)	0.0307(5)	2110(150)	1.19(13)	0.81(6)
OGLE-111[e]	?	0.12(28)	0.0470(10)	1040(160)	0.53(11)	$0.71(^{+9}_{-4})$
TrES-1[f]	?	0.00(4)	0.0393(11)	1180(140)	0.75(7)	0.77(4)
HD149026	2.0(8)	0.36(5)	0.042	1740	0.36(3)	0.52(4)

HD1492026b on a M-R diagram



HD1492026b's evolution





Probing deeper our own gas giants

Interior models: principles

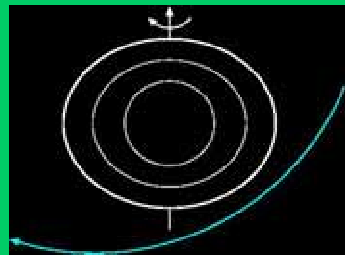
Equations of state

Opacities

Mass,
surface temperature…etc.

Hydrostatic equations
(rotation to 4th order)

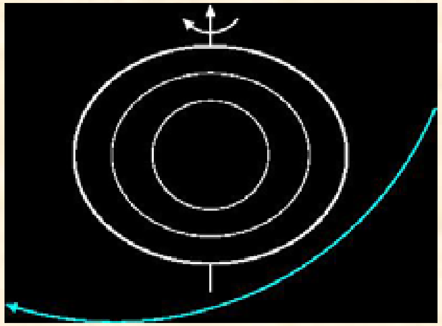
Measured gravitational
moments: J2, J4, J6



Theoretical gravitational
moments: J2, J4, J6,J8

Mass of the rock/ice core
Y+Z in the metallic region
Y+Z in the molecular region

Constraints from rotation

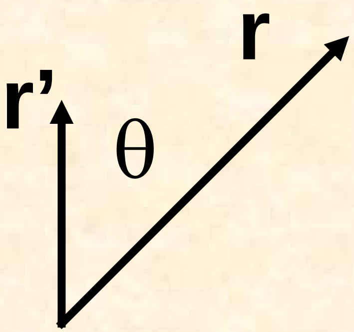


Measured: external gravity potential

$$V_{ext} = \frac{GM}{r}\left[1 - \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{2n} J_{2n} P_{2n}(\cos\theta)\right]$$

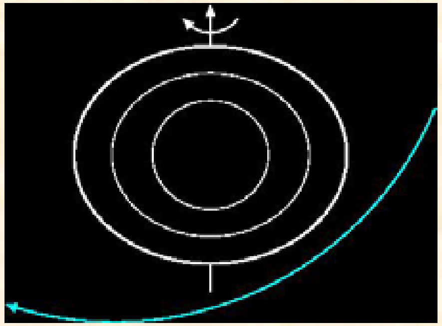
$$\frac{\nabla P}{\rho} = \nabla V - \Omega \times (\Omega \times \mathrm{r})$$

$$V = G\int \frac{\rho(\mathrm{r}')}{\left|\mathrm{r}-\mathrm{r}'\right|} d^3\mathrm{r}'$$



$$\frac{1}{\left|\mathrm{r}-\mathrm{r}'\right|} = \begin{cases} \dfrac{1}{r}\sum\left(\dfrac{r'}{r}\right)^{n} P_n(\cos\theta) & \text{if } r > r' \\ \dfrac{1}{r}\sum\left(\dfrac{r'}{r}\right)^{-n-1} P_n(\cos\theta) & \text{if } r < r' \end{cases}$$

Constraints from rotation



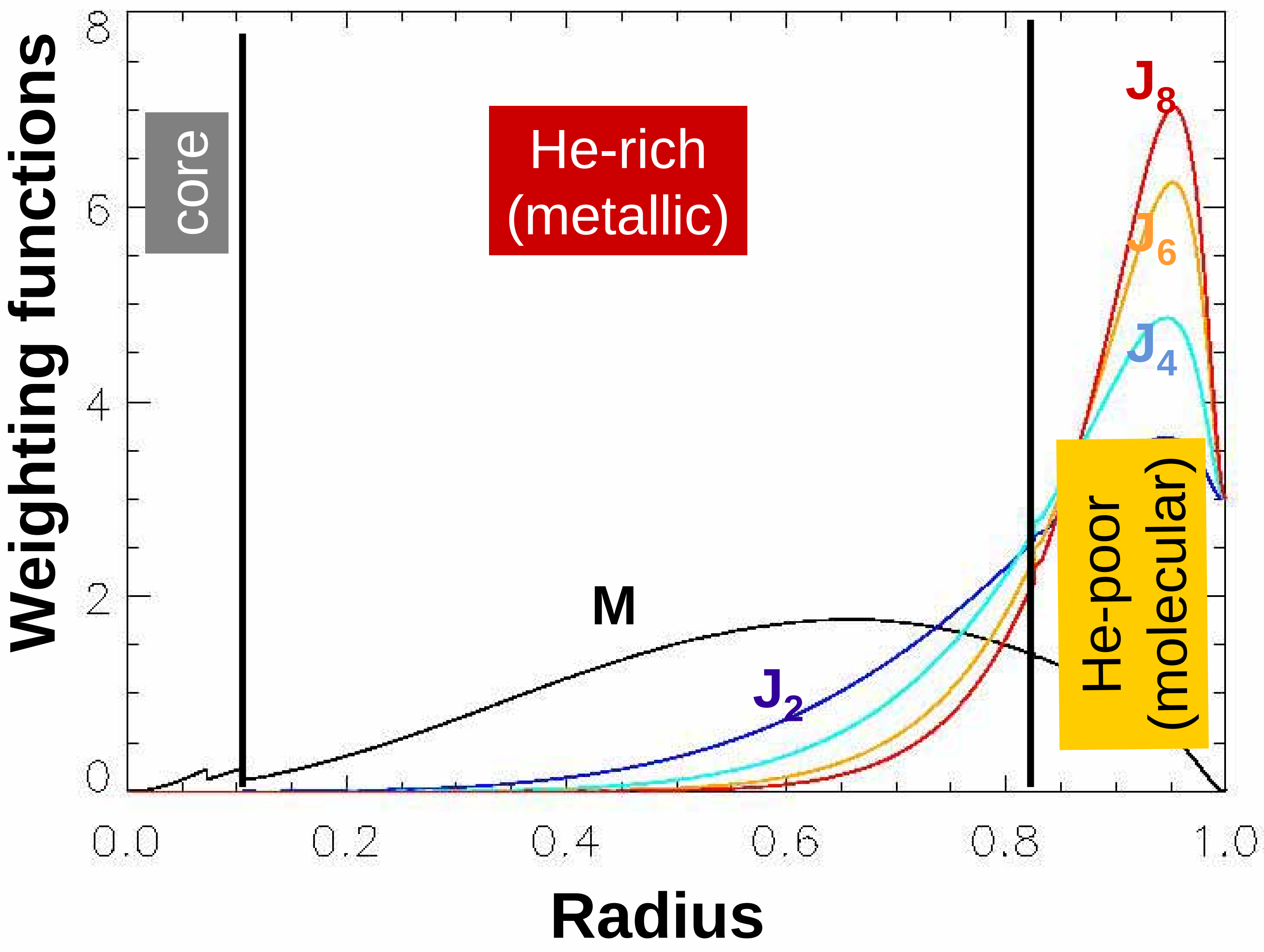
Measured: external gravity potential

$$V_{ext} = \frac{GM}{r}\left[1 - \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{2n} J_{2n} P_{2n}(\cos\theta)\right]$$

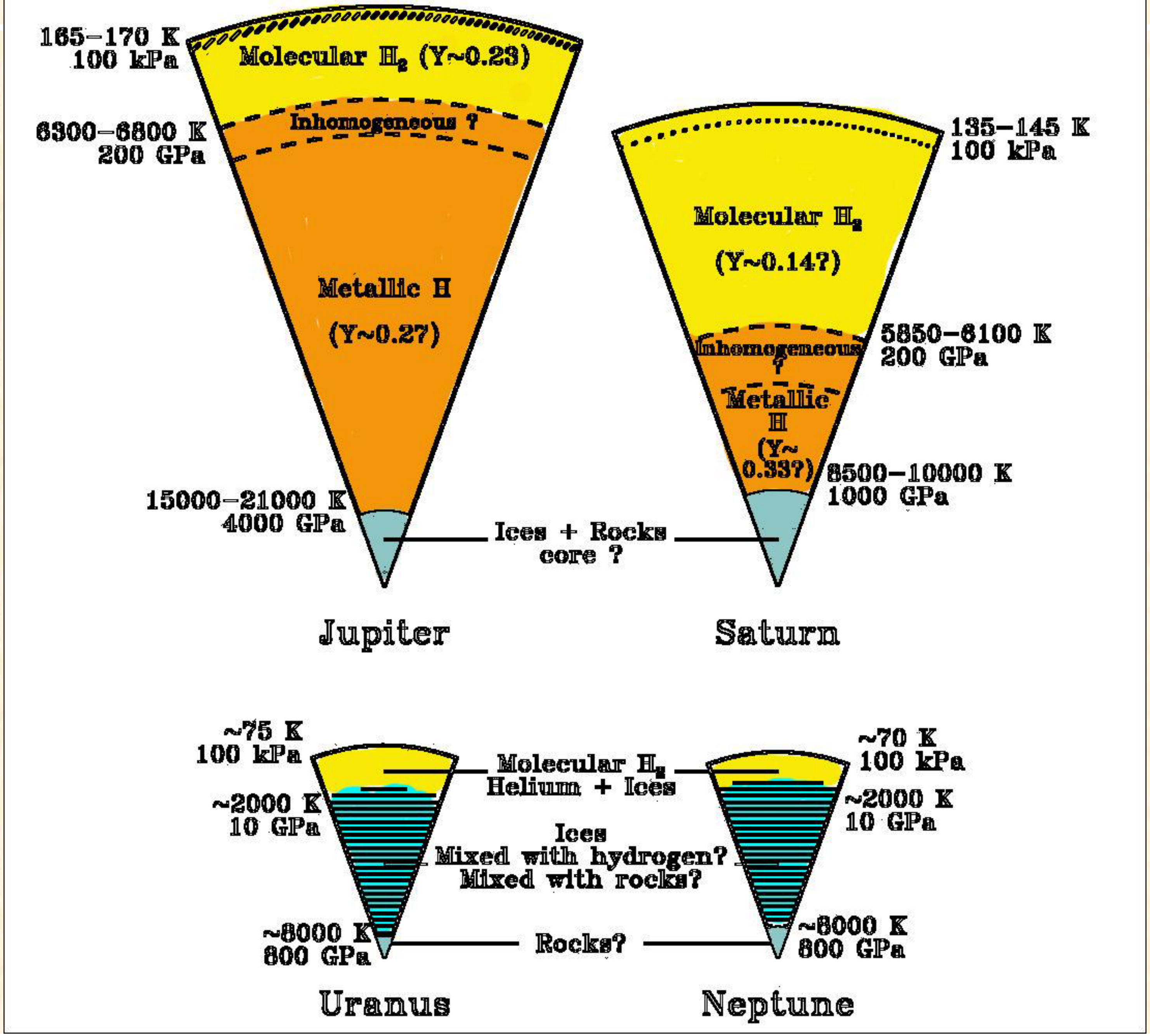
$$V_{\text{ext}} = \frac{G}{r}\sum r^{-2n}\int \rho r'^{2n} P_{2n}(\cos\theta) d^3 r'$$

$$J_{\text{2n}} = -\frac{1}{Ma^{2n}}\int \rho r'^{2n} P_{2n}(\cos\theta) d^3 r'$$

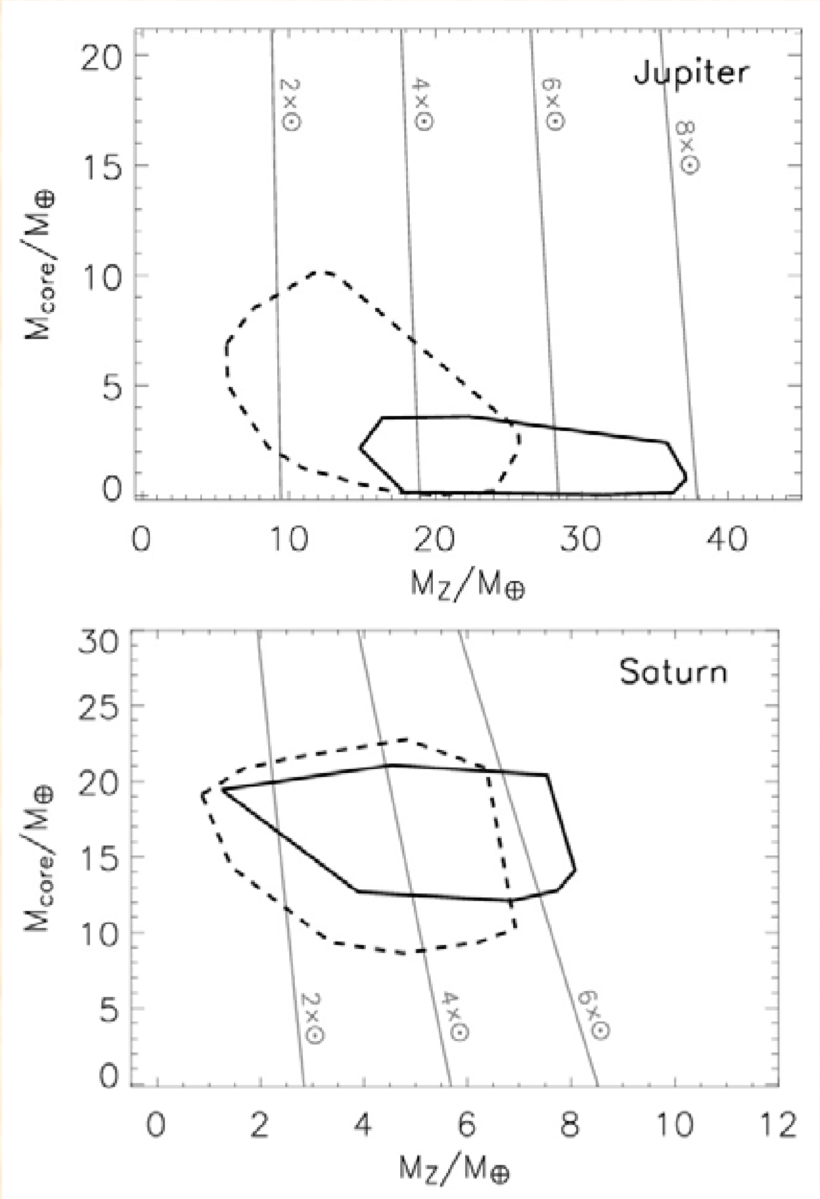
Constraints from gravity



Internal structures



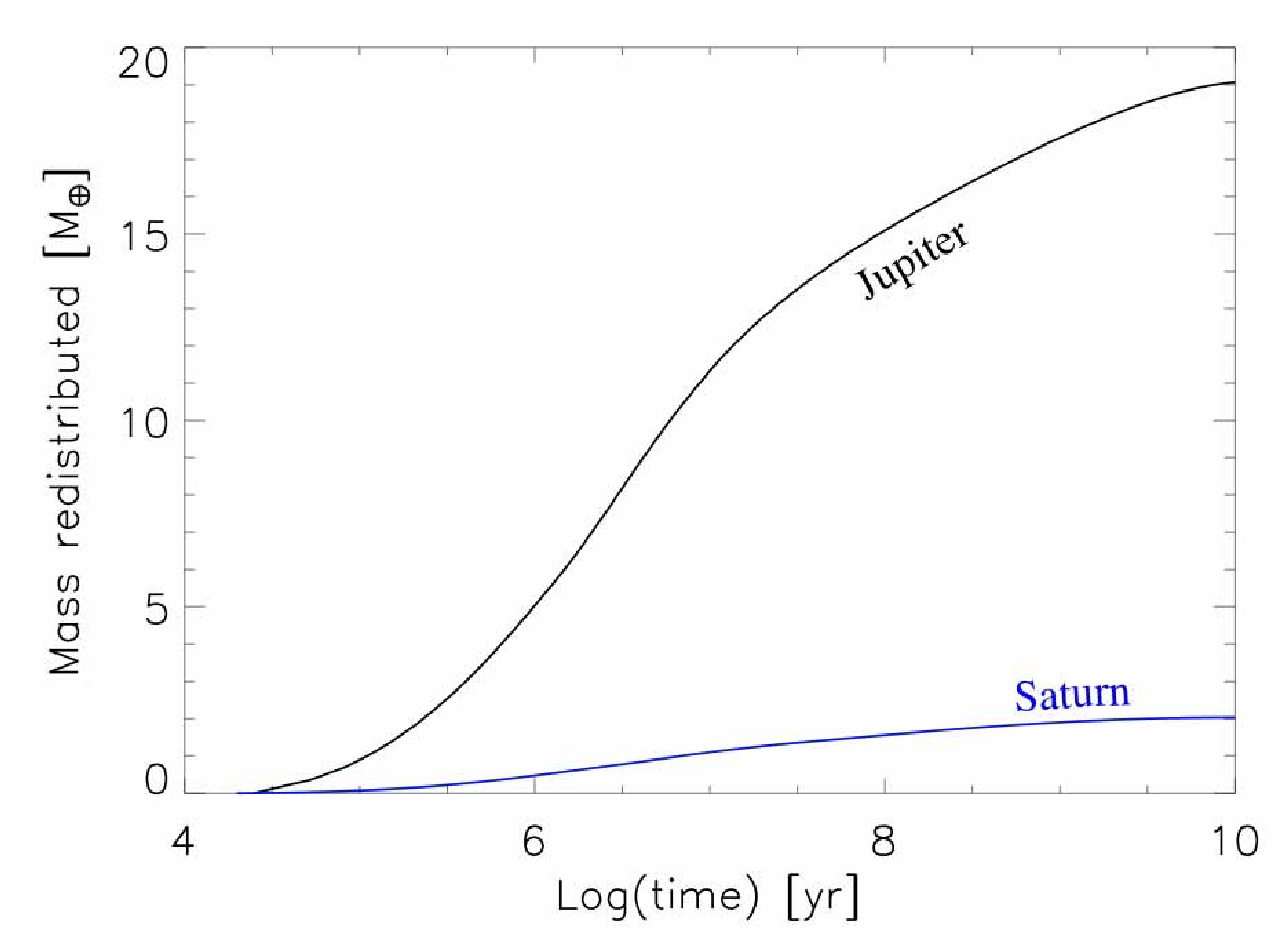
Jupiter & Saturn: M_{core} vs. M_Z



- Jupiter:
  - large uncertainties
  - small core (possible =0)
  - H/He phase separation?
- Saturn
  - tighter constraints
  - helium abundance?
  - H/He phase separation?

Saumon & Guillot, ApJ (2004)

Erosion: a possible explanation for Jupiter's small core



Conclusions

Future prospects

- The structure of giant planets remains mysterious
  - Global composition? Core size?
  - Internal rotation, meteorology?
  - Magnetic fields?
  - Cooling?
- Their formation is very fuzzy
  - Scenario?
  - Timing?
  - Location?
- A wealth of data awaits us...
  - Continuing ground base measurements
  - Spitzer, HST observations of transiting planets
  - COROT (2006), Kepler (2008) => 10-100's of transiting planets
  - Cassini + entended(?): Saturn's gravity field
  - Juno: a Jupiter orbiter to measure the deep abundance of water, differential rotation, heavy elements composition, magnetic field
  - Disk/planet formation connection