Particles in the Solar Nebula: Dust Evolution and Planetesimal Formation

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1. Behavior of isolated particles in the solar nebula

2. Gravitational Instability

3. Collective behavior of particles

4. Numerical modeling of particle layers

5. Implications for instability

6. 2-D models with particle coagulation and migration

7. Conclusions

Planetesimal Formation Mechanism(s)

The solar nebula was ~99% gas and 1% solids by mass. The solids were originally present as small grains, of order micrometer size. Somehow, these assembled into large km-scale planetesimals that were able to accrete by collisions into planetary bodies, held together by self-gravity. The early stage of growth, from grains to km-scale bodies, is still controversial. The principal argument is whether this stage of growth involved collisions and sticking by non-gravitational forces (e.g., van der Waals bonding, electrostatic, etc.), or whether this could be accomplished entirely by gravity. In either case, the solid particles were strongly influenced by the presence of gas.

The solar nebula has a radial pressure gradient, which partially supports it against the Sun's gravity. The nebular gas rotates at slightly less than the Kepler velocity. The fractional deviation from keplerian motion is approximately the ratio of thermal energy in the gas to the gravitational potential energy, of order 10^{-3}. This implies an absolute velocity difference of a few tens of meters per second.

$$V_{gas} = (1-\eta)V_K \qquad \Delta V = \eta V_K$$

where $$\eta = -(\partial P / \partial \ln R) / 2 V_K^2 \rho_g$$

Assumed Nebular Properties

For this discussion, I assume a nominal configuration for the nebula for quantitative examples. The qualitative conclusions are not dependent on the values chosen. For simplicity the surface density of the gas σ_g and temperature *T* are assumed to vary as power laws with heliocentric distance *R*:

$$\sigma_g = 2500\ R^{-1}\ g\ cm^{-2} \qquad T = 320\ R^{-1/2}\ {}^oK$$

where *R* is in AU. This model contains ~ 5% of a solar mass within 30 AU. η varies as $R^{1/2}$, giving a constant value of ΔV = 52 m s^{-1} at all *R*. Most results are obtained for *R* = 3 AU. The assumed abundance of solids (silicates and metal) is 0.0034 times that of the gas.

Solid bodies are not supported by the pressure gradient. In the absence of gas, they would move in keplerian orbits. They must move relative to the gas, and are subject to drag. Their dynamical behavior is controlled by the relative magnitudes of the gravitational and drag forces acting on them.

The residual gravitational acceleration (radial in the direction of the Sun) is equal to

$$\Delta g = 2\eta g_{Sun} = 2\eta GM_{Sun} / R^2$$

The vertical acceleration is

$$g_z = GM_{Sun} Z / R^3 + 4\pi G \int_0^Z (\rho_g + \delta_p) dZ$$

where δ_p is the spatial density of solid matter.

A particle's behavior is governed by the timescale of its response to a drag force, defined as

$$t_e = mV / F_d$$

where m is the particle's mass, V its velocity, and F_d is the drag force. For particles smaller than the mean free path of gas molecules, the Epstein drag law gives

$$t_e = d\rho_p / 2\rho_g c,$$

for a particle of diameter d and density ρ_p in gas with density ρ_g and sound velocity c. For larger particles, Stokes drag gives

$$t_e = \rho_p d^2 / 18\mu,$$

where μ is the molecular viscosity of the gas.

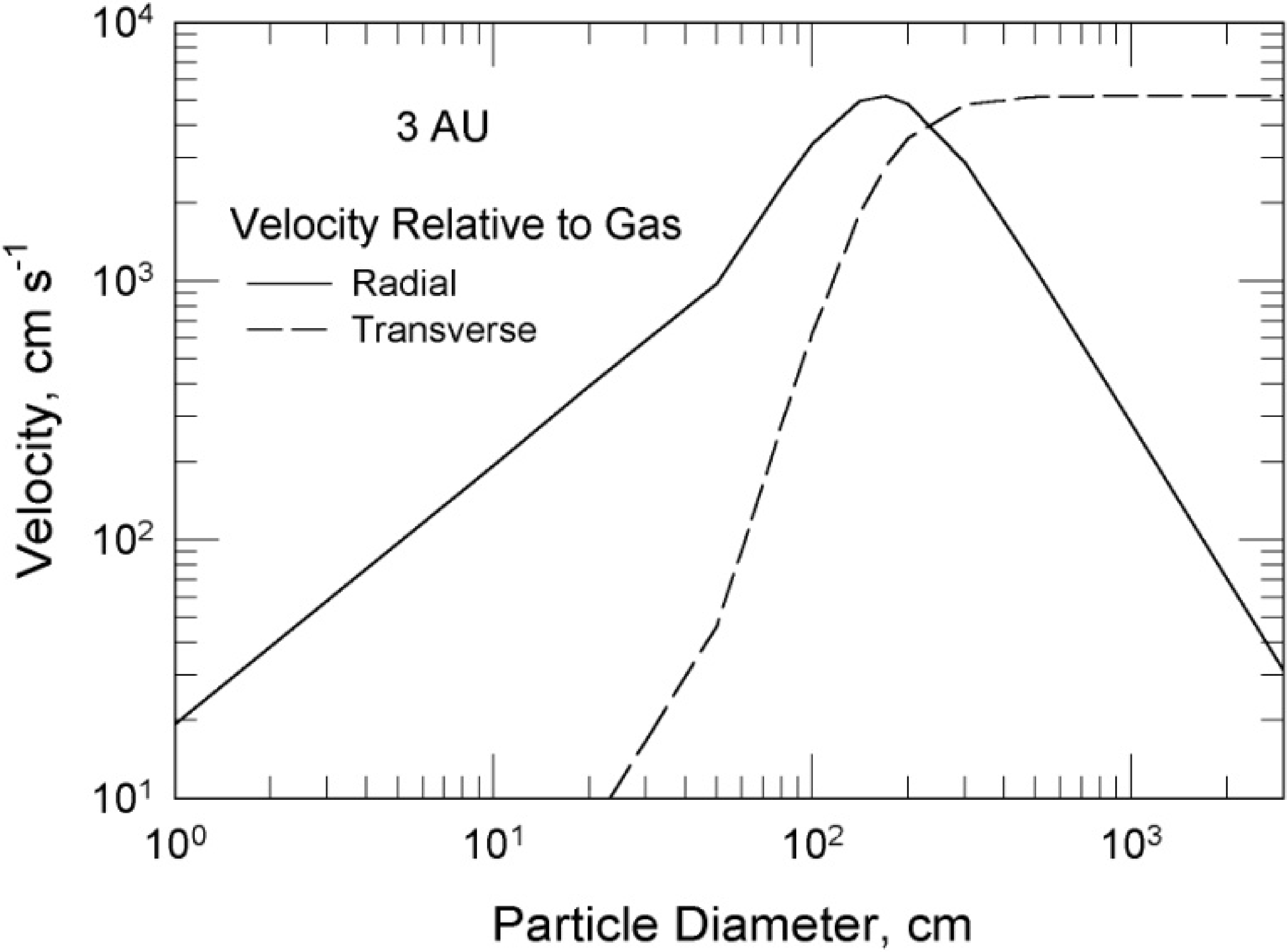
A particle is dynamically "small" or "large" if its response time is less than or more than the Kepler time, i. e., $t_e\Omega_K < 1$ or > 1. Small particles are constrained to move with the gas at its angular velocity. They feel a residual gravitational force that causes them to drift inward at a rate

$$V_r = t_e \Delta g = 2 t_e \eta G M_{sun} / R^2$$

Large particles move in keplerian orbits. They move faster than the pressure-supported gas, and experience a "headwind" of velocity $\Delta V = \eta V_K$. This causes their orbits to decay at a rate

$$V_r = 2\eta R / t_e$$

The radial velocity has a maximum value equal to ΔV When $t_e\Omega_K = 1$.



3 AU
Velocity Relative to Gas
Radial
Transverse
Velocity, cm s-1
10^4
10^3
10^2
10^1
10^0
10^1
10^2
10^3
Particle Diameter, cm

Gravitational Instability and Planetesimal Formation

Settling in a laminar nebula will concentrate particles into a layer in the midplane. If the layer becomes sufficiently dense, it is subject to gravitational instability; i.e., density perturbations will tend to collapse under their own gravity. Self-gravity exceeds the tidal force exerted by the Sun at a critical density, which is approximately

$$\delta_{crit} = 3M_{Sun}/2\pi R^3 = 3\Omega_K^2/2\pi G$$

This density is necessary, but not sufficient.

Collective Effects

Particles settling to the midplane form a layer with a density that eventually exceeds that of the gas ($\delta > \rho_g$). However, the density does not increase without limit. In the dust-dominant layer, particles drag the gas at a velocity closer to the Kepler velocity (although the layer never attains keplerian motion). The shear flow is unstable, and produces turbulence. This turbulence inhibits further settling, even if the nebula as a whole is laminar. The thickness of the particle layer, and its density vs. altitude Z, are determined by a balance between downward settling and turbulent diffusion upward along a concentration gradient.

For small particles, with sizes less than a few cm, the density of the layer due to shear-induced turbulence is significantly less than the critical value for gravitational instability, for plausible nebular parameters and normal abundance of solids. For example, in the nominal case at 3 AU, the critical density implies a solids/gas ratio of about 66, or ~ 2000 times the solar abundance of silicates
relative to hydrogen.

One solution is for the particles to grow by collisions to larger sizes so they are not affected by the turbulence, in which case gravitational instability is unnecessary. If particles cannot stick, some other means must be invoked to suppress this turbulence or augment the density of the layer.

Critical Density: Necessary, but not Sufficient

At 3 AU, $\delta_{crit} \sim 10^{-8}$ g cm^{-3}. The free-fall time for gravitational collapse is $t_{ff} \sim (3\pi/32G\delta)^{1/2} \sim 1$ year for $\delta = \delta_{crit}$.
If particles are small enough to be coupled to the gas, they cannot collapse in free fall, as this would be prevented by compression of the gas (Sekiya 1983). Instead, they must settle through the gas. The settling rate is $t_e g_r$, where $g_r = Gm/r^2 \sim 4\pi G\delta_{crit} r/3$. The settling timescale is $r/(t_e g_r) \sim (220/s\rho_s)$ years at 3 AU. For compact ($\rho_s = 2$ g cm^{-3}) chondrule-sized ($s = 0.1$ cm) particles, the settling timescale is ~ 10^3 years. During this time, a condensation would move through the gas at a velocity ~ ΔV. The drag force on a particle near the surface of condensation exceeds the gravitational attraction, so the condensation would shed mass.

The critical density can be expressed as a critical velocity, c^*. The dispersion relation for density perturbations is

$$F(\lambda) = \Omega_K^2 \lambda^2 - 4\pi^2 G\sigma\lambda + 4\pi^2 c^2$$

where λ is the wavelength of a perturbation, which grows with time if $F(\lambda)<0$. This requires $c < c^* = \pi G\sigma\Omega$. It is usually assumed that the half-thickness of the layer H is proportional to the velocity dispersion; $H \sim c/\Omega$. If $\delta \sim \sigma/H$, then $c^* \sim \pi G\sigma/\Omega$. $F(\lambda)$ has a minimum at $\lambda^* = 2\pi^2 G\sigma/\Omega^2$. Rotation stabilizes long wavelengths, and the velocity dispersion stabilizes short ones. An unstable layer will tend to break up into condensations of mass ~ $\sigma\lambda^{*?}$?

If the velocity dispersion is isotropic, then the density of the layer is inversely proportional to the mean particle velocity. It is usually assumed that the layer's half-thickness H is ~ c/Ω. The layer's density is then $\delta \sim \sigma/H$, and $\delta \sim \delta_{crit}$ when $c \sim c^* = \pi G\sigma/\Omega$. For typical conditions at 3 AU, σ = 2.83 g cm^{-2}, c^*~ 15 cm s^{-1}, λ^* ~ 2.5x10^9 cm, $\sigma\lambda^{*2}$ ~ 2x10^{19} g.

However, velocities driven by gas drag are ***not*** isotropic. Radial and transverse velocities are due to non-keplerian motion of the gas, while vertical velocities are due to turbulence. If the out-of-plane velocity dispersion is less than c^*, the layer's density may exceed the critical value, but the in-plane dispersion may still be too large. c^* is analogous to an escape velocity; a particle will not be gravitationally bound to a region of the layer.

Velocity Dispersion and Consequences

Drag-induced radial velocities exceed c^* for particles with sizes between ~ 1 cm and 50 m. This is not a problem for identical particles, as all would move together at the same rate. However, if particles can grow to such sizes by coagulation, they will have some distribution about the mean size. Since the velocities are size-dependent, there will also be a dispersion of velocities, which will act to inhibit gravitational instability. The velocity dispersion is expected to be comparable to the mean velocity. This implies that if gravitational instability is to be effective, it must occur among particles smaller than ~ 1 cm or larger than tens of meters.

Settling Rate

Particles settle toward the central plane of the nebula. Small particles have settling rate

$$dZ / dt = t_e g_z$$

Large bodies are in damped keplerian orbits. Their inclinations are damped by drag. The "settling velocity" is taken to be semimajor axis *a* times the rate of damping. From Adachi et al. (1976), this is

$$dZ \,/\, dt = (0.85\, Z \,/\, a + \eta) \,/\, 2\eta t_e$$

Turbulence and Particle Response

In turbulence with eddy frequency ω, the quantity $t_e\omega$ is called the Stokes number *St*. If the turbulence has velocity V_{turb}, the diffusion velocity of a particle is

$$V_{diff} = V_{turb}/(1+St)$$

The diffusion coefficient is

$$C_{diff} = (\pi/8)/V^2_{diff}/\omega = (\pi/8)V^2_{turb}/\omega(1+St)$$

The turbulence frequency ω is described by the Rossby number *Ro*, where $\omega = 2Ro\Omega_K$.

Ekman Length

A characteristic length scale for the thickness of a turbulent boundary layer of a disk rotating in a fluid is the Ekman length L_E , defined as

$$L_E = (\nu_t/\Omega)^{1/2}$$

where ν_t is the turbulent viscosity, and $\Omega = \Omega_K$ is the rotation frequency. After Cuzzi et al. (1993) we take $\nu_t \sim (\Delta V/Re^*)^2/\Omega_K$, where $Re^* \sim O(10^2)$ is a critical Reynolds number. Turbulence is assumed to decay exponentially over a distance L_E.

Richardson Number

The Richardson number *(Ri)* is a measure of the stability of a stratified shear flow. If a fluid element is displaced vertically, work is done against gravity and buoyancy, while kinetic energy is extracted from the flow due to the mismatch of velocity due to the shear. *Ri* is dimensionless, defined as

$$Ri = g_Z(-\partial\rho/\partial Z)/\rho(\partial V/\partial Z)^2$$

The flow becomes turbulent if $Ri < 0.25$.

Response of the Gas to Particle Loading

Nakagawa et al. (1986) solved coupled equations of motion for particles and gas in an inviscid layer without turbulence, for particles of arbitrary size. Defining $D = (\rho_g+\delta_p)/\rho_g t_e$, the radial and transverse velocities of particles (relative to particle-free pressure-supported gas) are:

$$V_{rp} = \frac{2\Omega_K \eta V_K}{t_e(D^2+\Omega_K^2)} \qquad V_{\phi p} = \frac{D\eta V_K}{t_e(D^2+\Omega_K^2)}$$

where radial velocity is defined as positive inward.

The corresponding gas velocities are:

$$V_{rg} = -(\delta_p / \rho_g) V_{rp} \qquad V_{\phi g} = -(\delta_p / \rho_g) V_{\phi p}$$

These result from transfer of momentum from the particles to the gas. The mass fluxes of particles and gas are equal and opposite; the particles move inward, while the gas moves outward. We refer to these motions as the laminar reaction flow.

For large (decimeter to meters) particles, the midplane concentration of solids can be quite high; $\delta_p >> \rho_g$, and V_{rp} and $V_{\phi p}$ can be large, so these gas velocities may be large.

If the gas is turbulent, then the turbulent viscosity ν_t produces another exchange of momentum, between elements of gas at different elevations above the midplane. Youdin and Chiang (2004) derive the stress tensor. Assuming axial symmetry, the stress $P_{z\phi}$ is due to the vertical gradient of azimuthal velocity V_ϕ:

$$P_{Z\phi} = (\rho_g + \delta_p)\nu_t(\partial V_\phi / \partial Z)$$

and the radial velocity induced by this stress is proportional to the gradient of that stress:

$$V_{r,turb} = \frac{-R}{(\rho_g + \delta_p)} \frac{\partial P_{Z\phi} / \partial Z}{\partial \Omega_K R^2 / \partial R}$$

Youdin and Chiang assumed the particles are perfectly coupled to the gas. For larger particles with imperfect coupling, for δ_p we substitute $\delta_p/(1+St)$.

The gas in the midplane rotates more rapidly than that at larger Z, so $P_{Z\phi} < 0$. The shear removes angular momentum from the gas at small values of Z and transfers it to the gas at larger elevations. The resultant profiles of radial velocity show inflow near the midplane and outward flow of gas near the top of the particle layer. The net radial motion of the gas at any value of Z is the sum of the laminar reaction term and the velocity due to turbulent shear.

Richardson Number and Turbulence Structure

Cuzzi et al. (1993) developed a computational fluid dynamical model of 2-phase particle-gas system for large (10-60 cm radius) particles. These cases had high midplane densities and steep velocity gradients, with fully developed turbulence and *Ri* << 0.25. They argued that the Rossby number was large (20-80), and eddy frequency ω ? $2Ro\,\Omega_K$.

Sekiya (1998) assumed small particles well coupled to the gas (*St << 1)*, and argued that the onset of turbulence would prevent further settling; the vertical density profile would keep *Ri* at the critical value of 0.25. The incipient turbulence would have an eddy timescale imposed by the rotation of the nebula, $\omega \sim \Omega_K$.

Numerical Modeling of Particle Layers

- Divide layer into a series of levels, with assumed particle abundance at $t = 0$.
- Compute velocity of gas from mass loading
- Compute Richardson number *Ri*
- Assume turbulent velocity proportional to velocity difference between local gas and particle-free gas
- Assume eddy timescale is a function of *Ri*; compute turbulent diffusivity
- Distribute particles vertically by settling and diffusion
- Iterate until a steady state is reached

The numerical model assumes that the turbulent velocity at any level depends on the velocity difference between the local gas and the particle-free gas at large *Z*, and the Richardson number:

$$V_{turb} = F(Ri)(V_{\phi g}^2 + V_{rg}^2)^{1/2}$$

where

$$F(Ri) = 2Ro(1-4Ri)^2$$

For *Ri* < 0.25, and *F(Ri)* = 0 for *Ri* > 0.25. To fit the values of ω used by Cuzzi et al. and Sekiya in the limits of small and large *Ri*, I assume

$$\frac{1}{\omega} = \frac{1}{2Ro\Omega_K} + [\frac{1}{\Omega_K} - \frac{1}{2Ro\Omega_K}](4Ri)^2$$

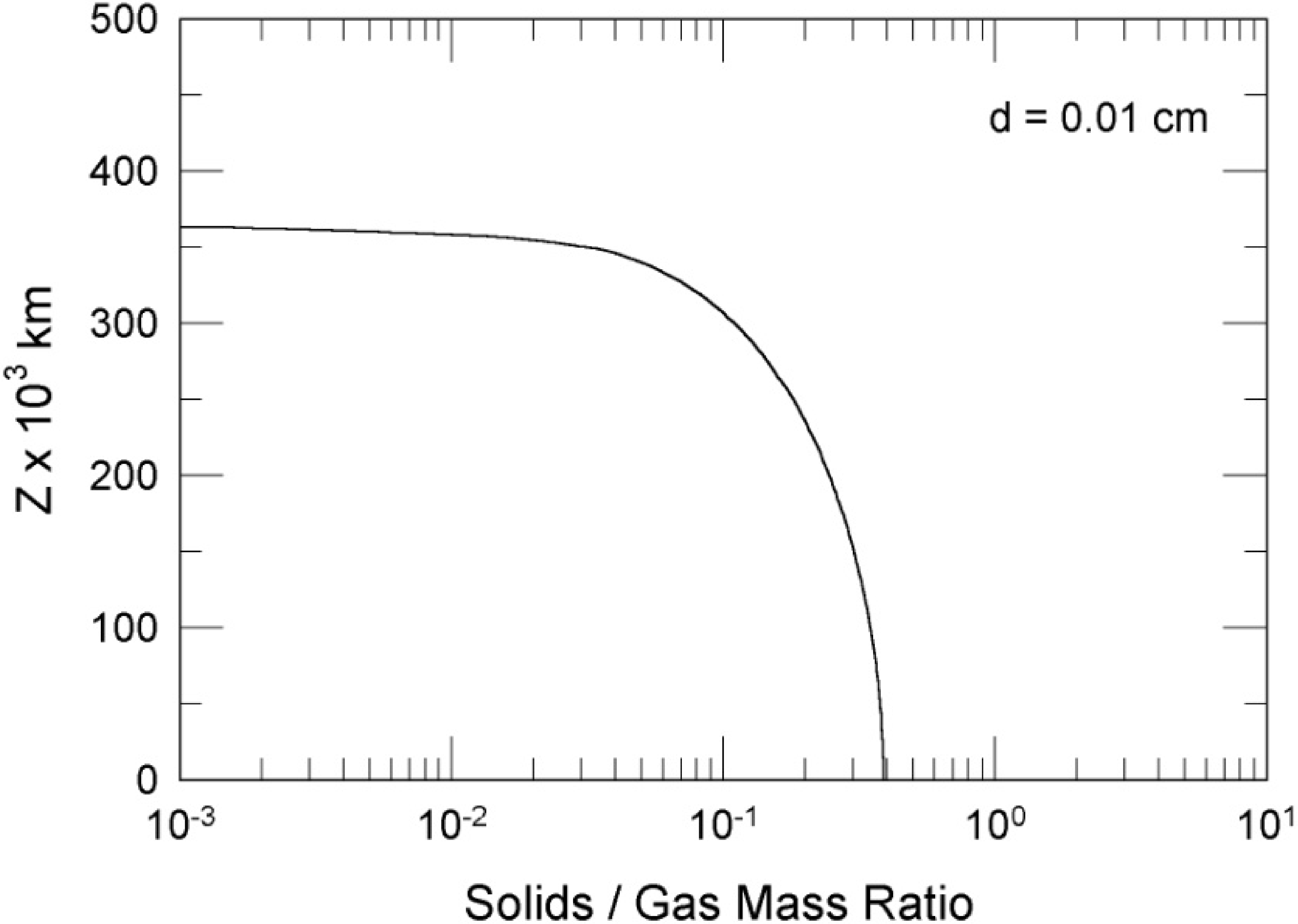
Caveats

For numerical stability, the Richardson number used is a mass-weighted cumulative average, rather than the local value.

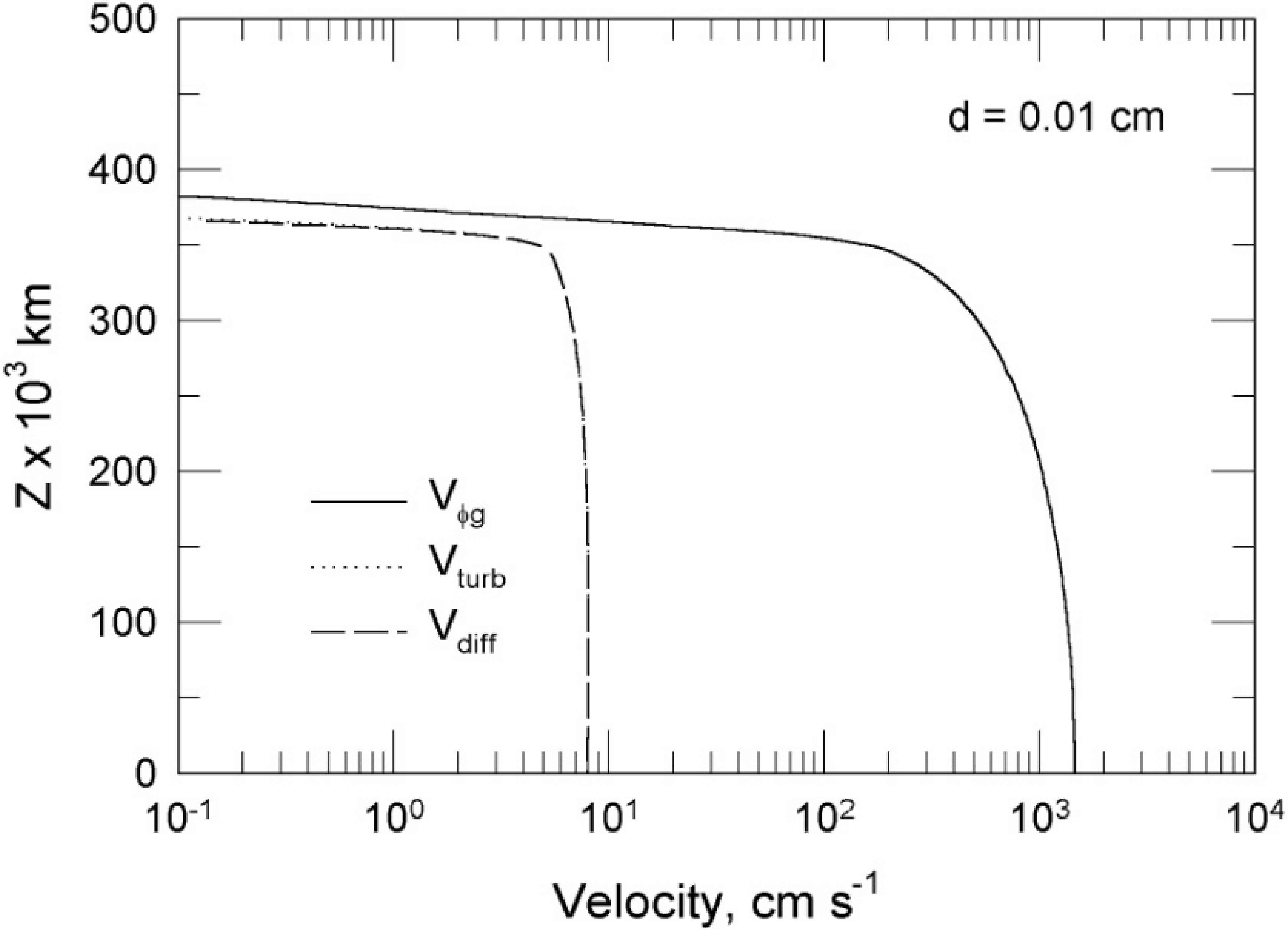
To evaluate the turbulent stress tensor $P_{z\phi}$, the velocity gradient is smoothed over a distance L_E

Turbulence generated locally at a given level is compared
with that generated at other levels, assumed to decay exponentially on scale L_E; the largest V_t and ω are used.

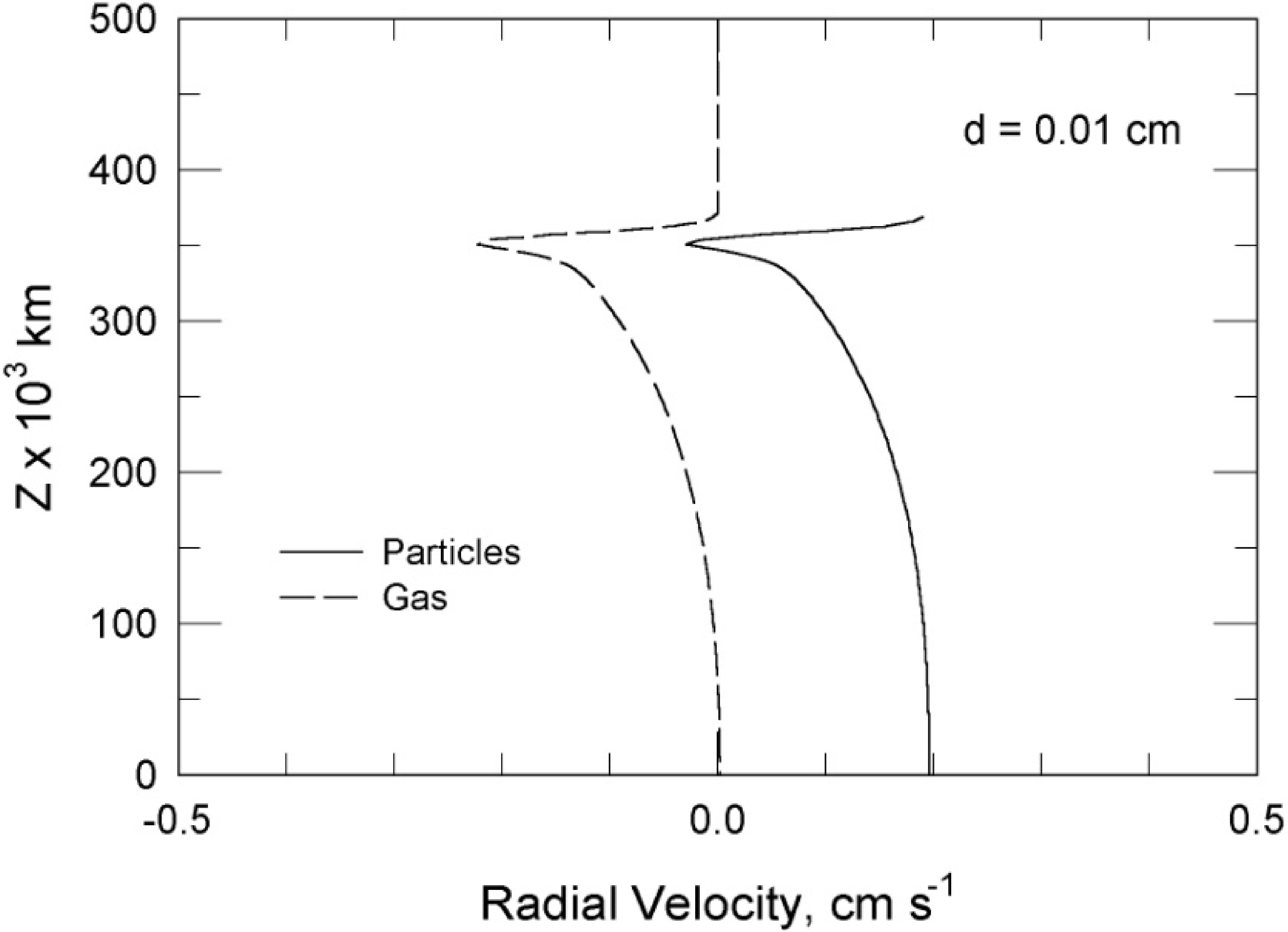
The computed radial and transverse velocities of the gas are also smoothed over L_E



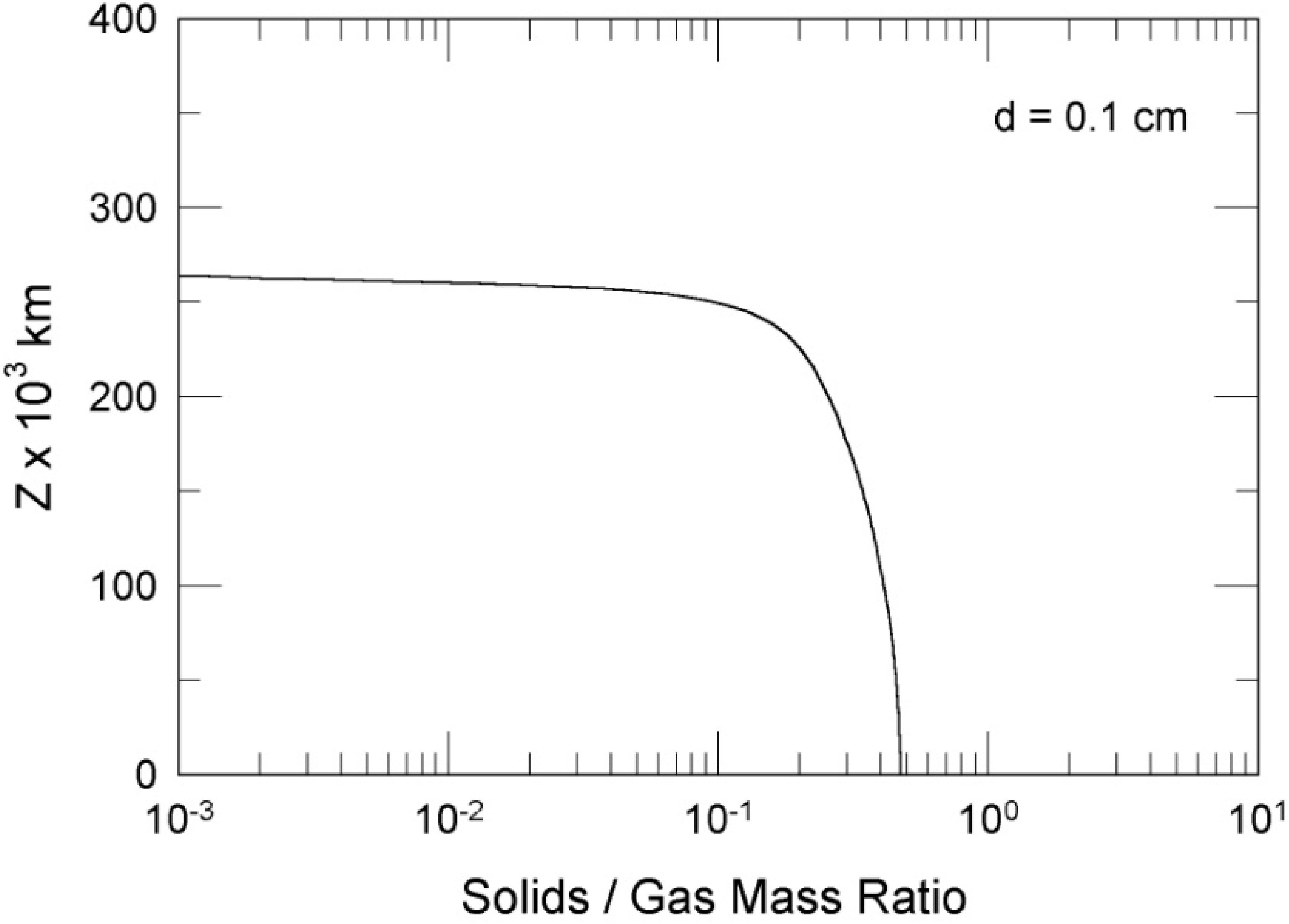
500
400
300
200
100
0
Z x 10^3 km
d = 0.01 cm
10^-3
10^-2
10^-1
10^0
10^1
Solids / Gas Mass Ratio



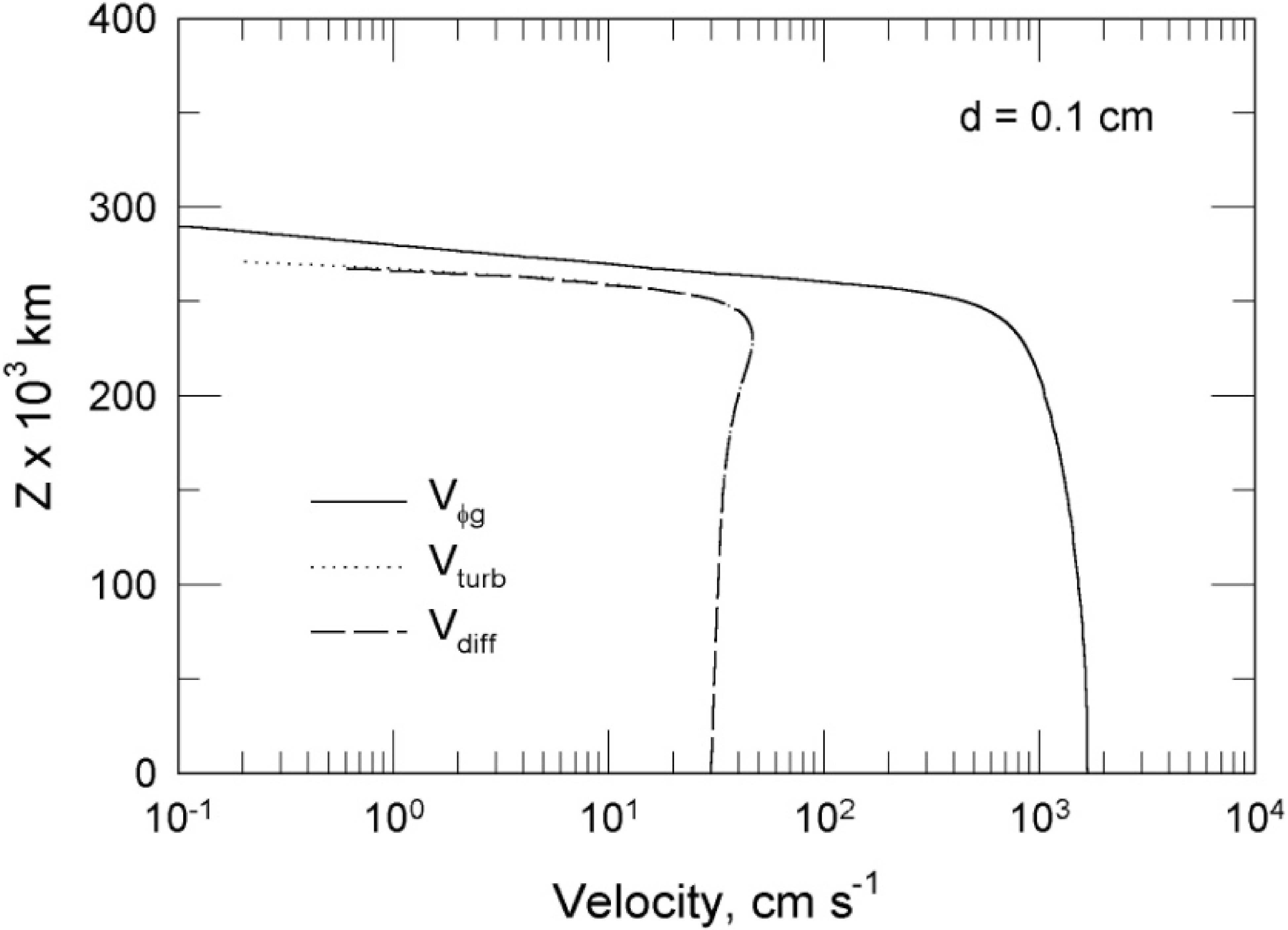
d = 0.01 cm
$V_{\phi g}$
V_{turb}
V_{diff}
Z x 10³ km
Velocity, cm s⁻¹
500
400
300
200
100
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}



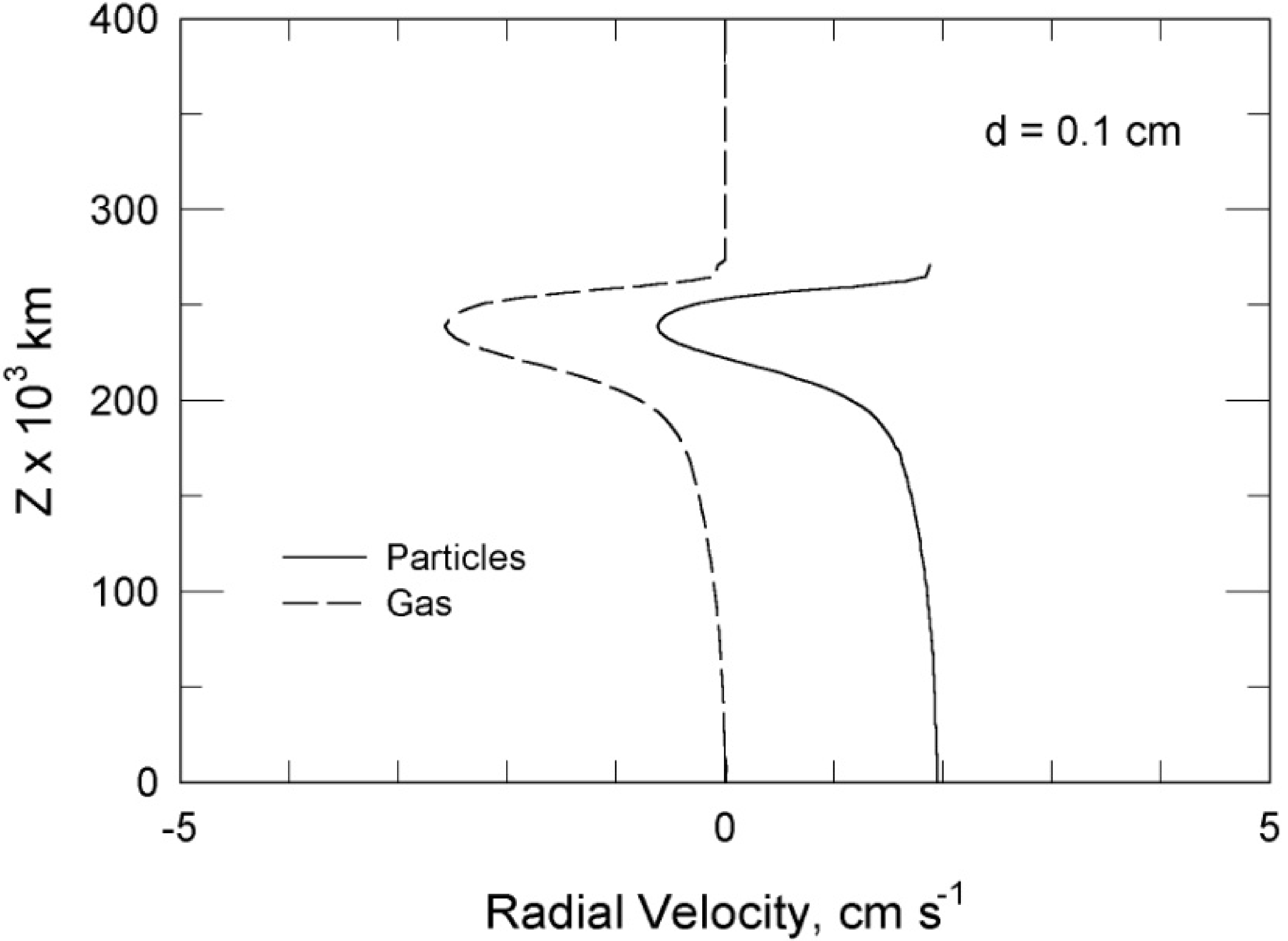
500
400
300
200
100
0
Z x 10^3 km
d = 0.01 cm
Particles
Gas
-0.5
0.0
0.5
Radial Velocity, cm s^{-1}

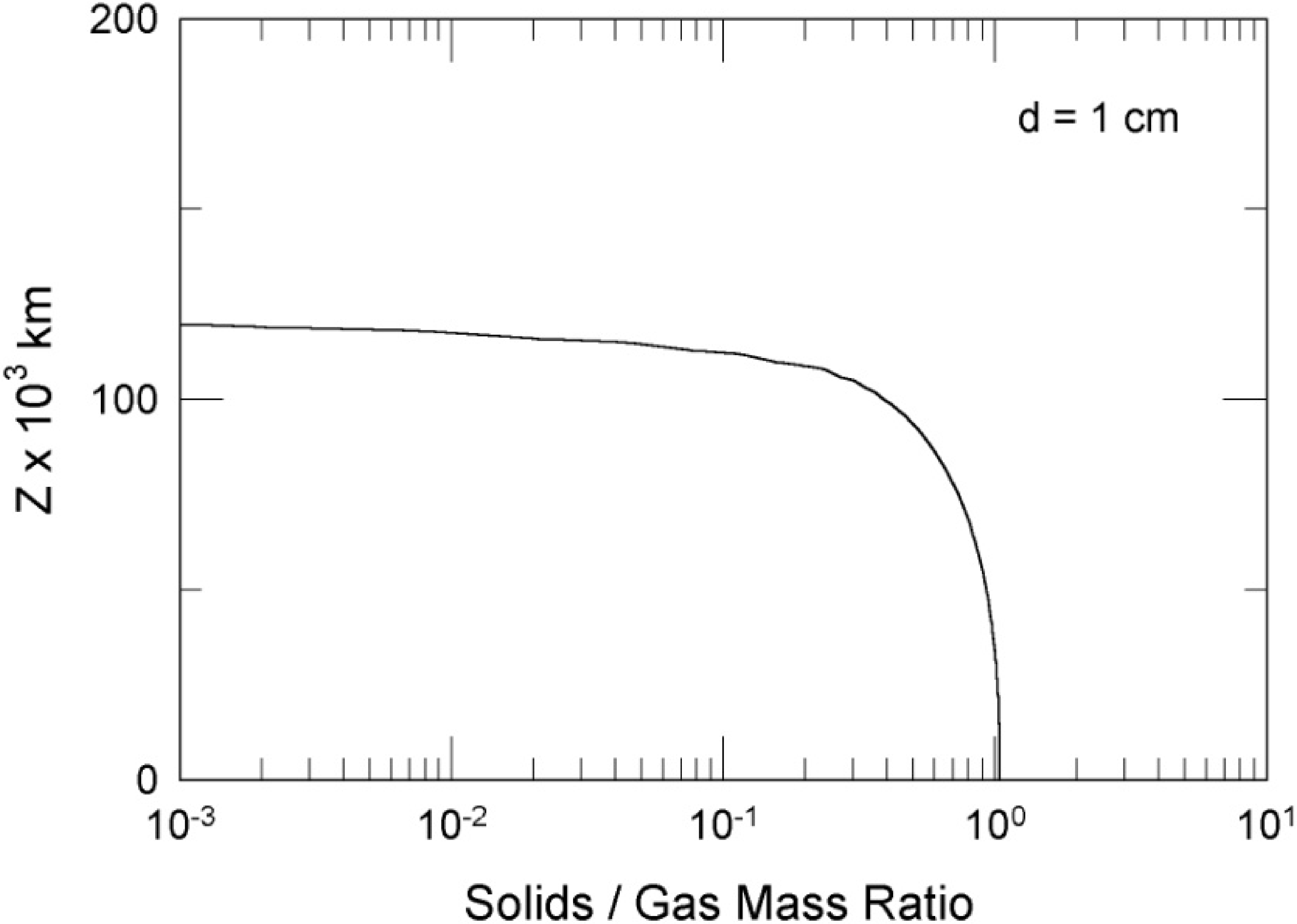


400
300
200
100
0
Z x 10^3 km
d = 0.1 cm
10^-3
10^-2
10^-1
10^0
10^1
Solids / Gas Mass Ratio

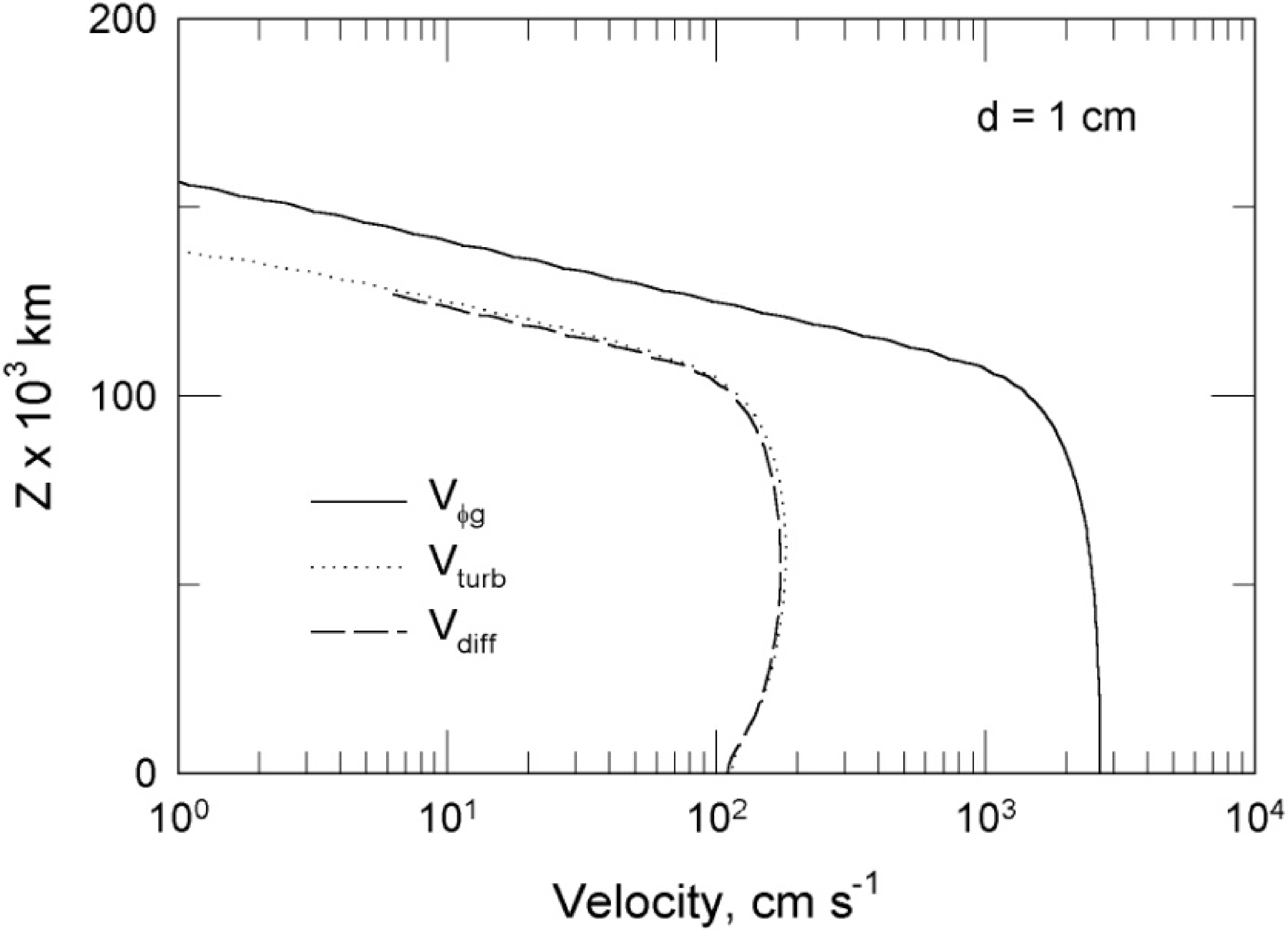


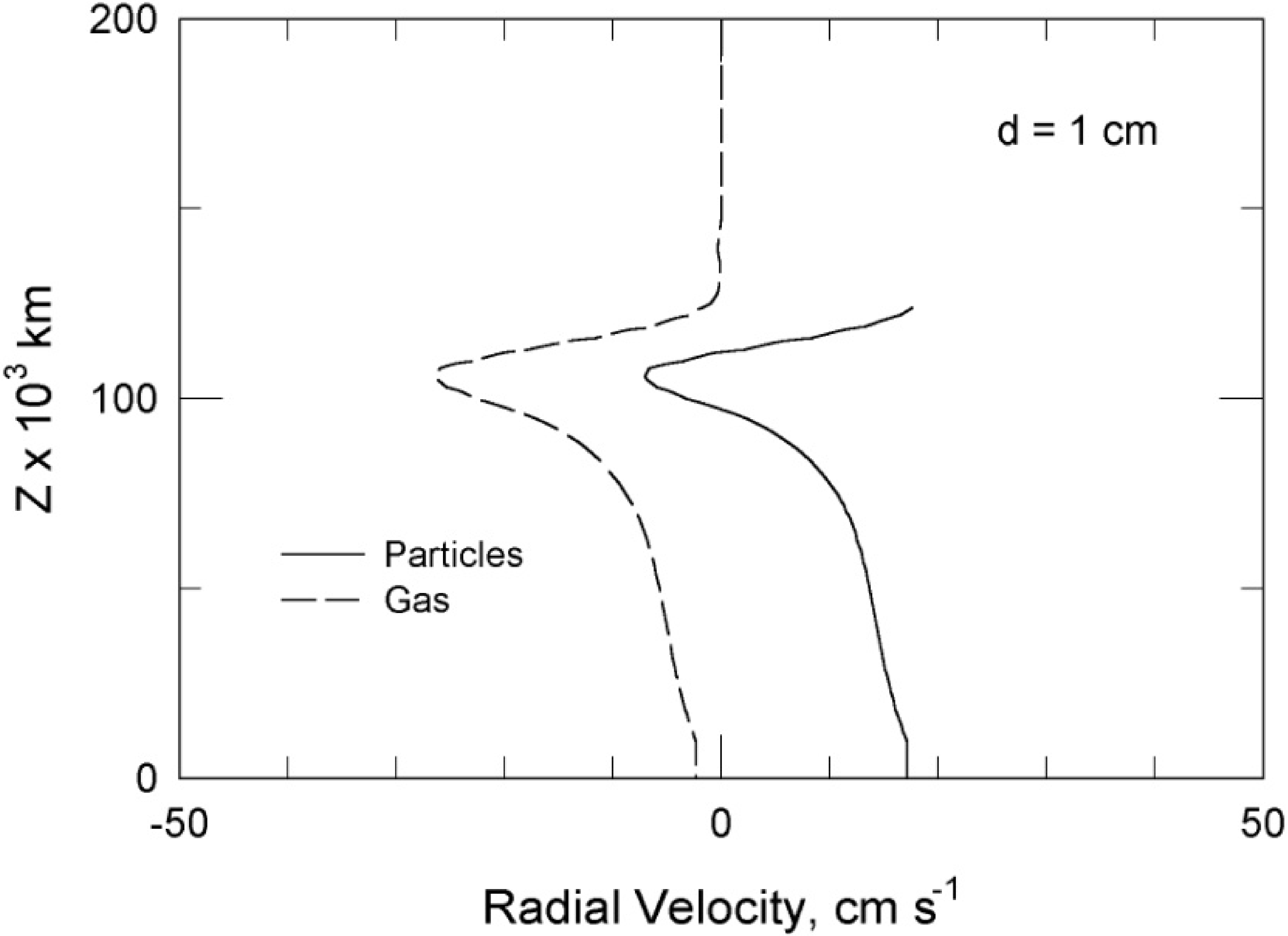
d = 0.1 cm
$V_{\phi g}$
V_{turb}
V_{diff}
400
300
200
100
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Z x 10^3 km
Velocity, cm s^{-1}


400
300
200
100
0
-5
0
5
Z x 10^3 km
Radial Velocity, cm s^-1
d = 0.1 cm
Particles
Gas

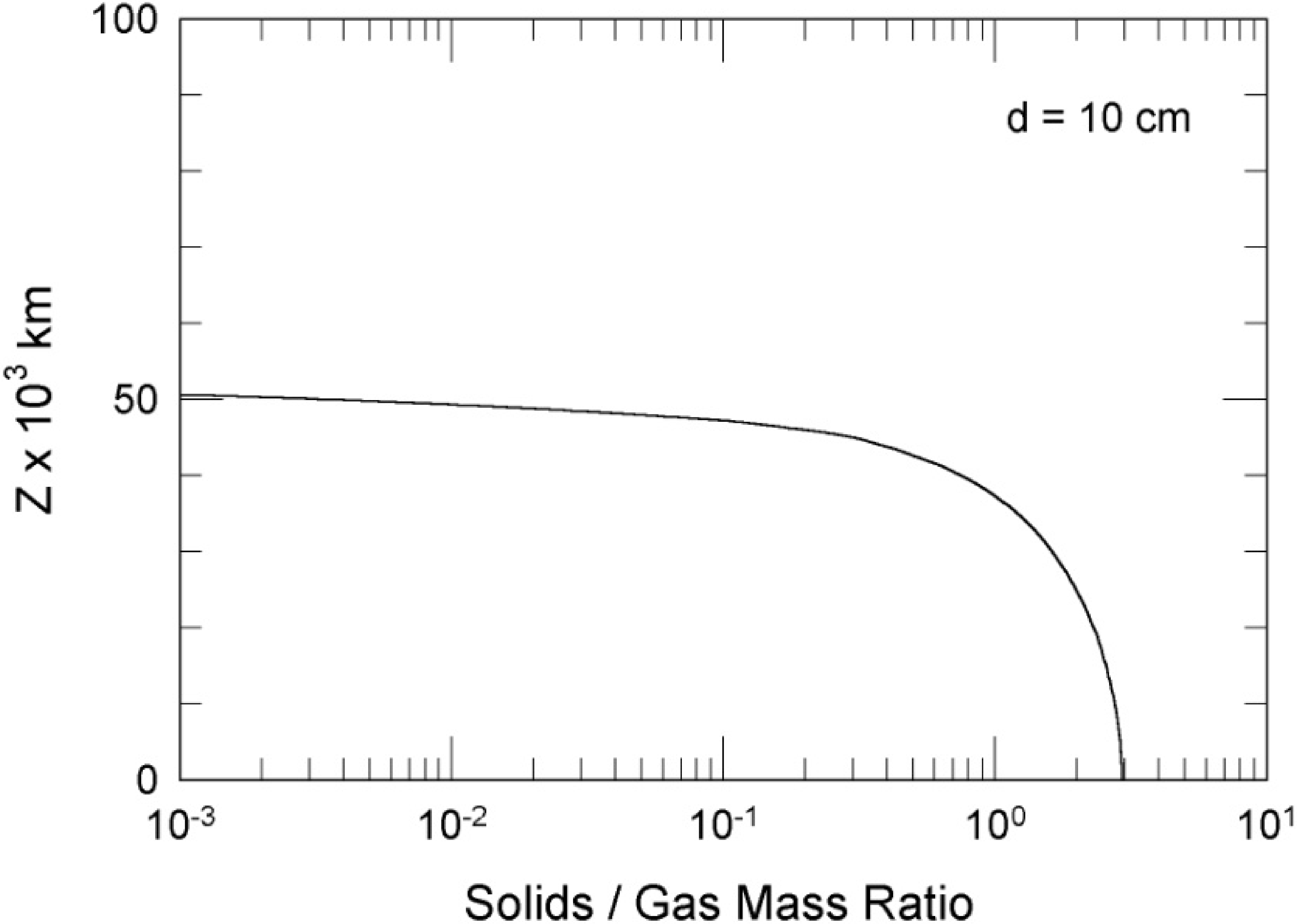


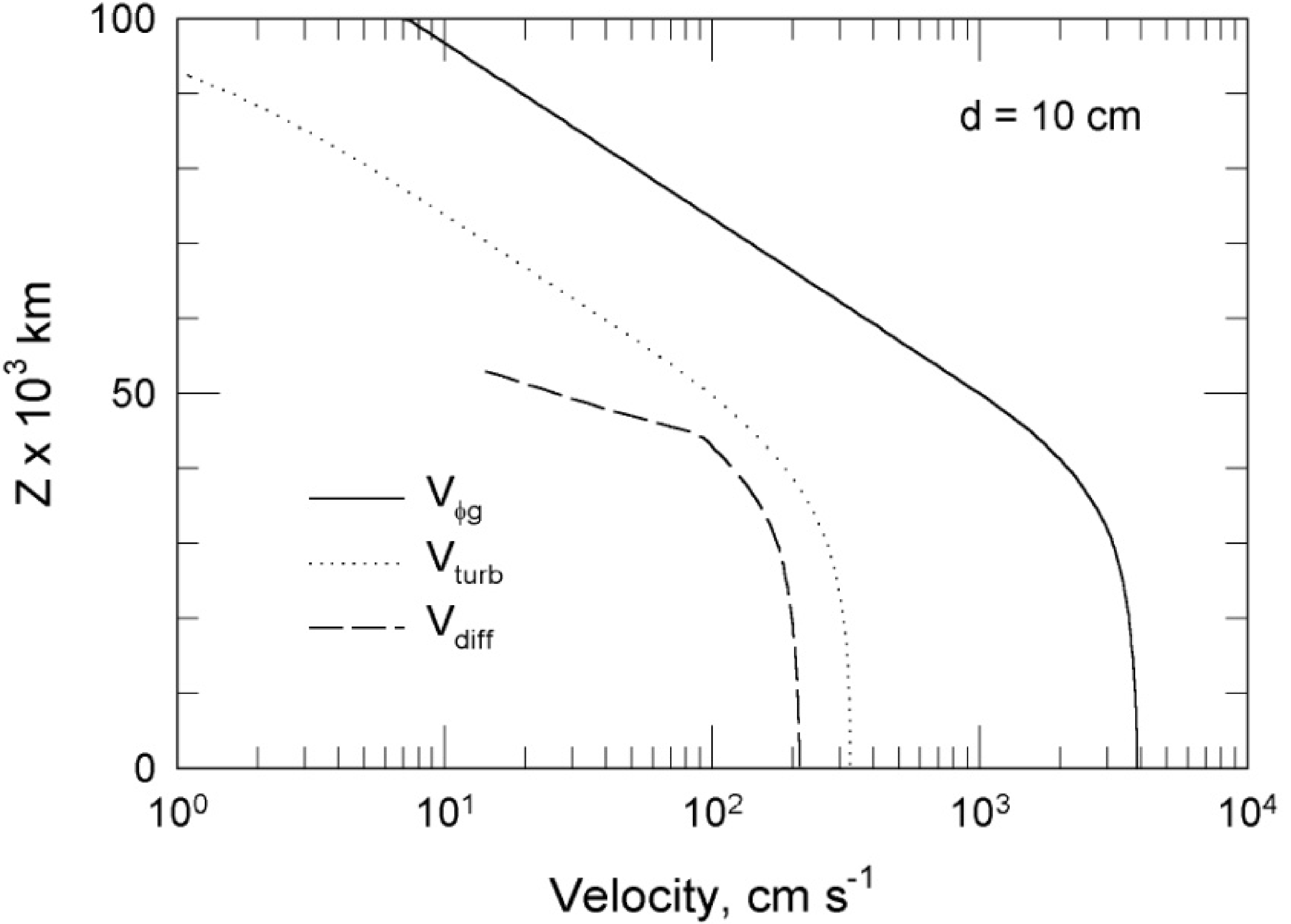
200
100
0
Z x 10^3 km
d = 1 cm
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
Solids / Gas Mass Ratio


200
100
0
Z x 10^3 km
d = 1 cm
V_φg
V_turb
V_diff
10^0
10^1
10^2
10^3
10^4
Velocity, cm s^-1

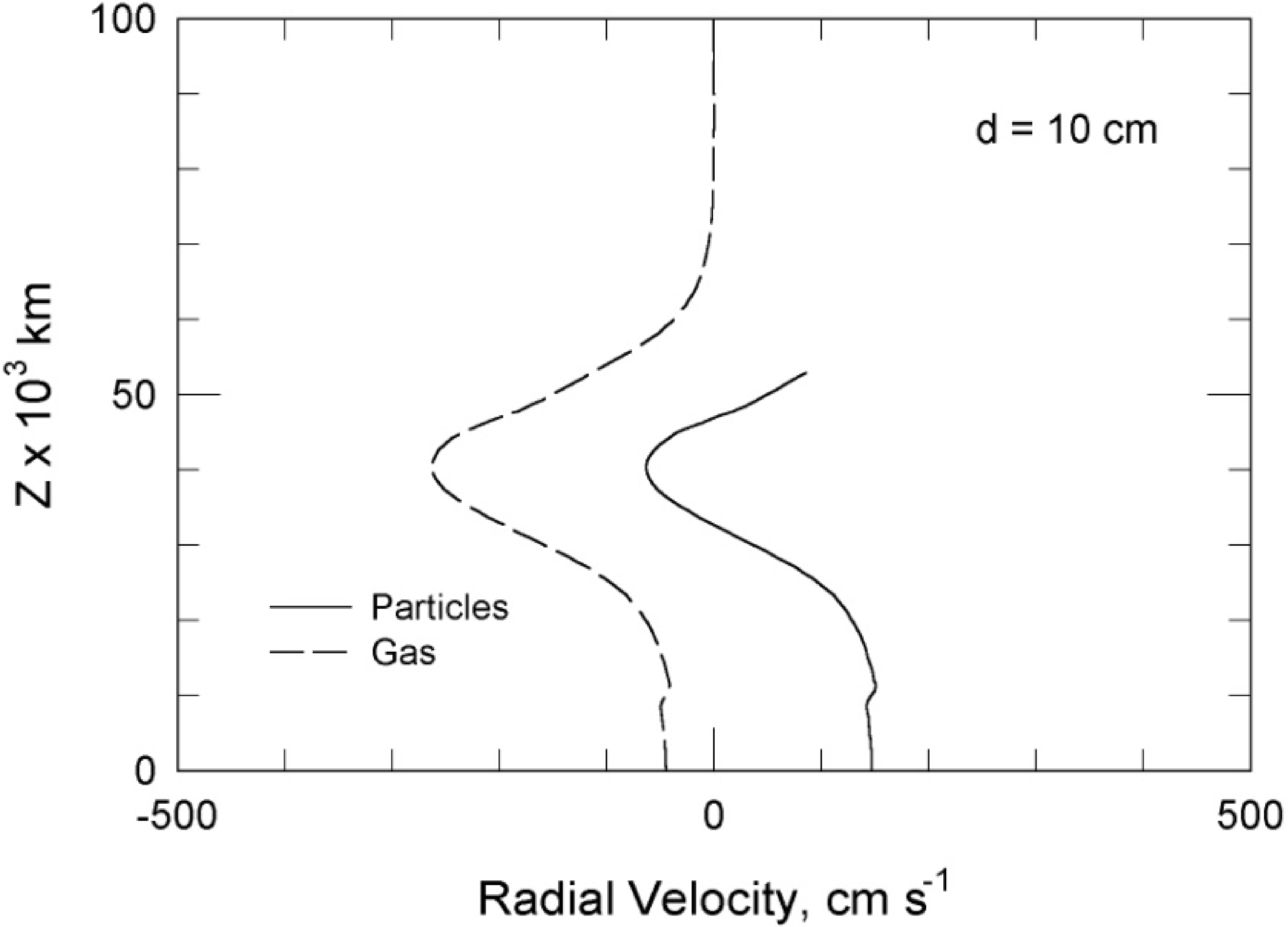


200
100
0
-50
0
50
d = 1 cm
Particles
Gas
Z x 10^3 km
Radial Velocity, cm s^-1

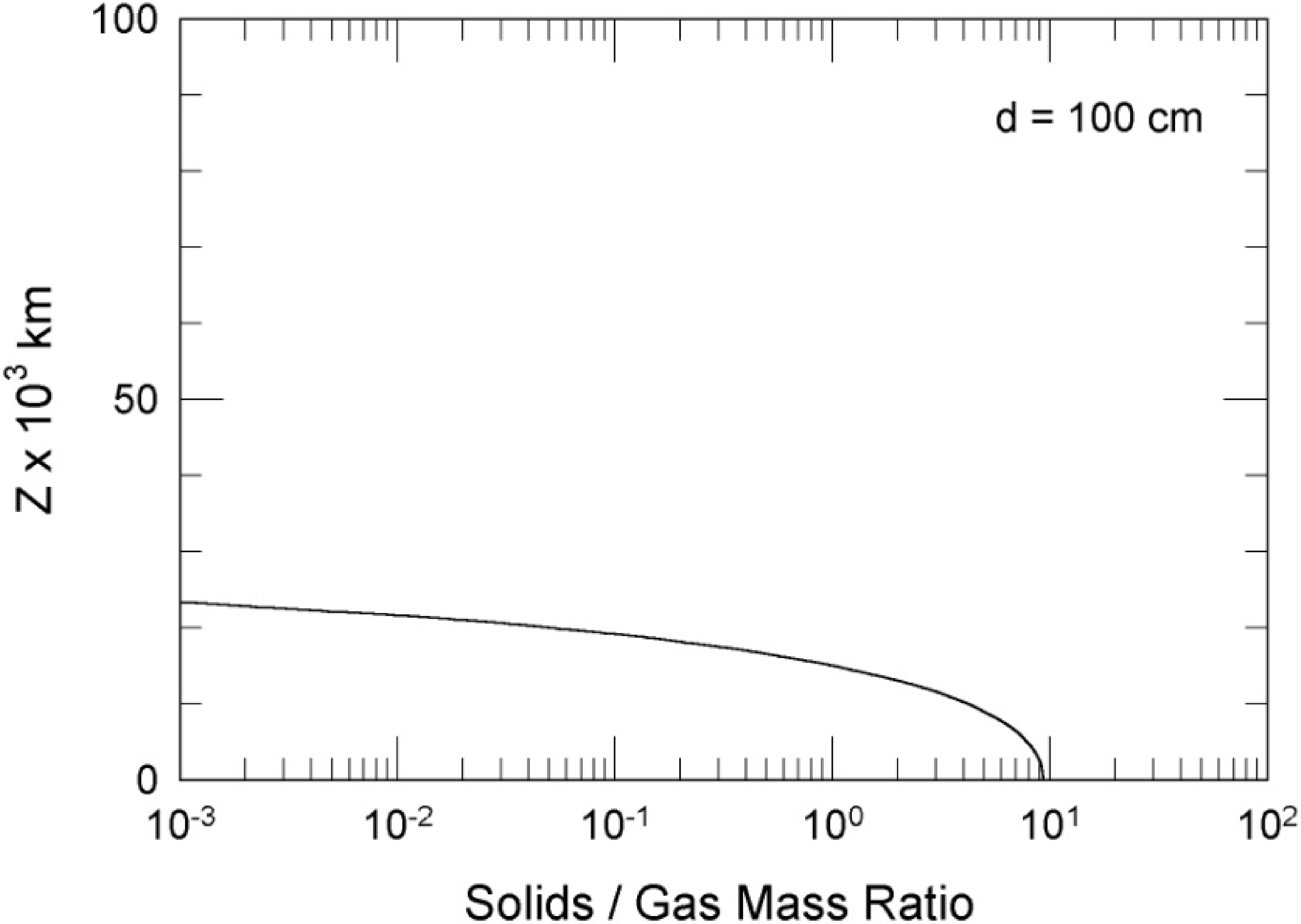

d = 10 cm
100
50
0
Z x 10^3 km
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
Solids / Gas Mass Ratio



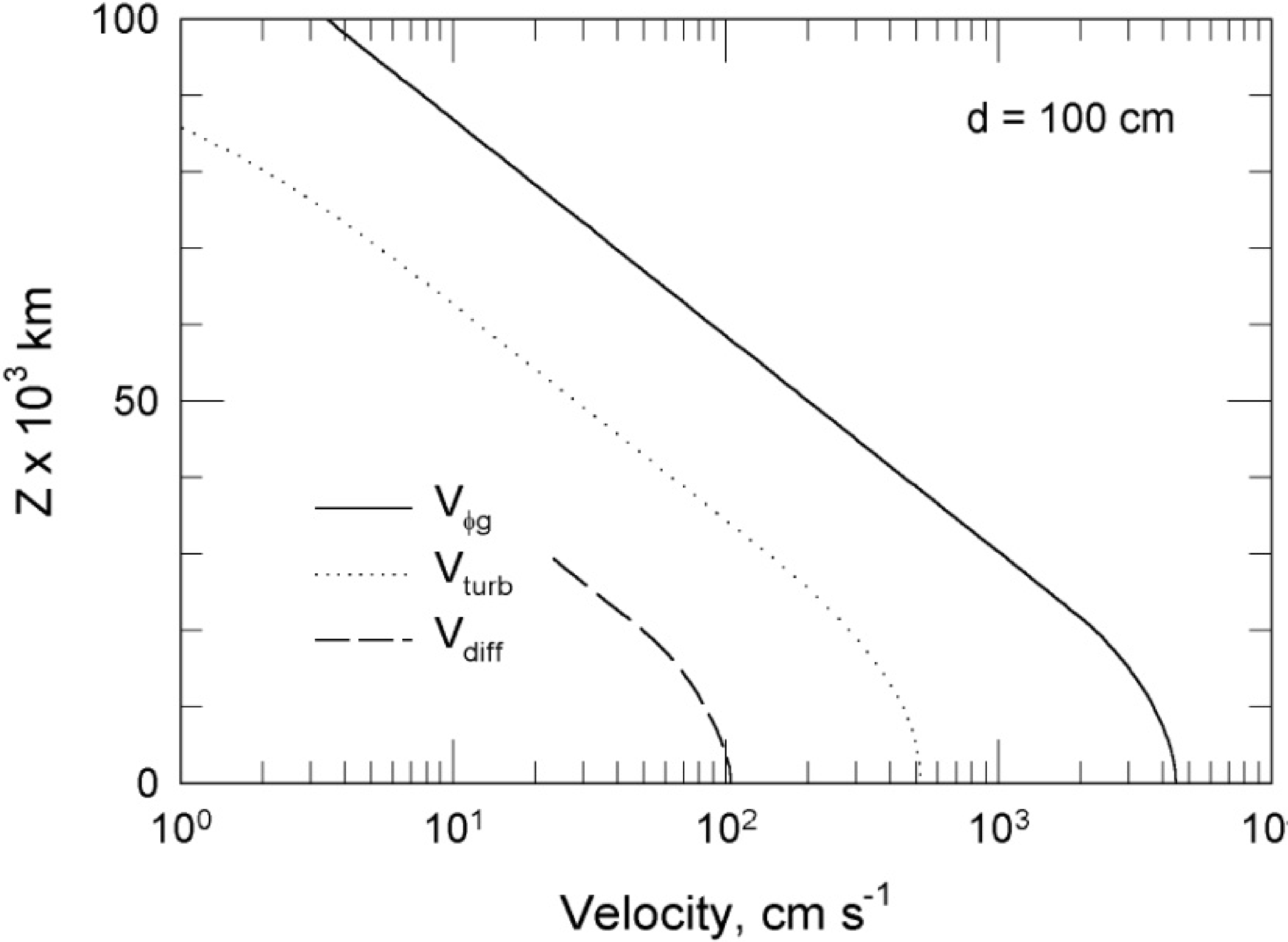
d = 10 cm
$V_{\phi g}$
V_{turb}
V_{diff}
100
50
0
$Z \times 10^3$ km
10^0
10^1
10^2
10^3
10^4
Velocity, cm s^{-1}



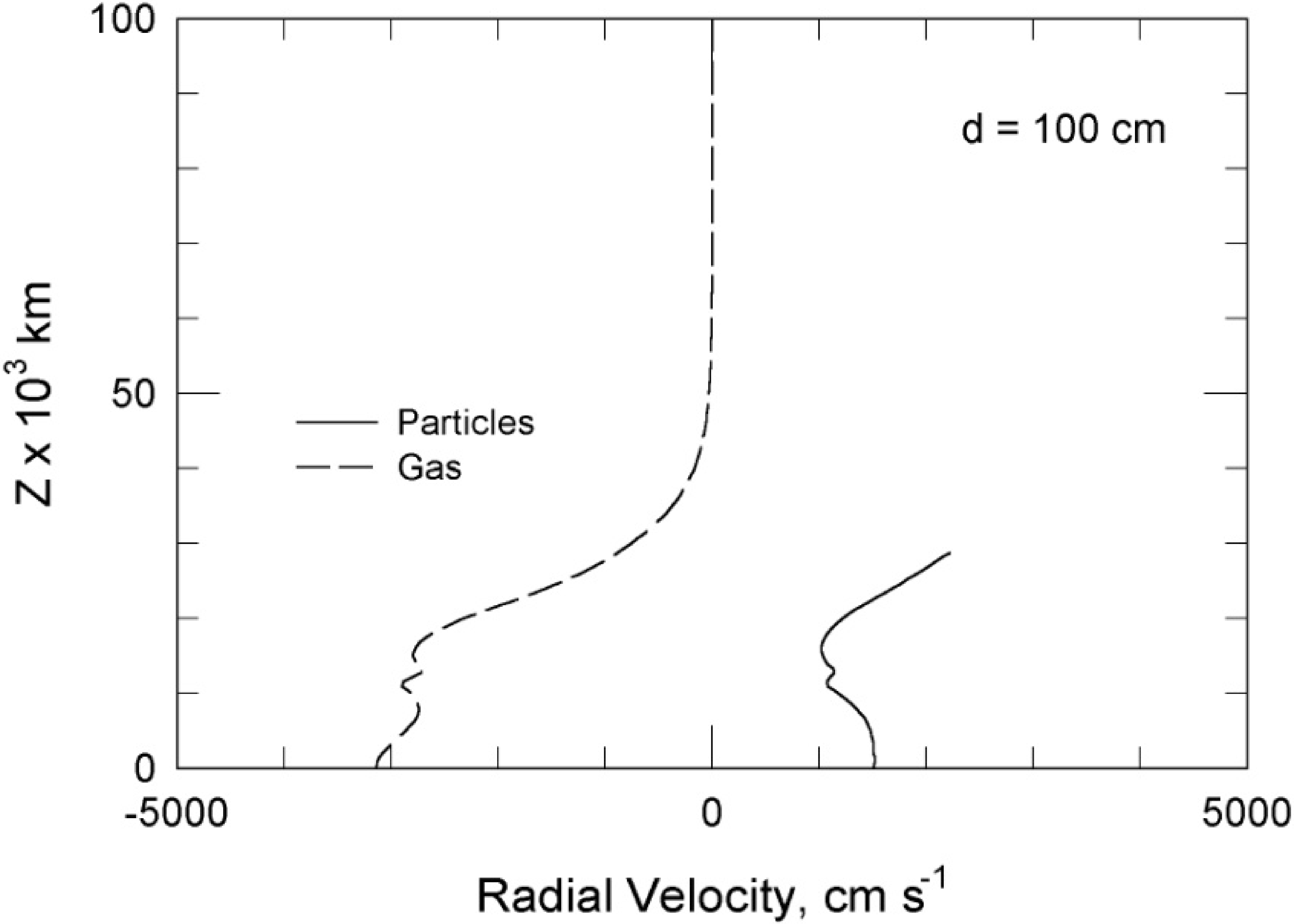
100
50
0
-500
0
500
Z x 10³ km
Radial Velocity, cm s⁻¹
d = 10 cm
Particles
Gas

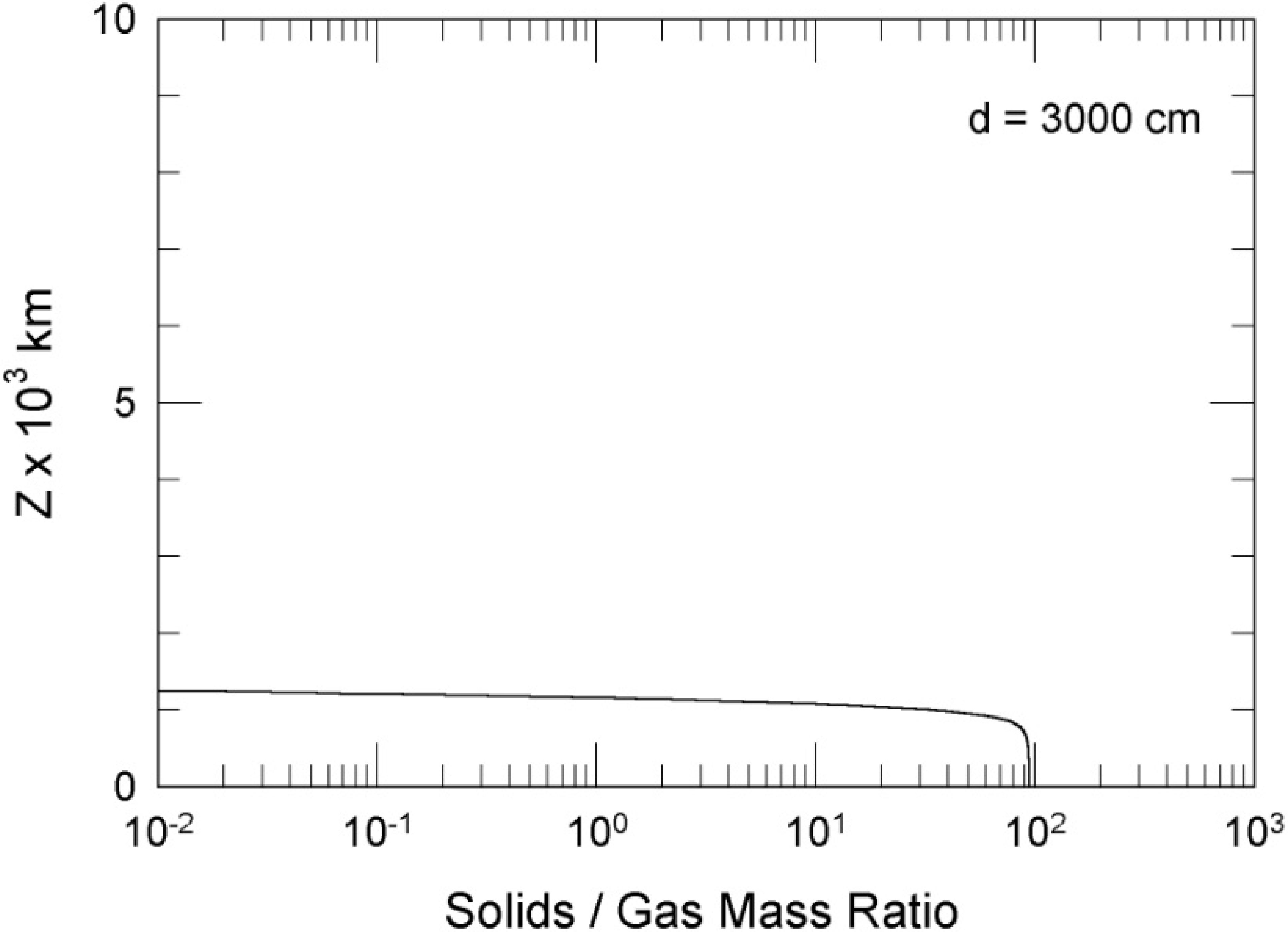


100
50
0
d = 100 cm
Z x 10^3 km
10^-3
10^-2
10^-1
10^0
10^1
10^2
Solids / Gas Mass Ratio

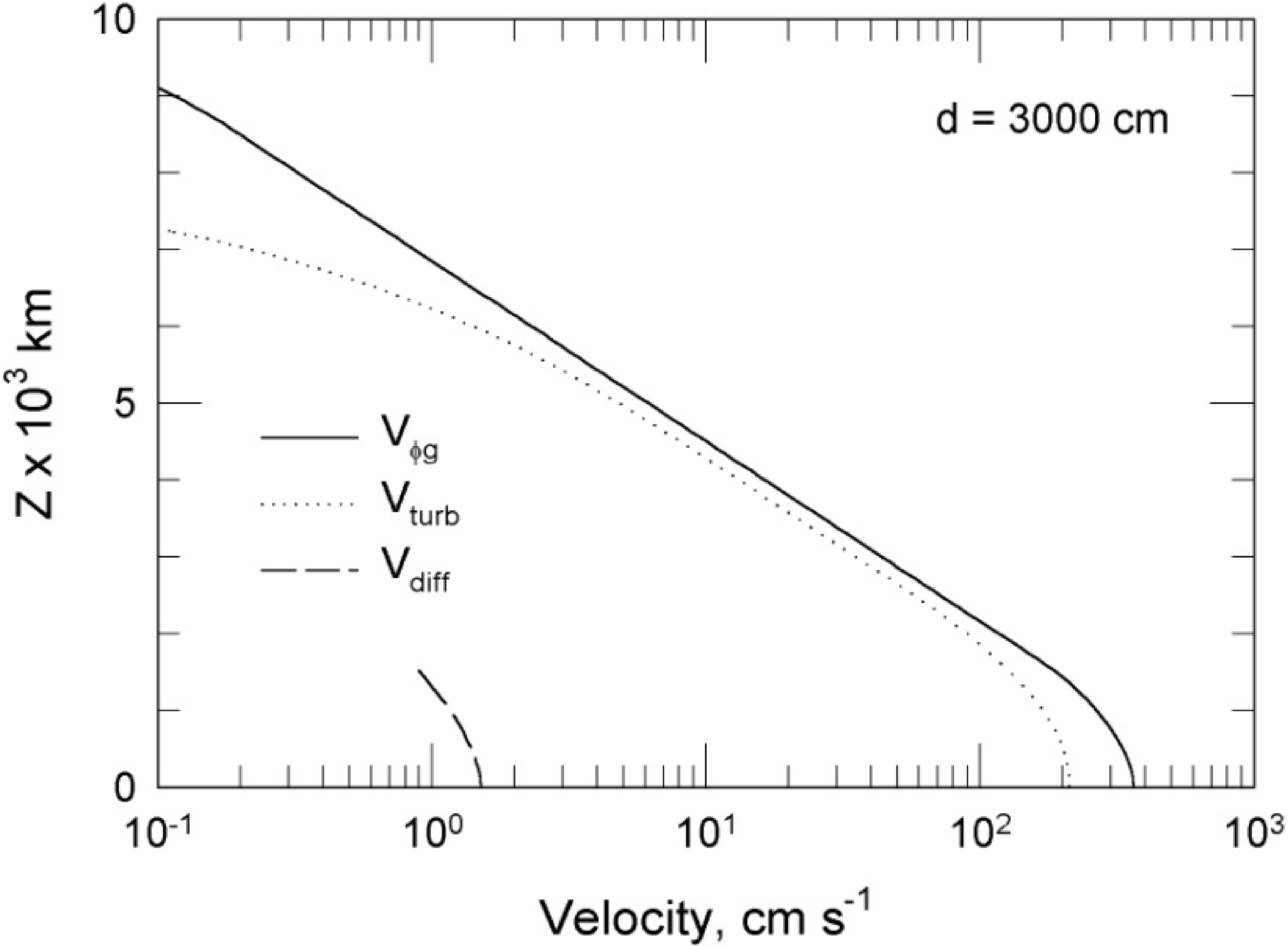


d = 100 cm
$V_{\phi g}$
V_{turb}
V_{diff}
100
50
0
$Z \times 10^3$ km
10^0
10^1
10^2
10^3
10^4
Velocity, cm s^{-1}

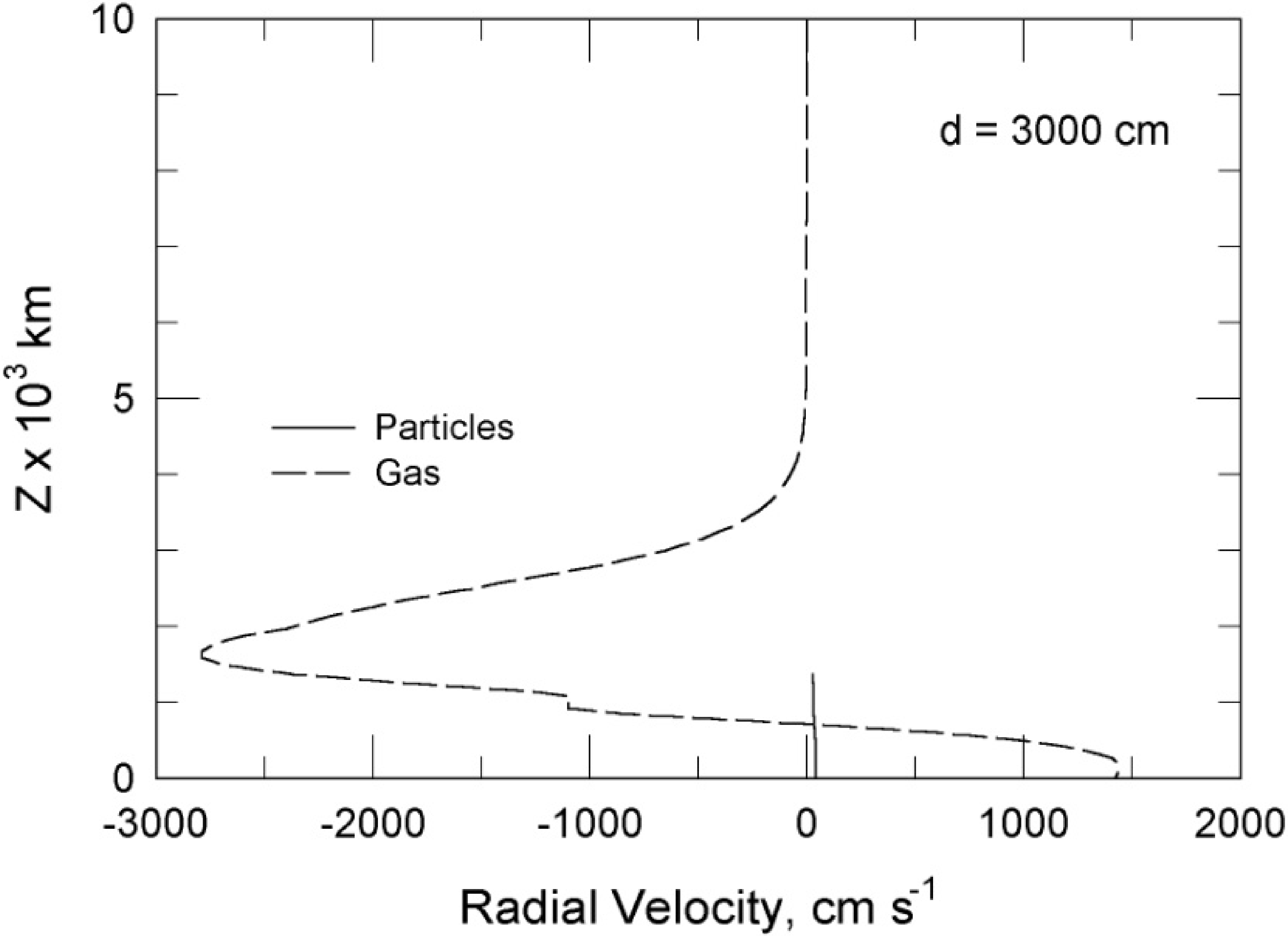

100
50
0
-5000
0
5000
Z x 10^3 km
Radial Velocity, cm s^-1
d = 100 cm
Particles
Gas



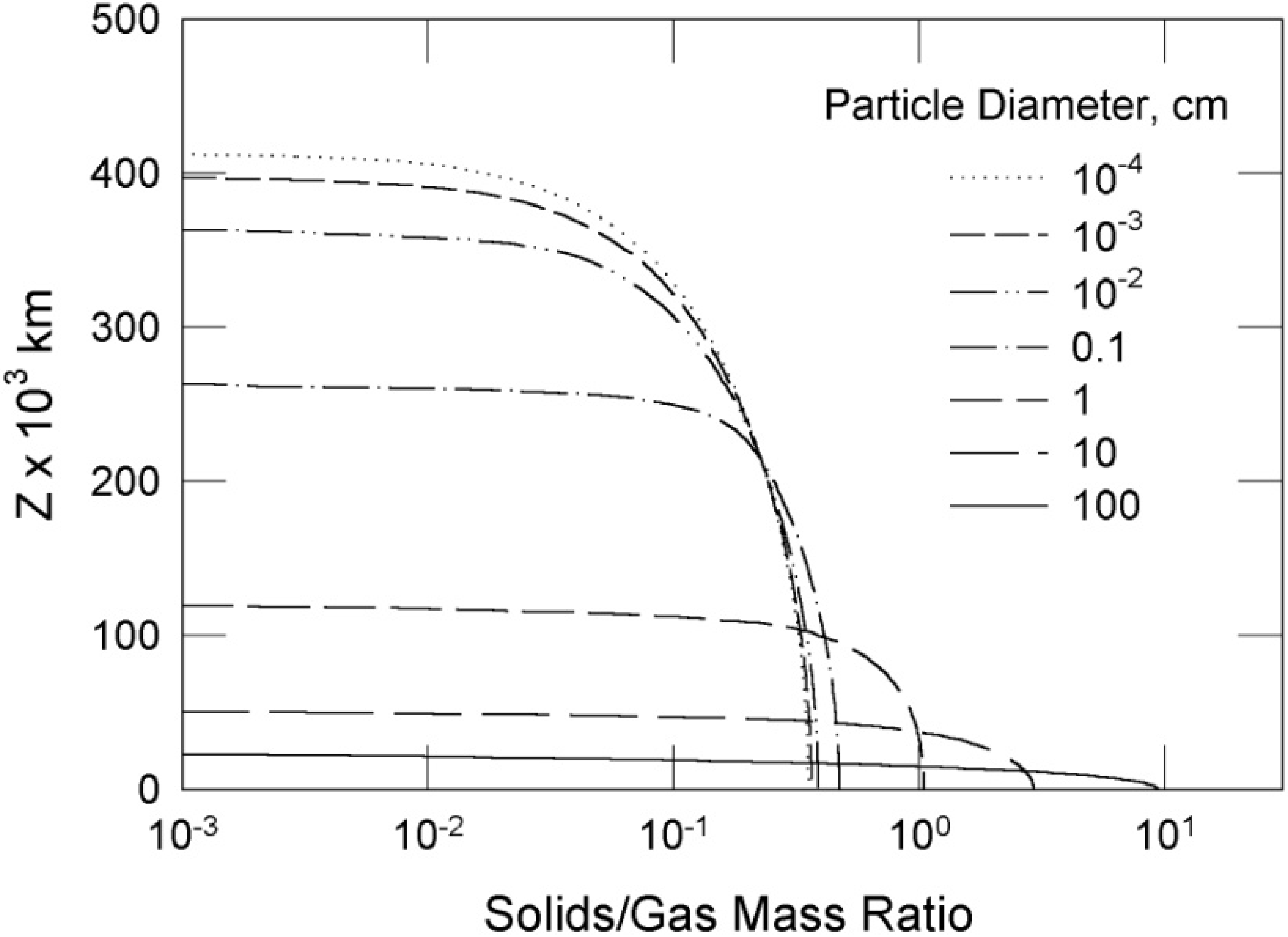
10
5
0
Z x 10^3 km
d = 3000 cm
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
Solids / Gas Mass Ratio



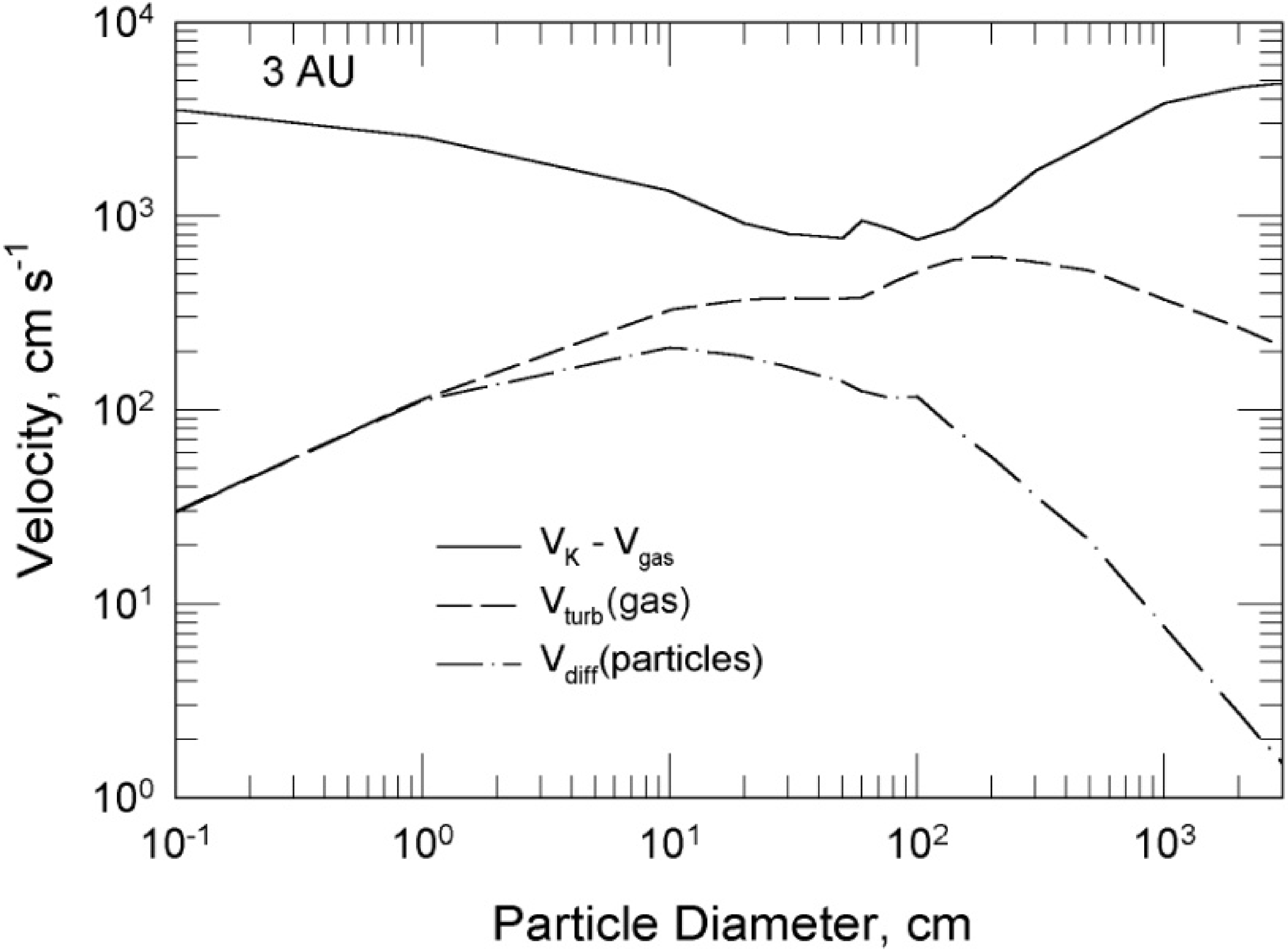
d = 3000 cm
$V_{\phi g}$
V_{turb}
V_{diff}
Z x 10^3 km
Velocity, cm s^{-1}
10
5
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}



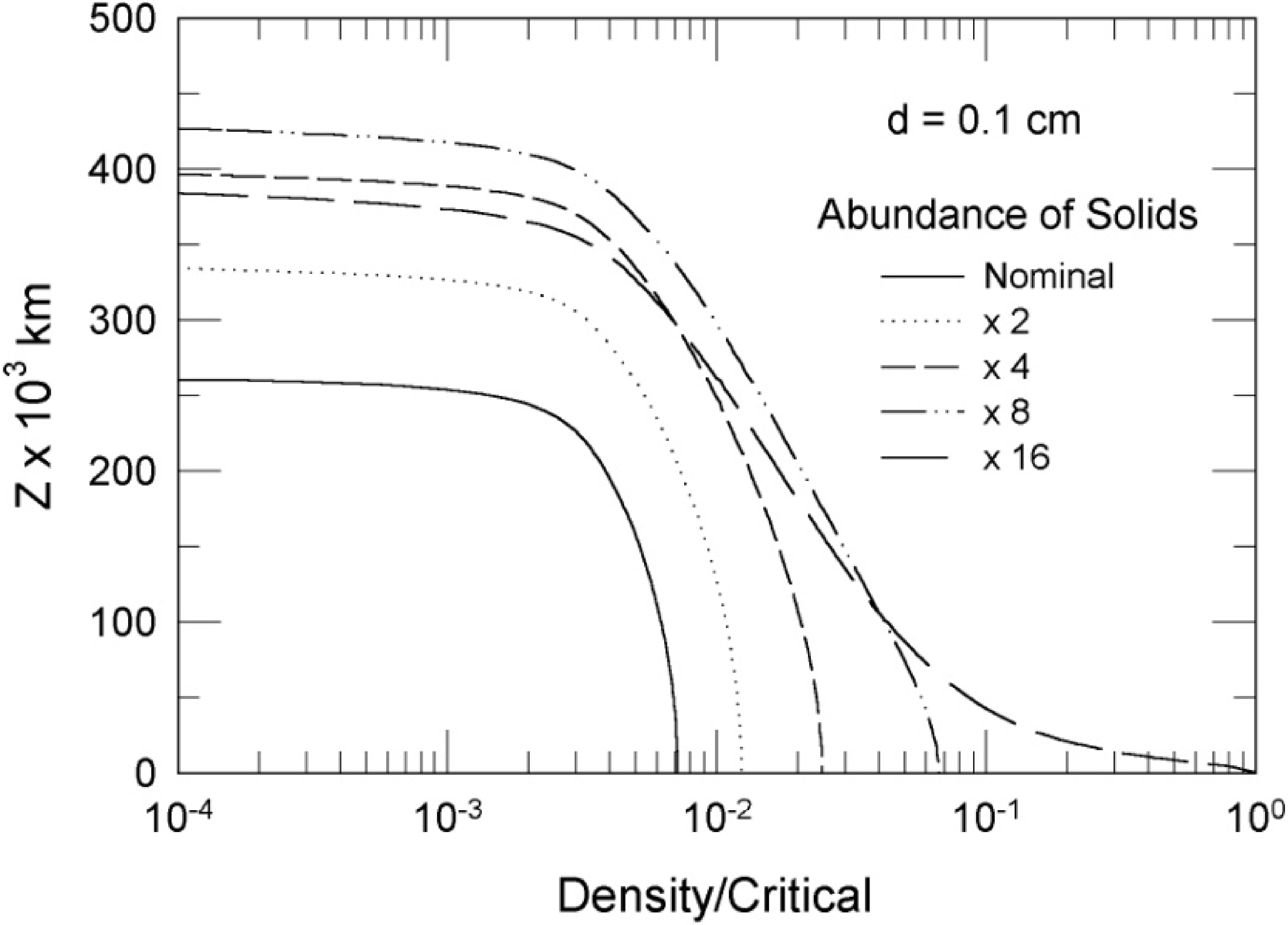
10
5
0
Z x 10^3 km
d = 3000 cm
Particles
Gas
-3000
-2000
-1000
0
1000
2000
Radial Velocity, cm s^{-1}



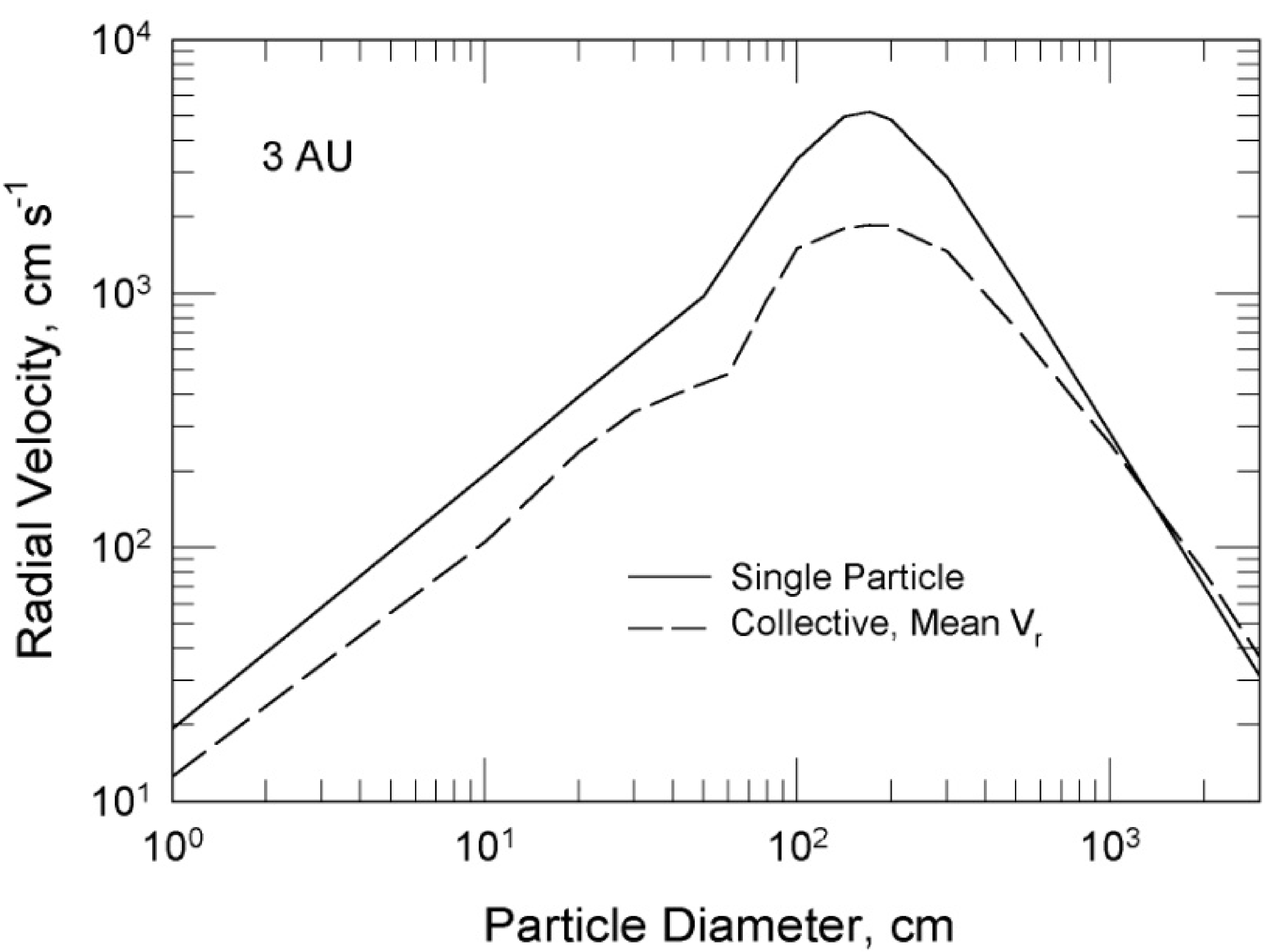
Particle Diameter, cm
10^{-4}
10^{-3}
10^{-2}
0.1
1
10
100
Z x 10^3 km
500
400
300
200
100
0
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Solids/Gas Mass Ratio



3 AU
V_K - V_gas
V_turb(gas)
V_diff(particles)
Velocity, cm s^-1
Particle Diameter, cm
10^4
10^3
10^2
10^1
10^0
10^-1
10^0
10^1
10^2
10^3

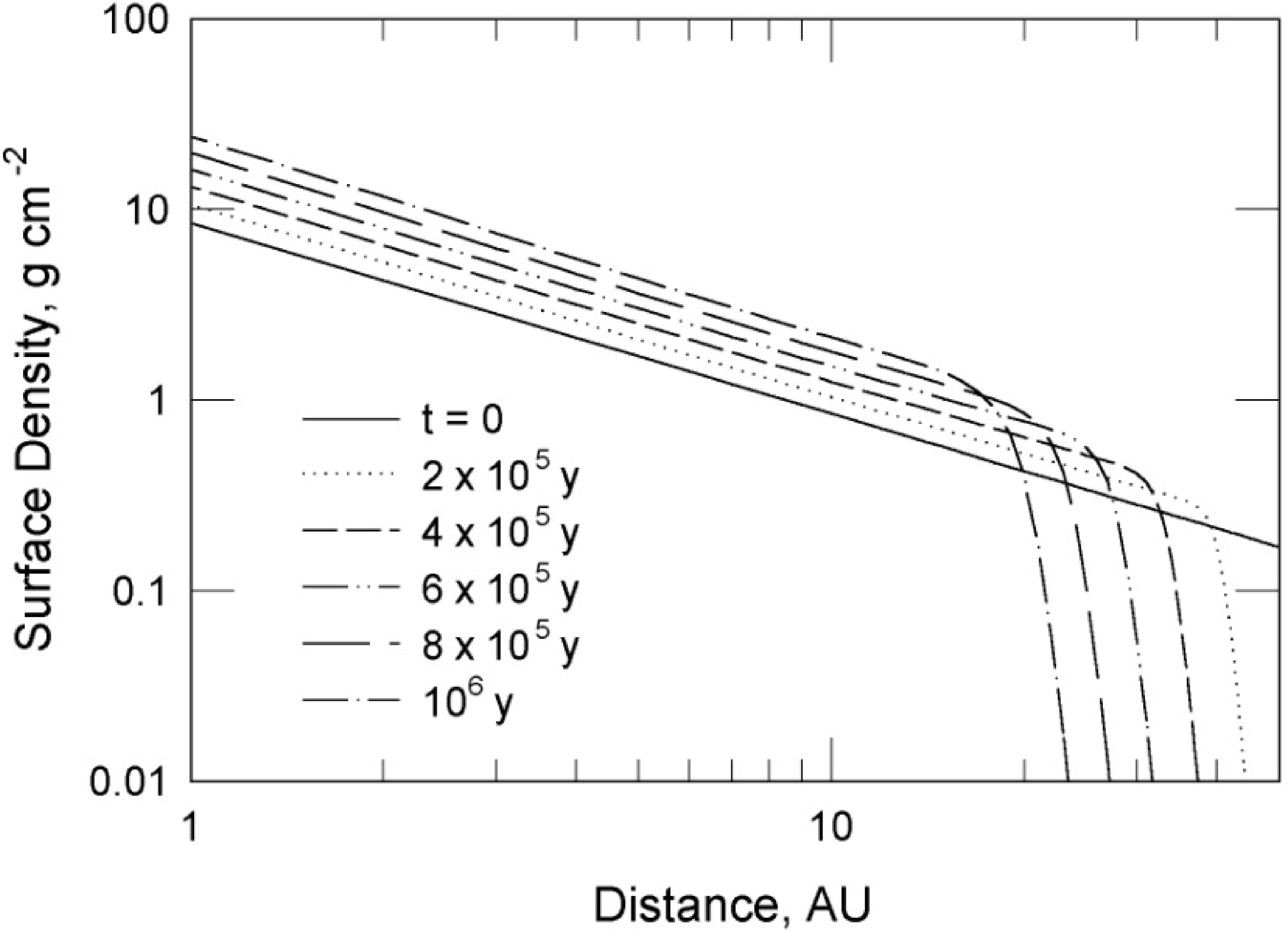


500
400
300
200
100
0
Z x 10^3 km
10^-4
10^-3
10^-2
10^-1
10^0
Density/Critical
d = 0.1 cm
Abundance of Solids
Nominal
x 2
x 4
x 8
x 16

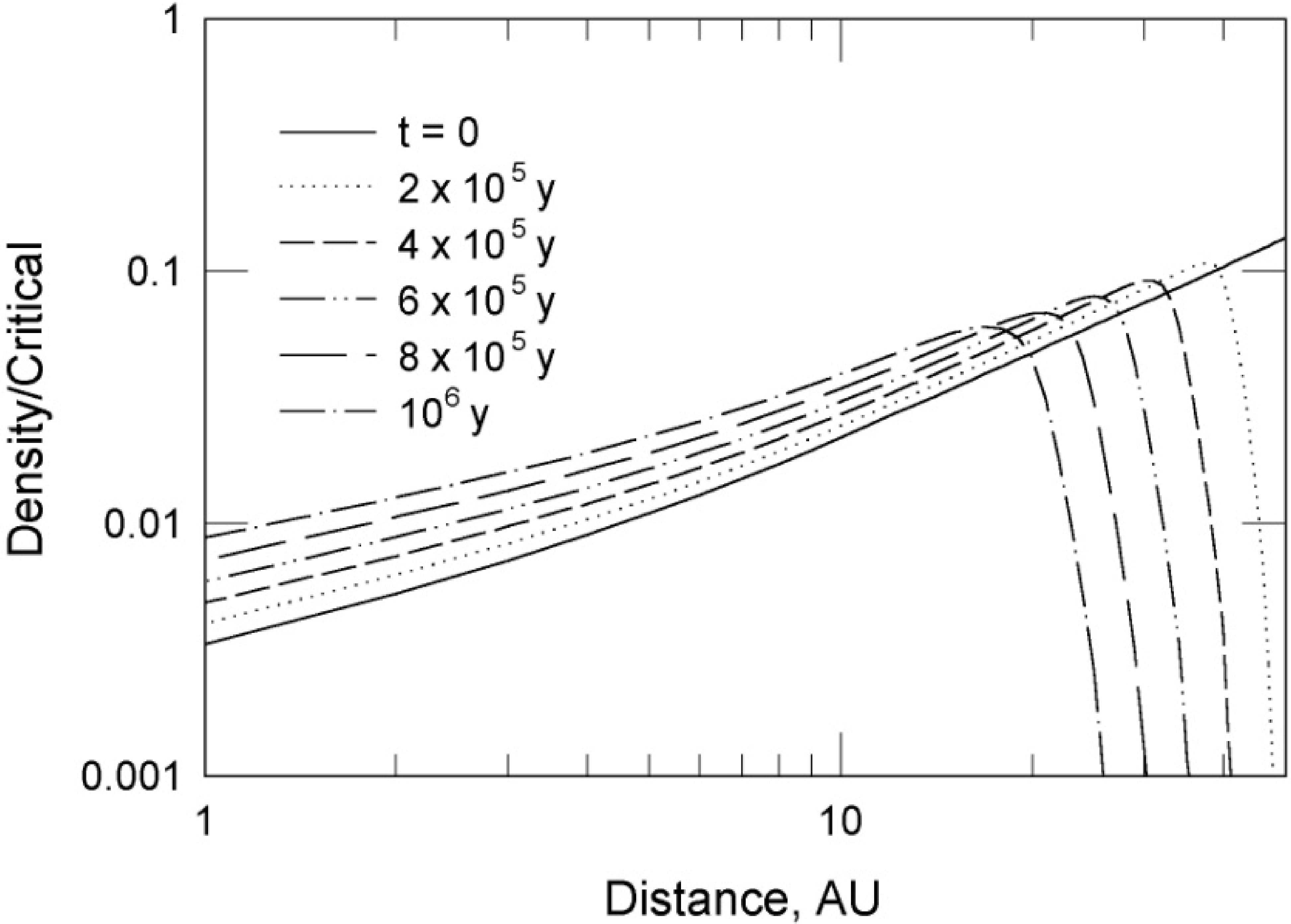


3 AU
Radial Velocity, cm s-1
Particle Diameter, cm
Single Particle
Collective, Mean Vr
10^4
10^3
10^2
10^1
10^0
10^1
10^2
10^3

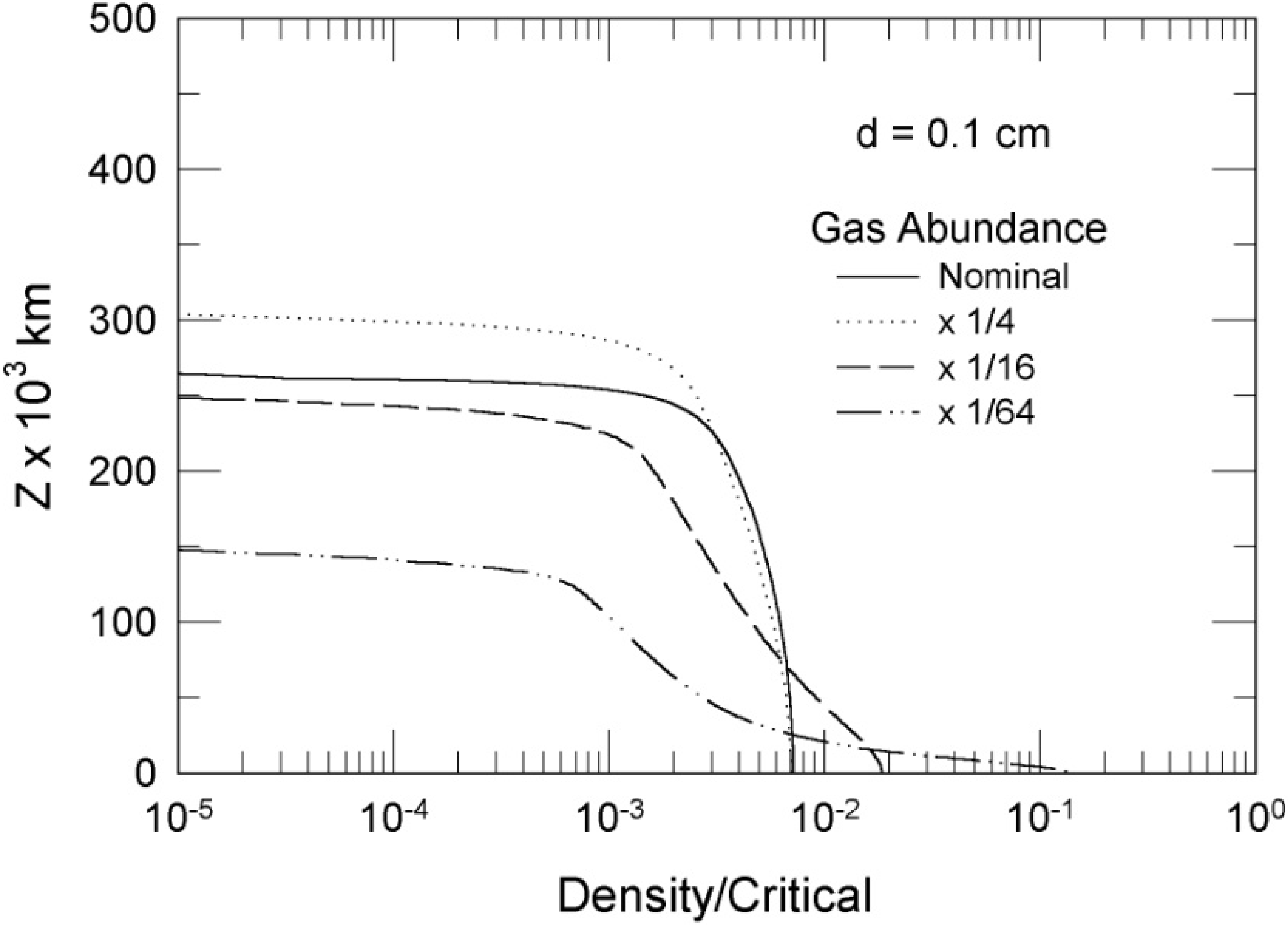
The effective radial velocity is defined as the net mass flux, integrated over all values of Z, divided by the surface density. The numerical model shows that for a layer of small particles ($t_e\Omega < 1$), the effective radial velocity is proportional to particle size. The collective drift velocity varies because the turbulence must have the proper strength to counteract particle settling. Also, the net radial velocity has a significant component due to drift of particles through the gas within the layer, which is proportional to size (or t_e). The effective velocity is somewhat less than that of an isolated particle of that size. This is due to the laminar reaction to the inward drift and the decrease of ΔV resulting from the mass loading.



100
10
1
0.1
0.01
1
10
Surface Density, g cm^-2
Distance, AU
t = 0
2 x 10^5 y
4 x 10^5 y
6 x 10^5 y
8 x 10^5 y
10^6 y



1
0.1
0.01
0.001
Density/Critical
t = 0
2 x 10^5 y
4 x 10^5 y
6 x 10^5 y
8 x 10^5 y
10^6 y
1
10
Distance, AU



d = 0.1 cm
Gas Abundance
Nominal
x 1/4
x 1/16
x 1/64
500
400
300
200
100
0
Z x 10^3 km
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
Density/Critical

Removal of the gas can lead to increased density of the particle layer; however, the effect does not depend simply on the solids/gas ratio. Removing gas decreases its density, increasing the response time t_e and the Stokes number - the particles behave as if they are larger. removing 99% of the gas makes mm-sized particles behave like decimeter-sized bodies in the standard nebula. Their drift velocities are correspondingly larger, as are the turbulent velocities produced by shear. If the particles are not identical, the velocity dispersion due to size differences also increases. If more gas is removed, eventually even small particles will have $t_e\Omega_K >$, and velocities will decrease.

Plate Drag Approximation

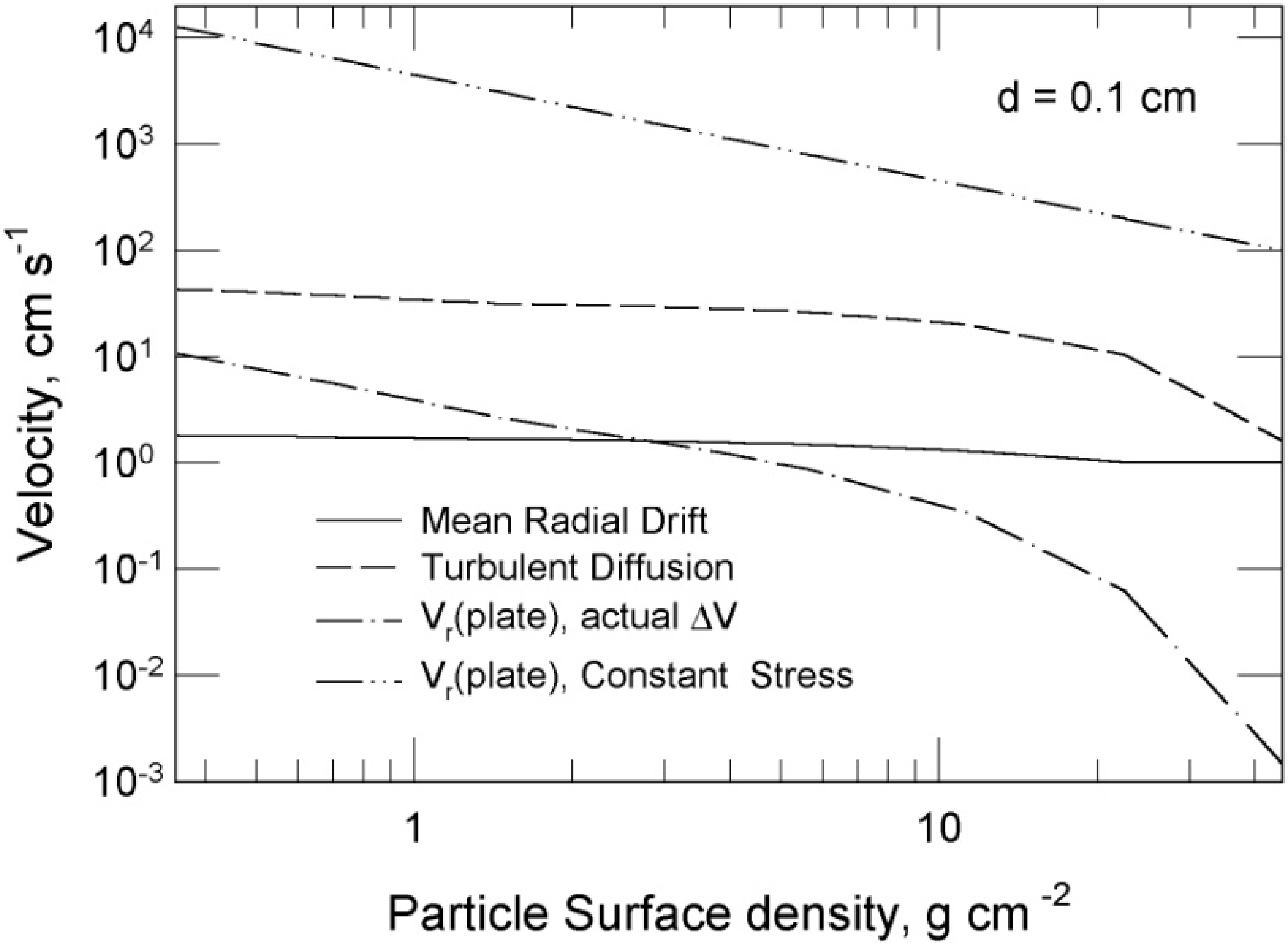
Plate drag assumes that the layer can be treated as an opaque solid rotating disk. Usually it is assumed to be rotating at the Kepler velocity, with a turbulent boundary layer of thickness equal to the Ekman length L_E. The turbulent velocity in the boundary layer is ~ $\Delta V/Re^*$, giving a turbulent viscosity $\nu_t \sim \Delta V L_E/Re^*$. The turbulent stress is then $S \sim \rho_g \Delta V^2/Re^*$. This stress acting on the disk removes angular momentum, causing it to move inward at a velocity $dR/dt \sim S/\sigma_p\Omega_K$.

Note that the plate drag model implies that the radial velocity is independent of particle size, and varies inversely with the surface density of the layer (mass flux is independent of σ_p).

Drag Instability in the Particle Layer?

Goodman and Pindor (2000) proposed that drag acting on a particle layer could produce secular instability. If the plate drag model is applicable, the radial velocity of the layer varies inversely with surface density. If a region has a slightly higher density, then that region migrates inward more slowly. Particles from a less dense region farther out will overtake it, adding to the density. A linear stability analysis suggested that a particle layer would rapidly separate into dense rings with widths comparable to the thickness of the layer.

This analysis depends on the plate drag assumption, and neglects mixing due to nonuniform particle sizes.



d = 0.1 cm
Velocity, cm s-1
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
1
10
Particle Surface density, g cm -2
Mean Radial Drift
Turbulent Diffusion
Vr(plate), actual ΔV
Vr(plate), Constant Stress

The model shows no significant variation in effective drift velocity with surface density of the particle layer (less than a factor of 2 for σ_p varied by factor 100). Any increase in surface density changes the structure of the layer and the strength of turbulence in a manner that keeps the mean velocity constant (mass flux is proportional to σ_p rather than constant). There is no tendency for particles to pile up at density perturbations in the layer. The drag instability mechanism appears to depend on the plate drag assumption.

2-D Models with Coagulation

Nebula divided into radial zones of heliocentric distance. Each zone is divided into a series of levels, from the midplane to 2 scale heights of the gas (assume gaussian density profile with Z), with finer resolution closer to the midplane to resolve the structure of the particle layer (cf. Weidenschilling 1997).

Particles settle toward the midplane, and migrate radially between zones. Most motion is downward and inward, but diffusion occurs along concentration gradients in turbulence.

Particle size distribution modeled by logarithmic diameter bins, from 10^{-4} cm to ~ 1000 km.
Particles have low-density fractal structure at $d < 1$ cm.

Start with all solids present as μm-sized grains, mixed with gas (uniform solid/gas ratio) at all *R, Z.*

Particles collide due to thermal motions, differential settling, radial and transverse motions due to drag, and turbulence where present.

Simulations in outer nebula, beyond the "snow line," assume solids/gas ratio 0.015.

Collisional Outcomes

Outcomes of collisions depend on particle sizes and impact velocities.

Small particles have a velocity threshold for perfect sticking, according to the model of Dominik and Tielens
(ApJ 480, 647-673, 1997).

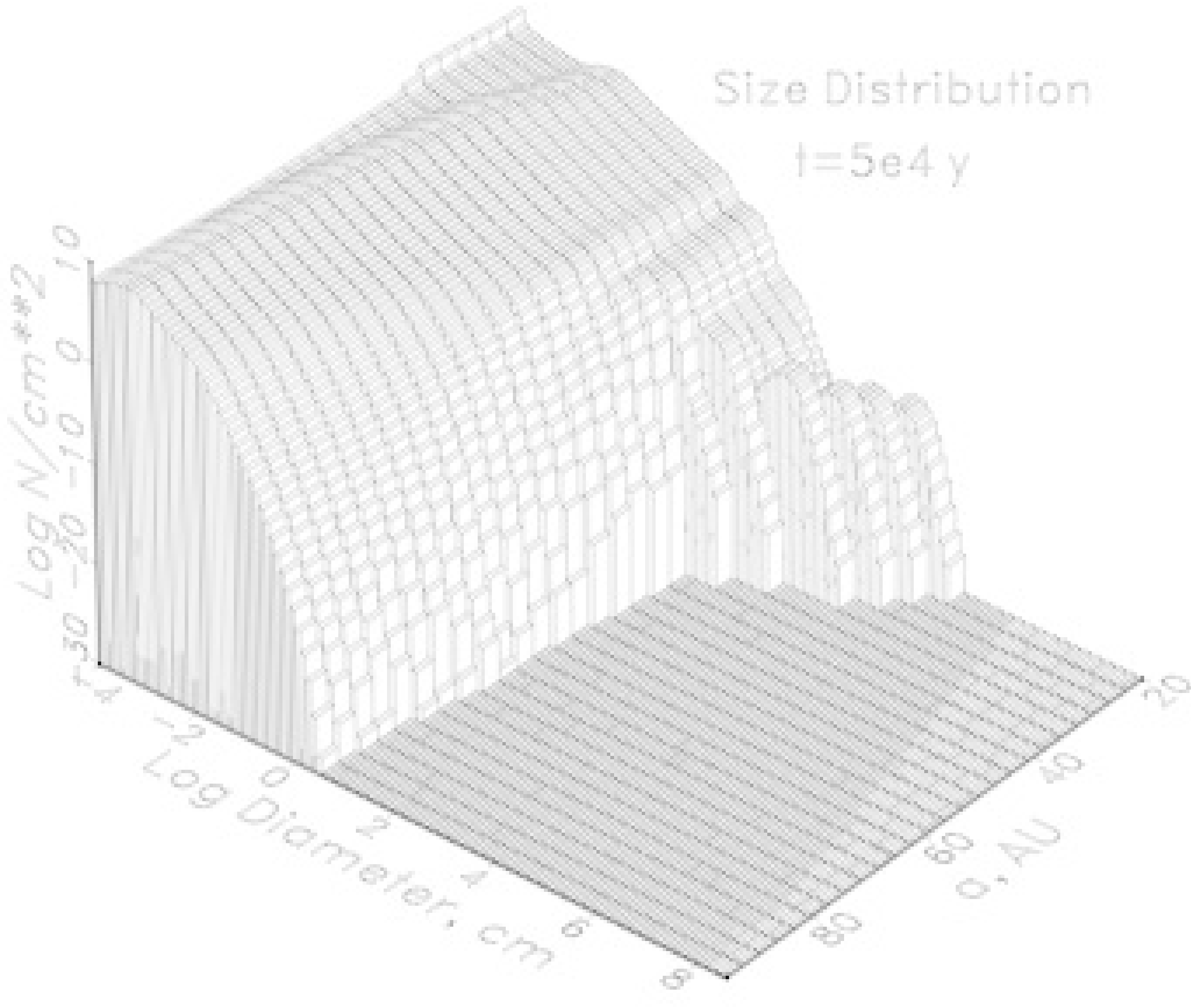
Particles have an assumed impact strength (erg/g) for collisional disruption. Disrupted bodies are assumed to have a power law fragment size distribution.

Projectile mass is added to target, and mass proportional to impact energy escapes as fragments. There is a critical velocity for transition from net mass gain to net loss.

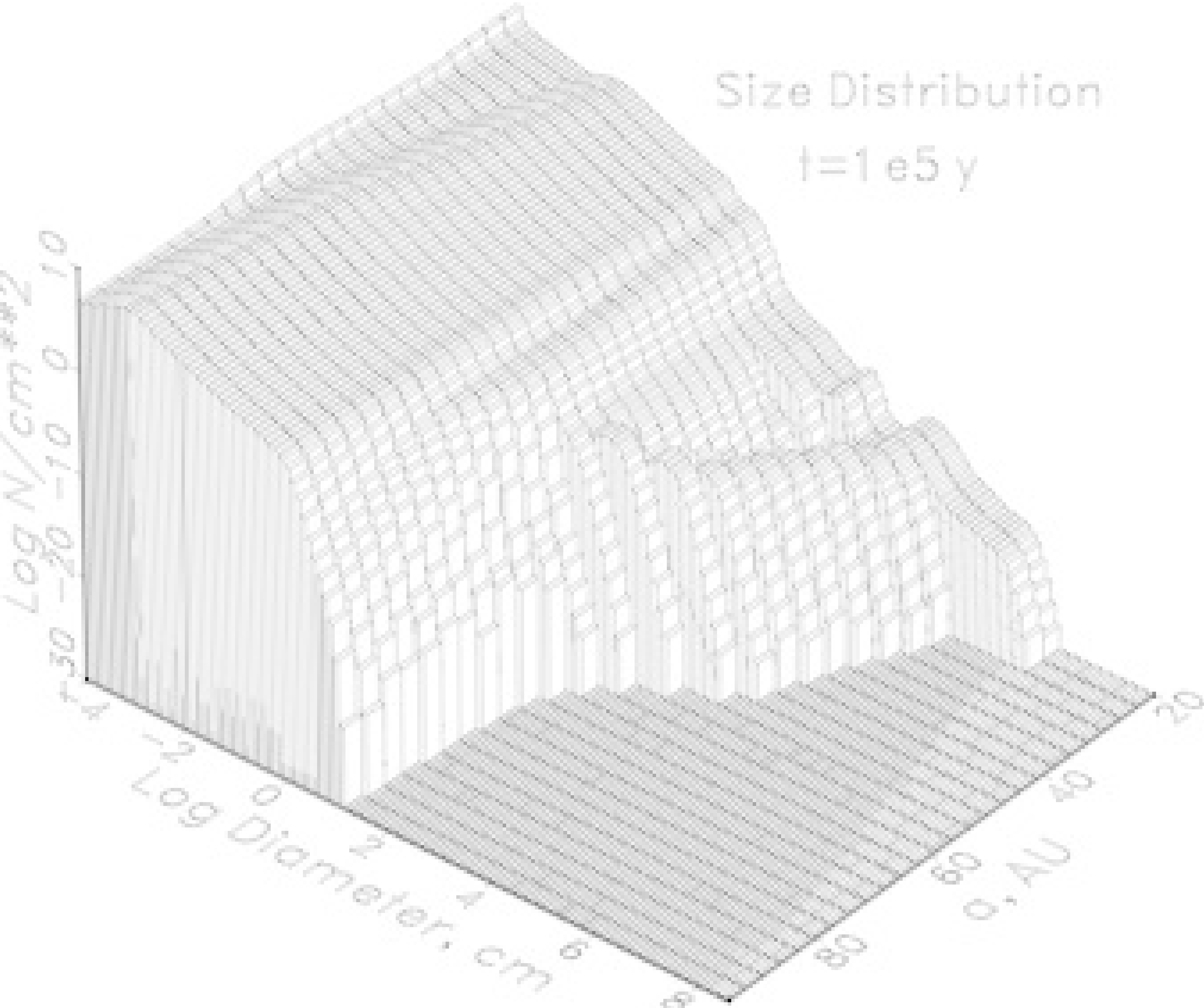
In most simulations, mass in each size bin is transported between zones at a rate proportional to the radial drift velocity due to gas drag.

The following shows results of a simulation without radial drift. The coagulation and settling are computed within each zone, but no mass is transported between zones.

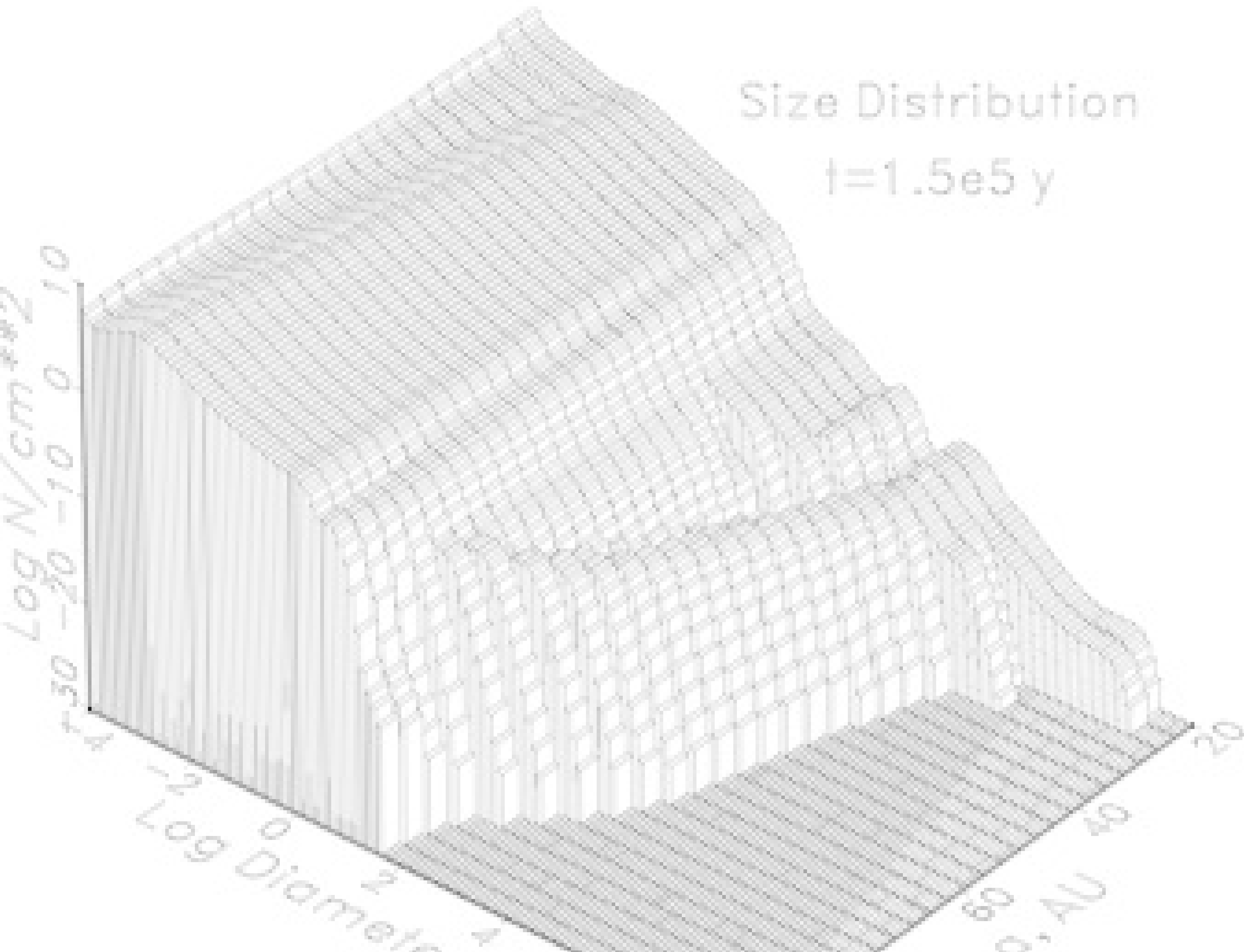
The nebula is assumed to be laminar, except for turbulence generated locally in the midplane by shear.



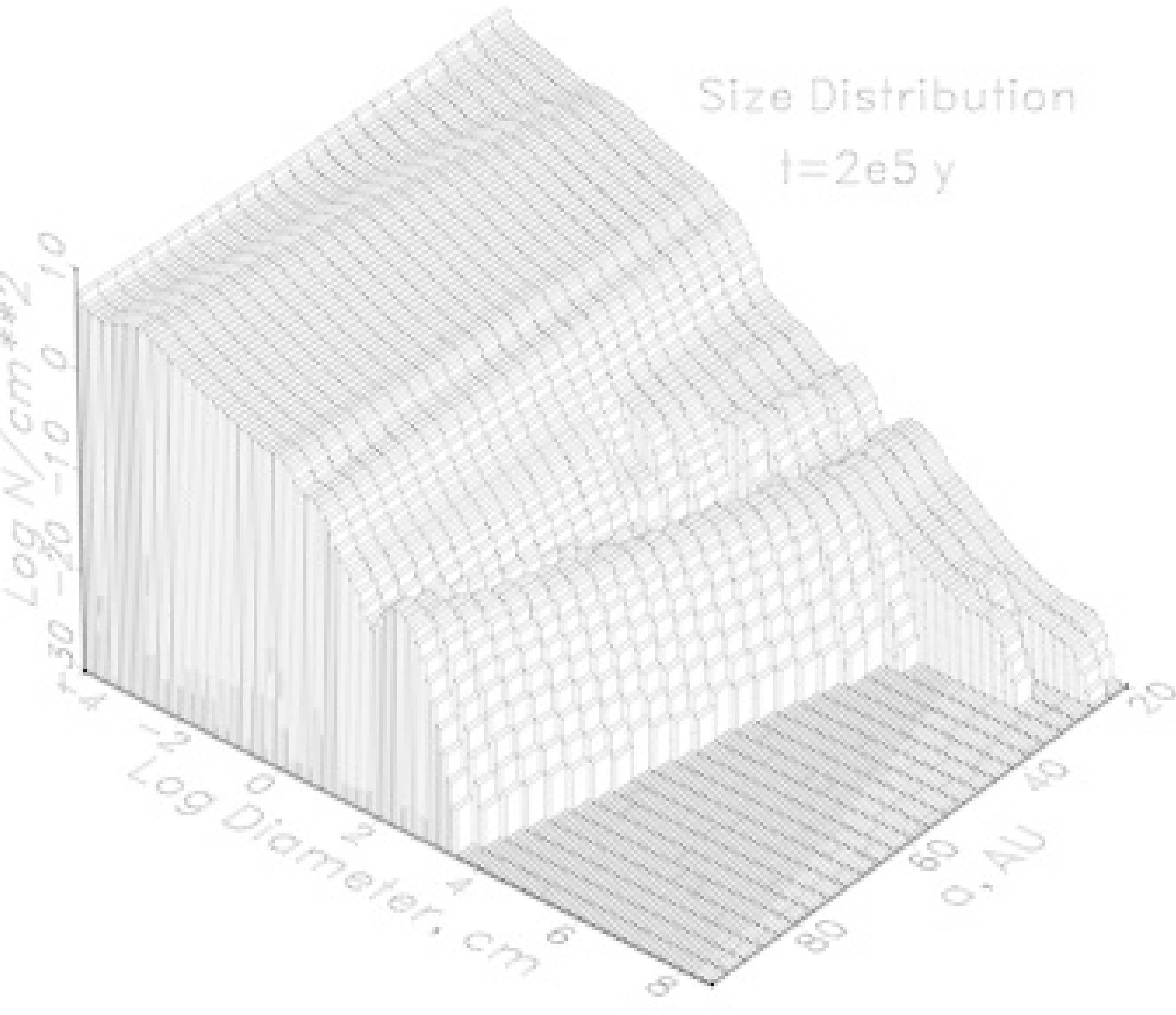
Size Distribution
t=5e4 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU



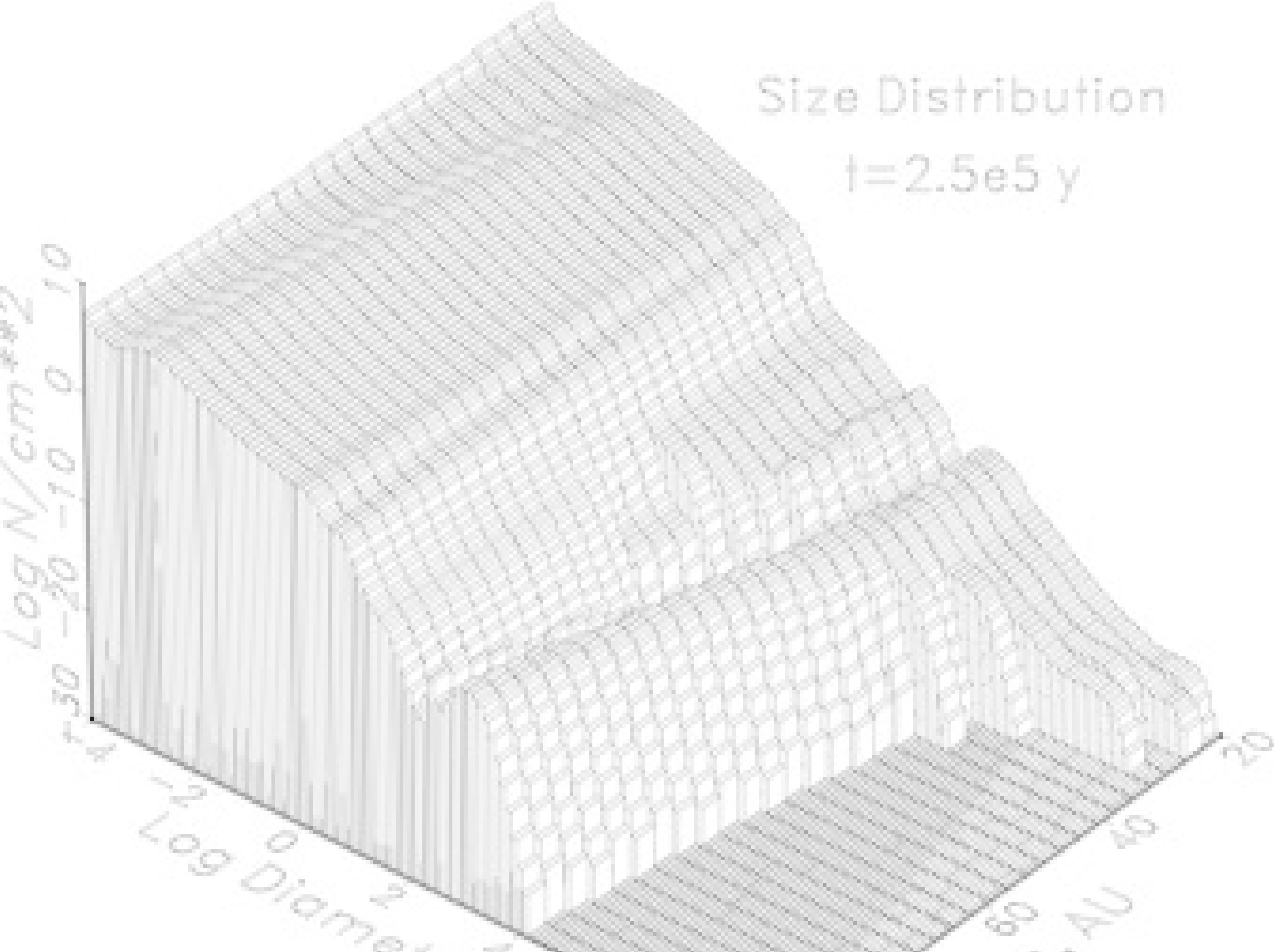
Size Distribution
t=1 e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU



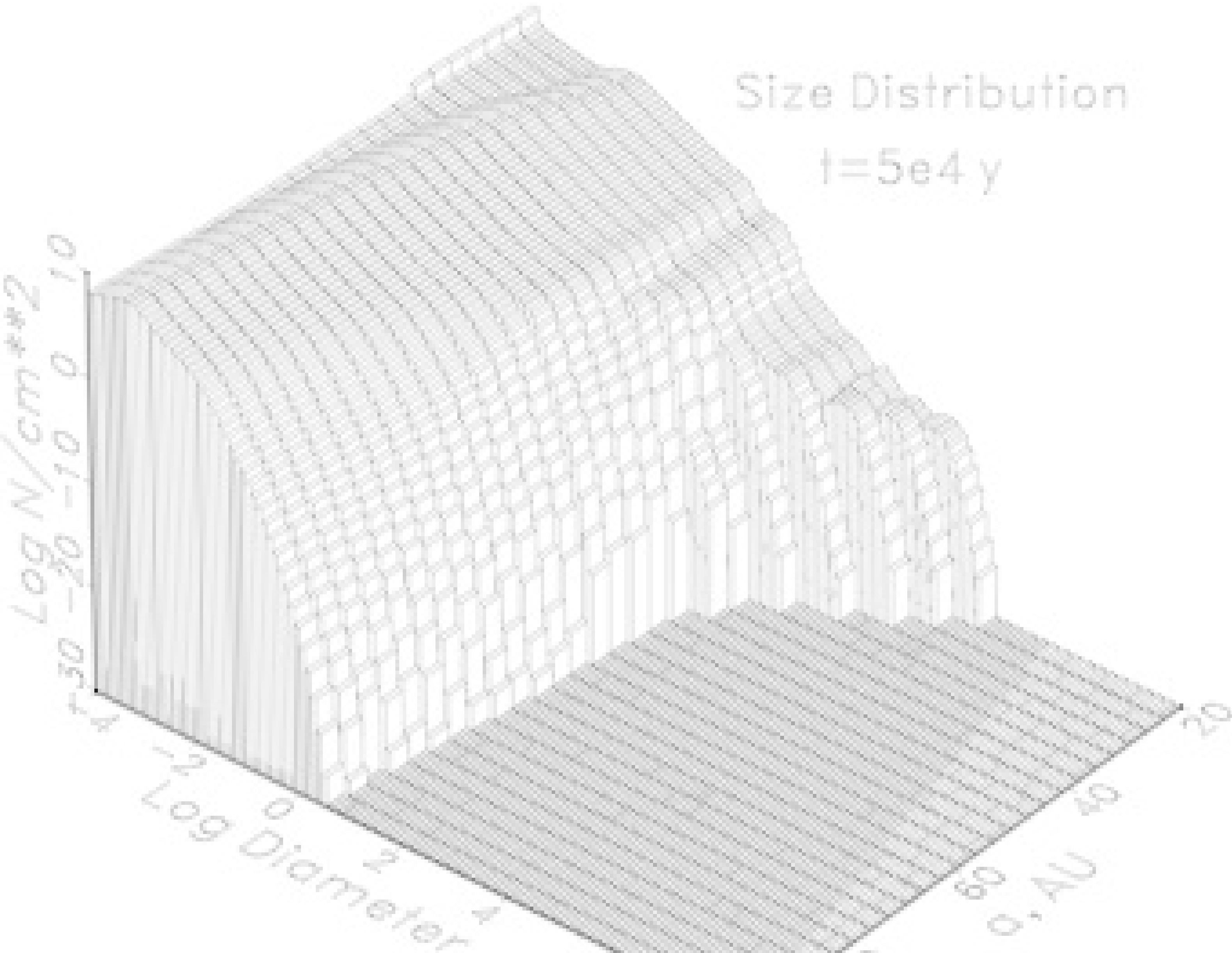
Size Distribution
t=1.5e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU



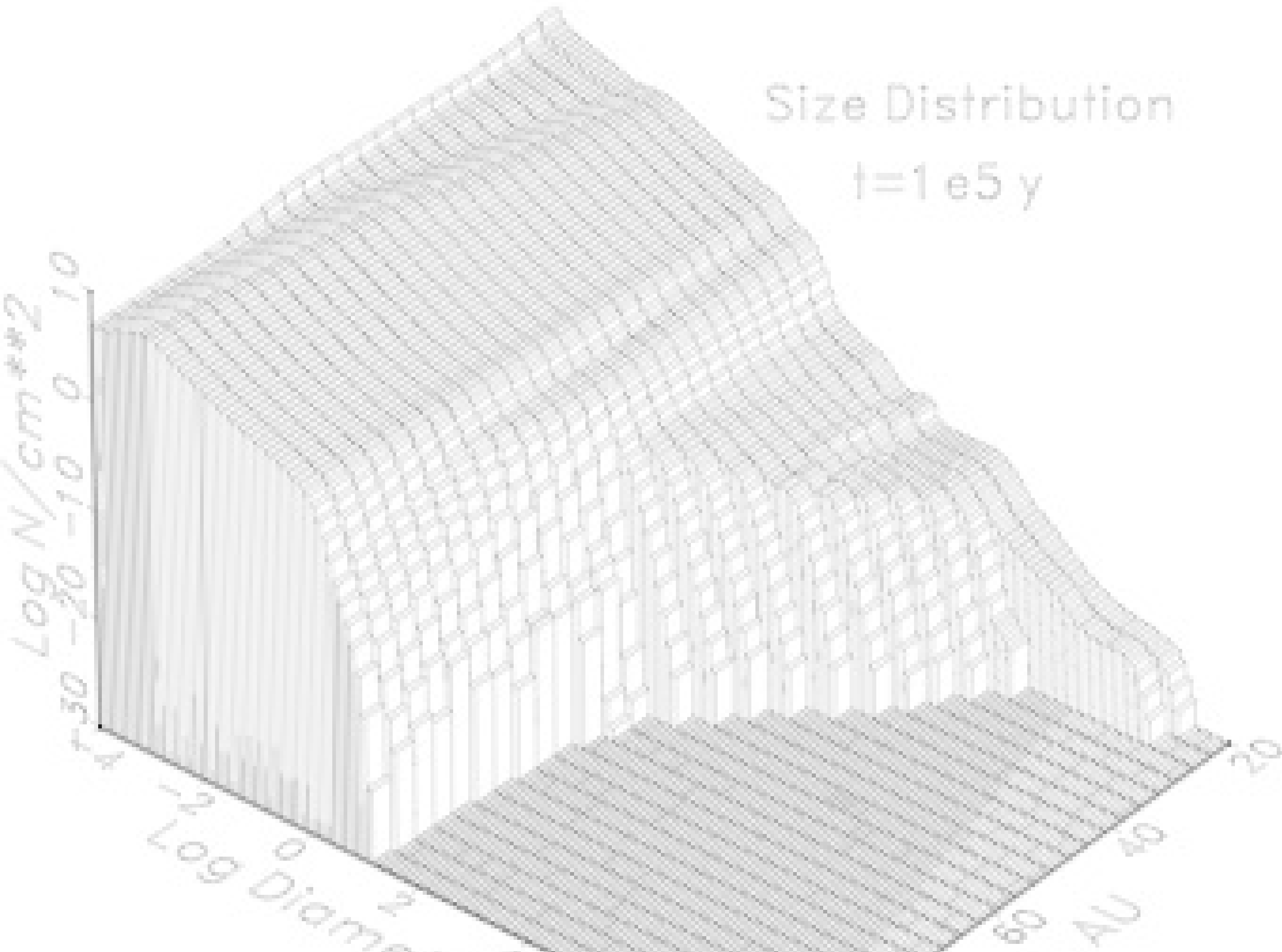
Size Distribution
t=2e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU


Size Distribution
t=2.5e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU

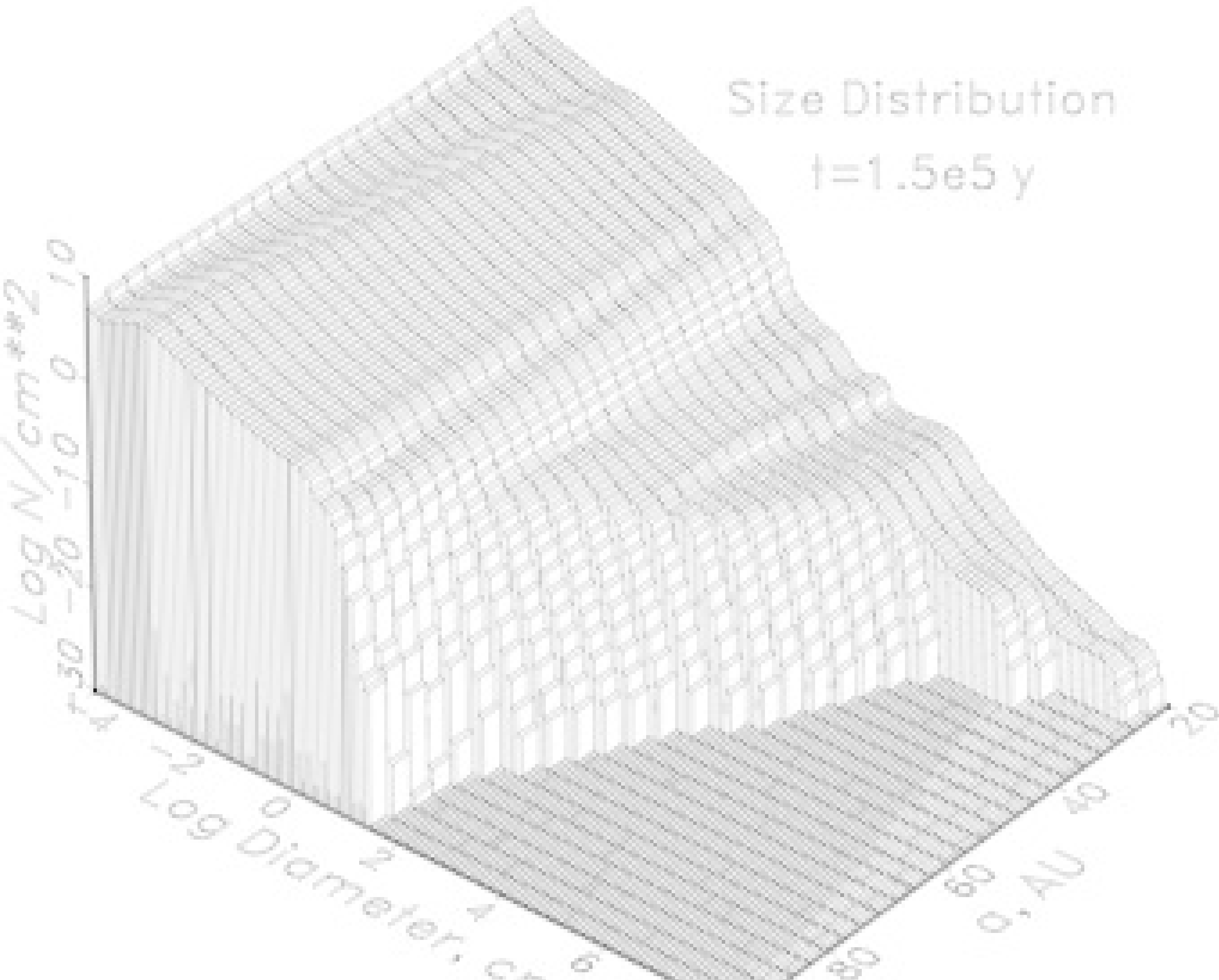
Here we see a simulation with the same parameters, but radial mass transport is included.

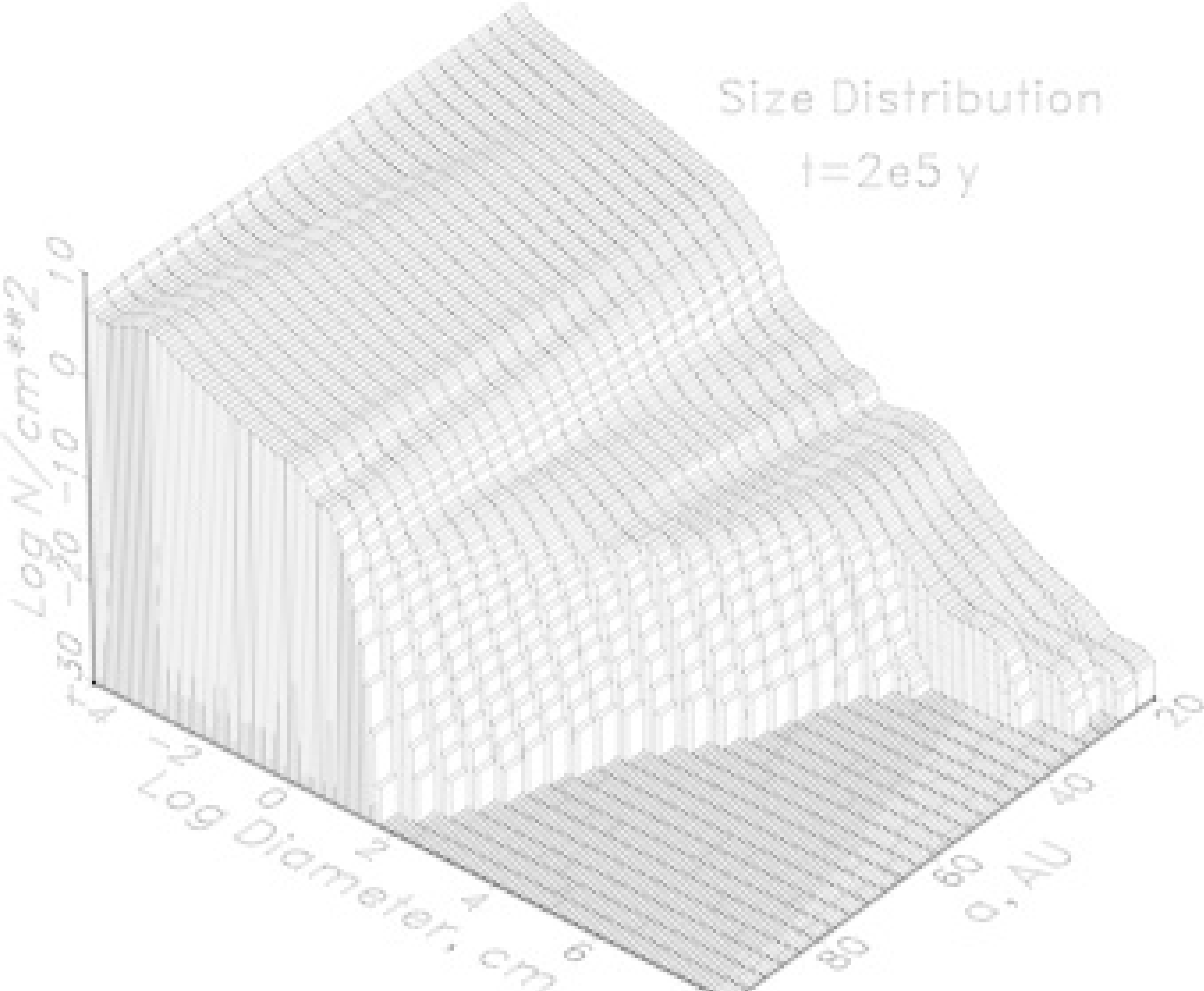


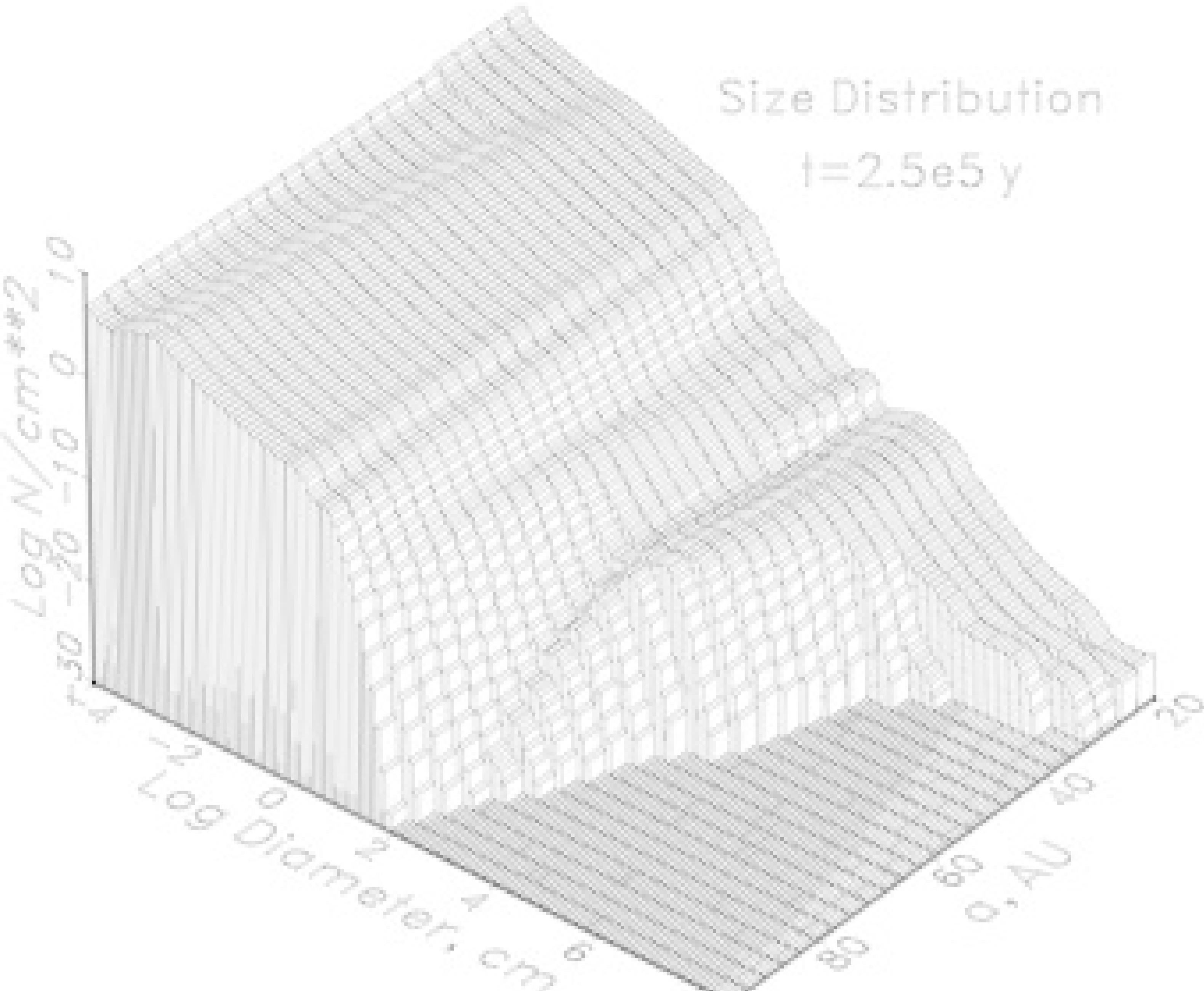
Size Distribution
t=5e4 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU



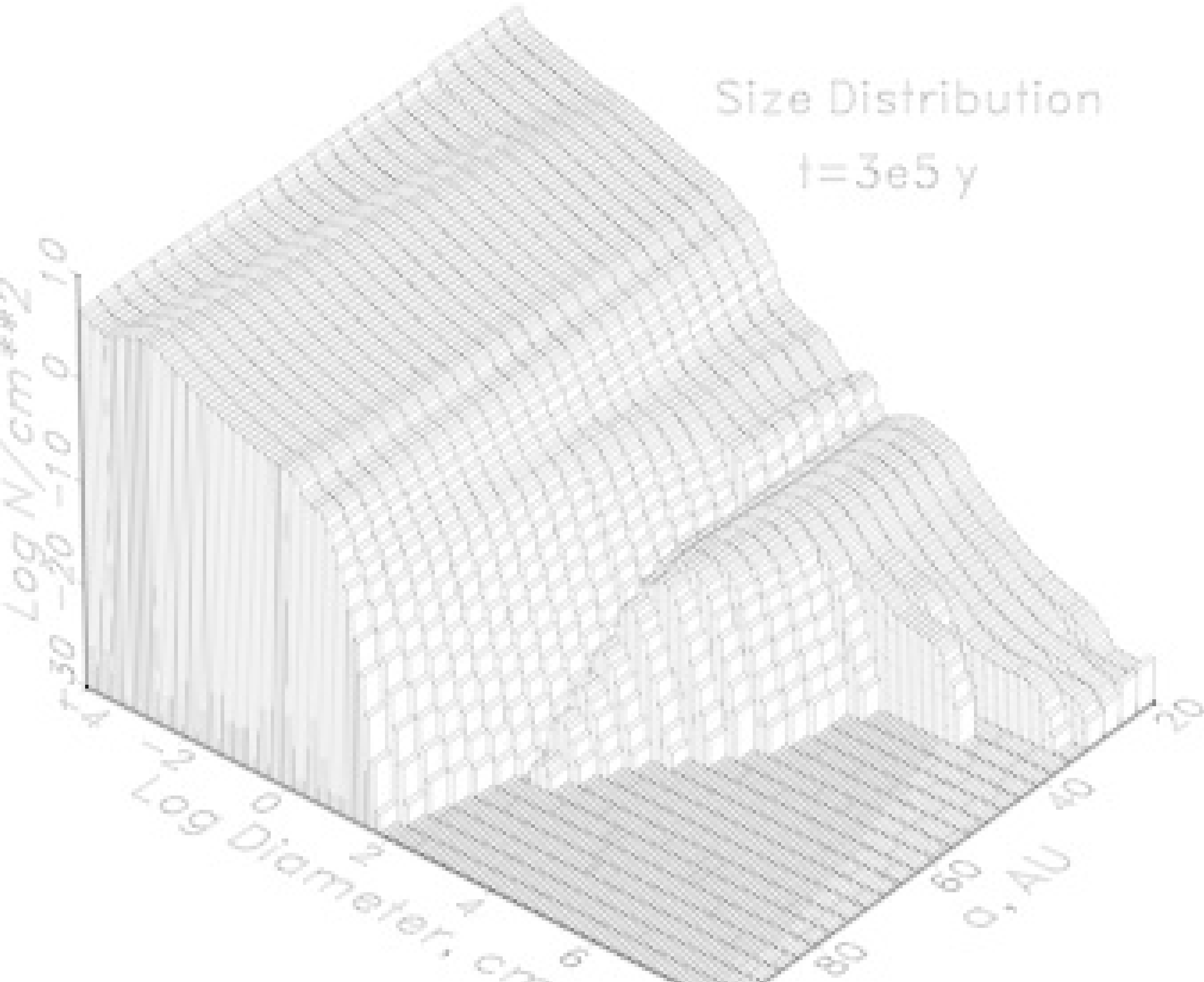
Size Distribution
t=1e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU

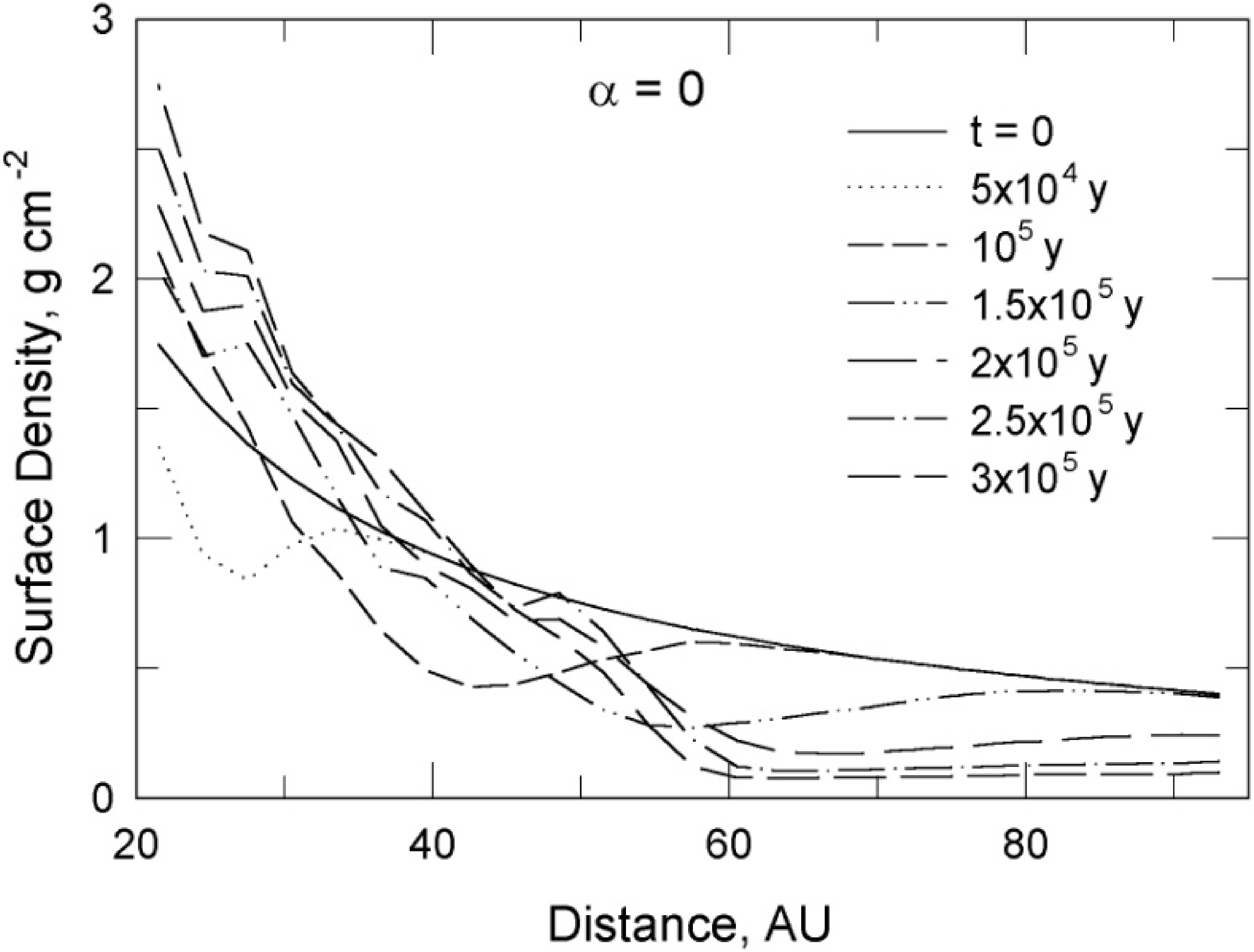

Size Distribution
t=1.5e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU


Size Distribution
t=2e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU



Size Distribution
t=2.5e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU


Size Distribution
t=3e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU



α = 0
t = 0
5x10^4 y
10^5 y
1.5x10^5 y
2x10^5 y
2.5x10^5 y
3x10^5 y
Surface Density, g cm^-2
Distance, AU
0
1
2
3
20
40
60
80

Radial Migration and Redistribution of Mass

Meter-sized bodies move inward at radial velocity ΔV ~ 50 m/sec ~ 1 AU/century. Unless growth through this size range is rapid, such bodies travel a distance comparable to the size of the nebula, and/or may be lost into the Sun.

ΔV and the size at which $t_e \Omega_K \sim 1$ do not vary significantly with heliocentric distance. However, growth times increase with distance due to the lower density of matter. Thus, bodies that start at larger R move inward greater distances before growing large enough to be unaffected by drag.

Radial migration of growing ~m-sized bodies depletes the outer nebula of mass and produces a surface density distribution of solids that is steeper than that of the gas.

At some distance, bodies can grow large enough (~ km)
to stop migrating. Because the particle layer is very thin in a laminar nebula, these bodies become efficient traps for the smaller ones that are drifting inward from larger distances.

Mass piles up, producing a distinct “edge” in the disk of planetesimals.

The planetesimal disk is significantly smaller than the extent of the original gas/dust nebula.

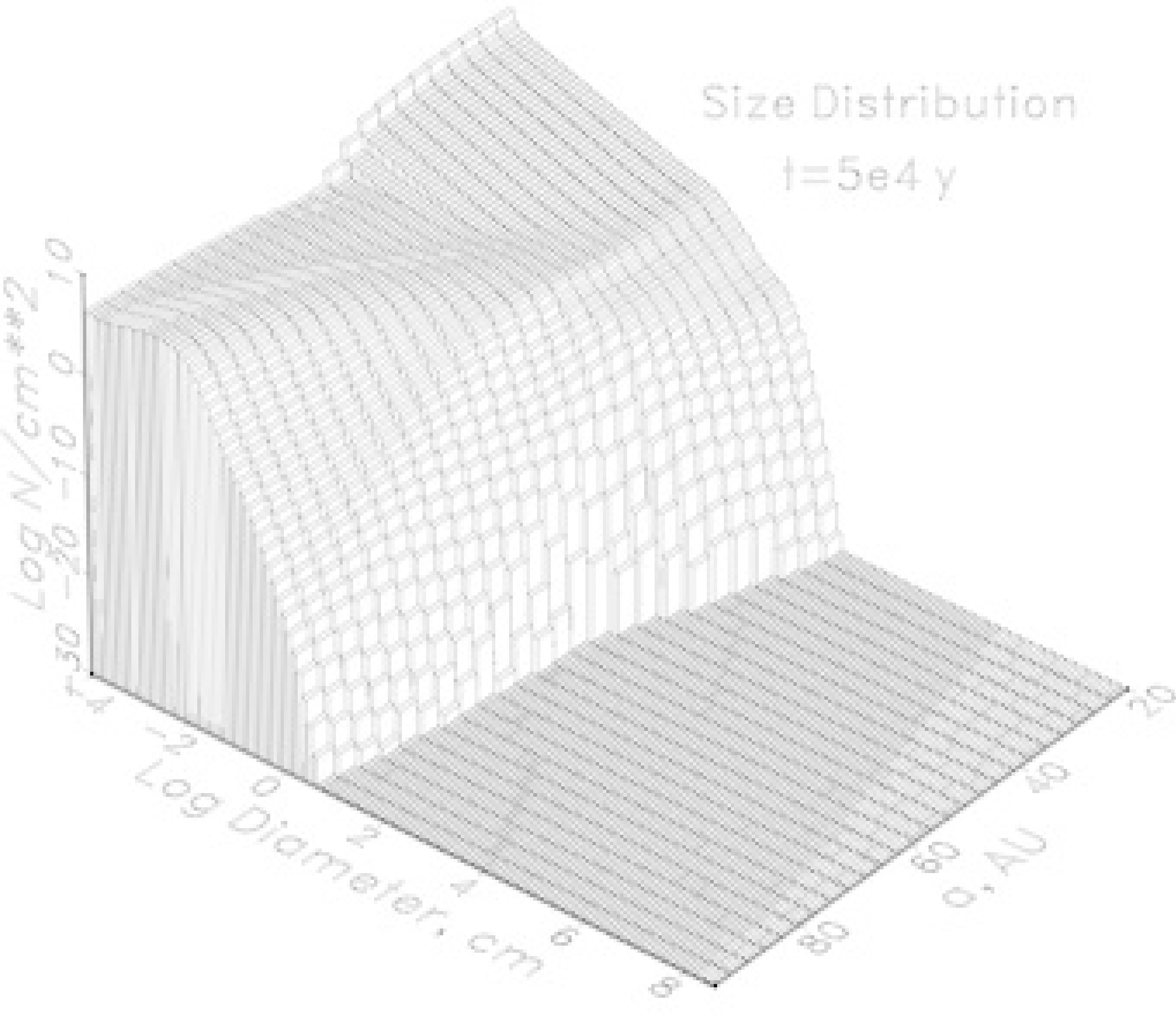
Migration in a Turbulent Nebula

Suppose the nebula has a source of turbulence in addition to shear in the midplane particle layer, characterized by a parameter α, such that

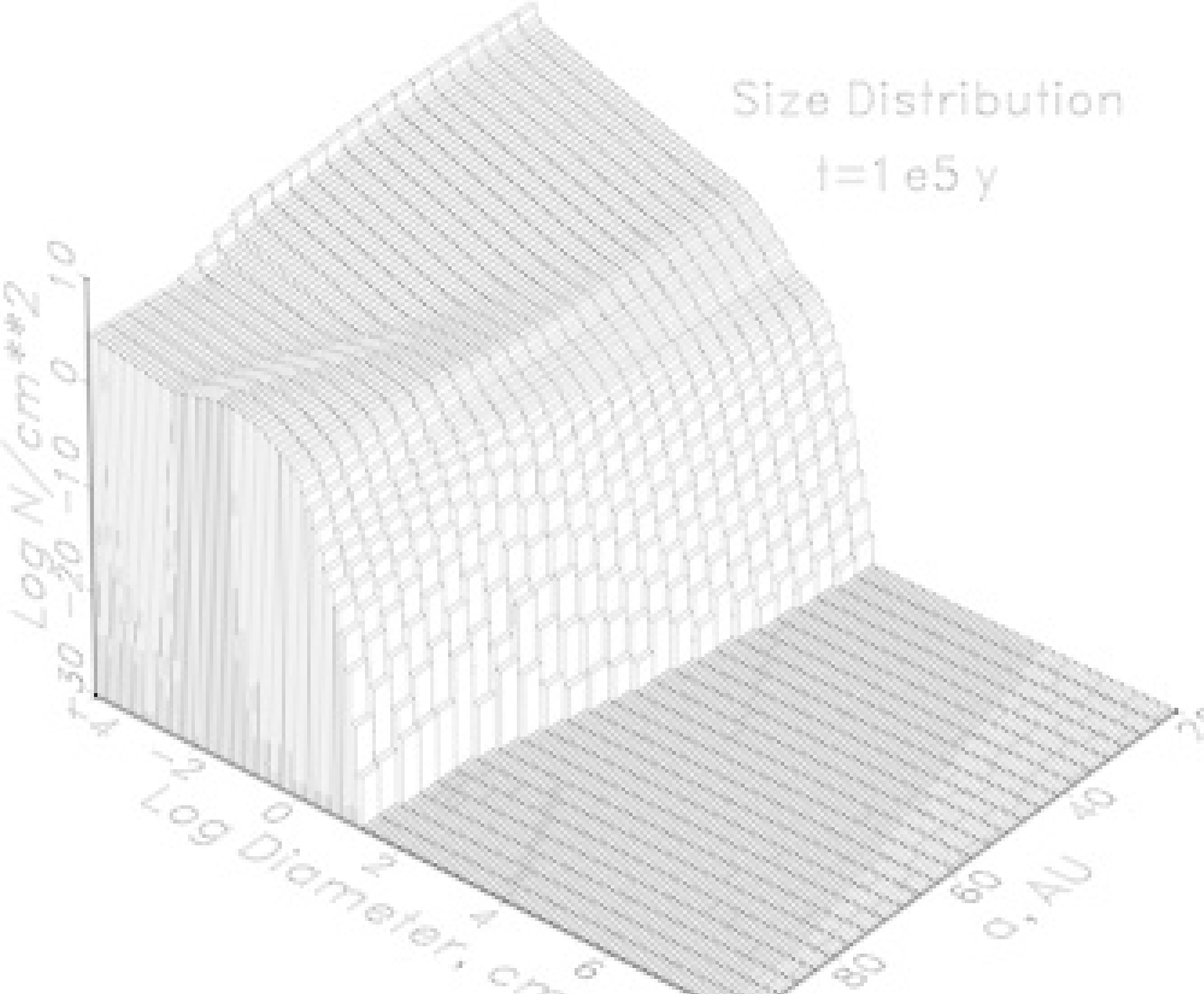
$$V_{turb} = c\alpha^{1/2} \qquad \omega = \Omega_K$$

For $\alpha \ll 1$, turbulence does not affect collision velocities significantly, but stirs the particle layer and decreases its density. This slows the growth rate of bodies through the meter size range, causing them to migrate farther.

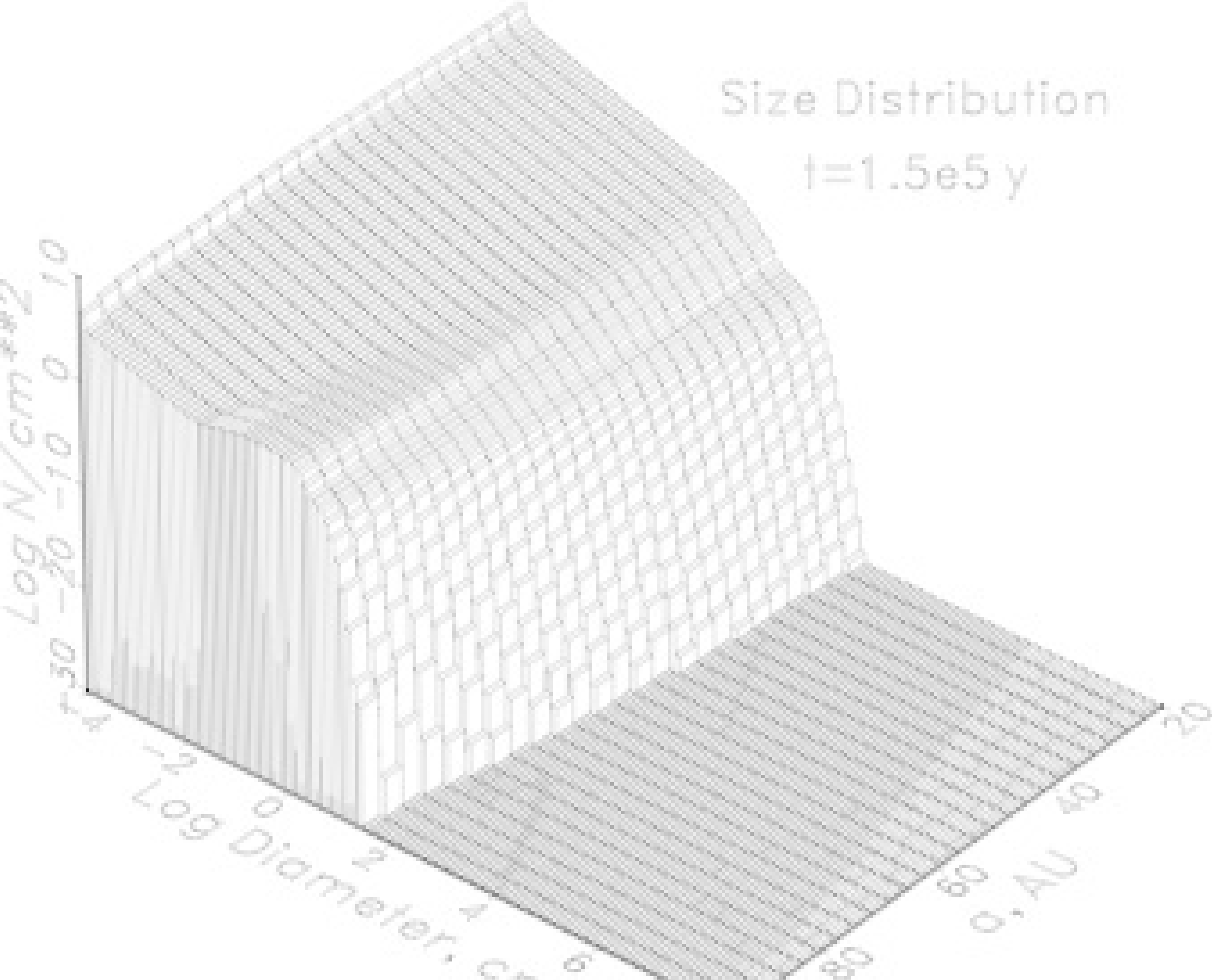
$\alpha = 10^{-6}$ gives results similar to $\alpha = 0$.
The following shows the evolution of solids for $\alpha = 10^{-4}$.

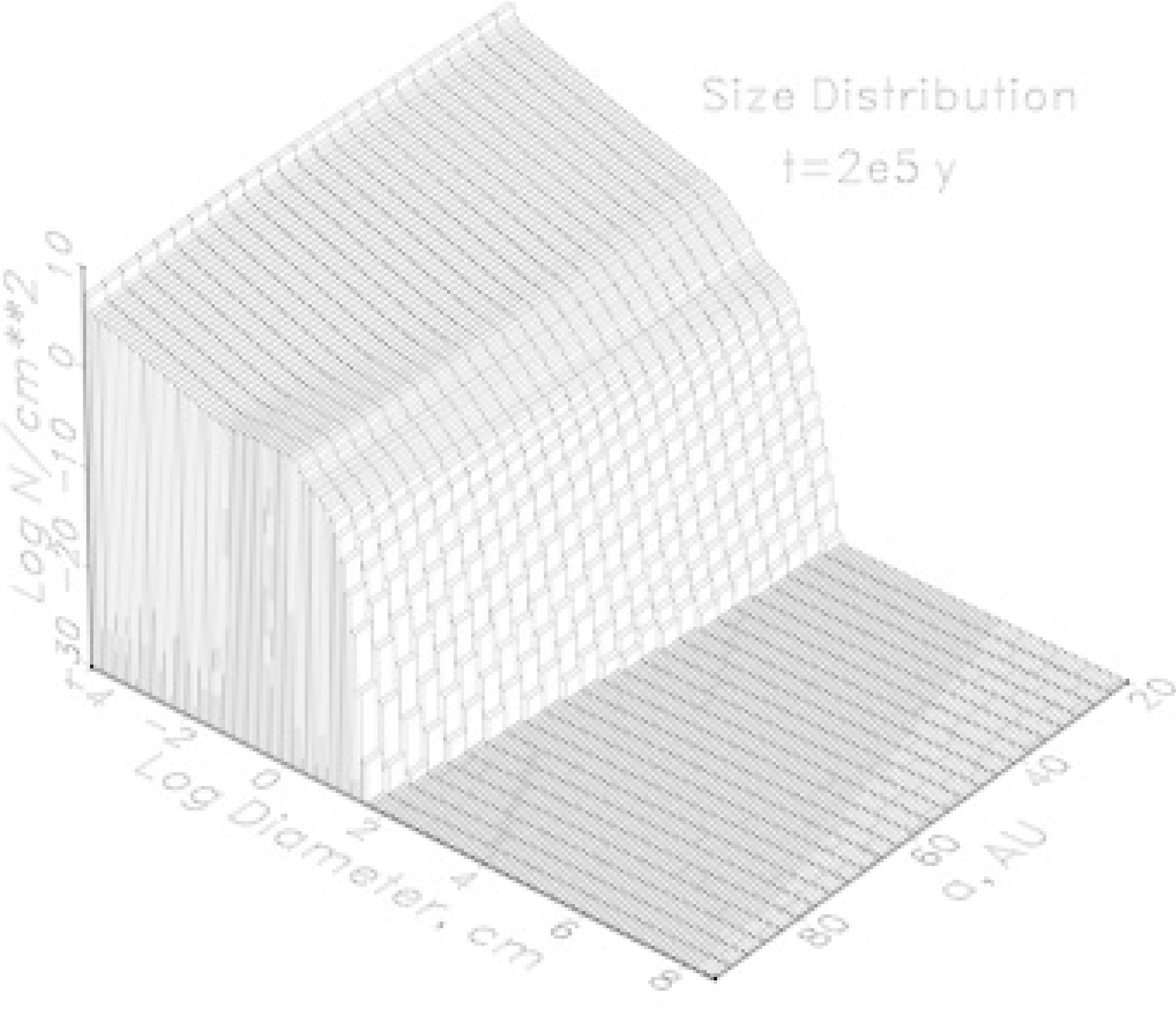


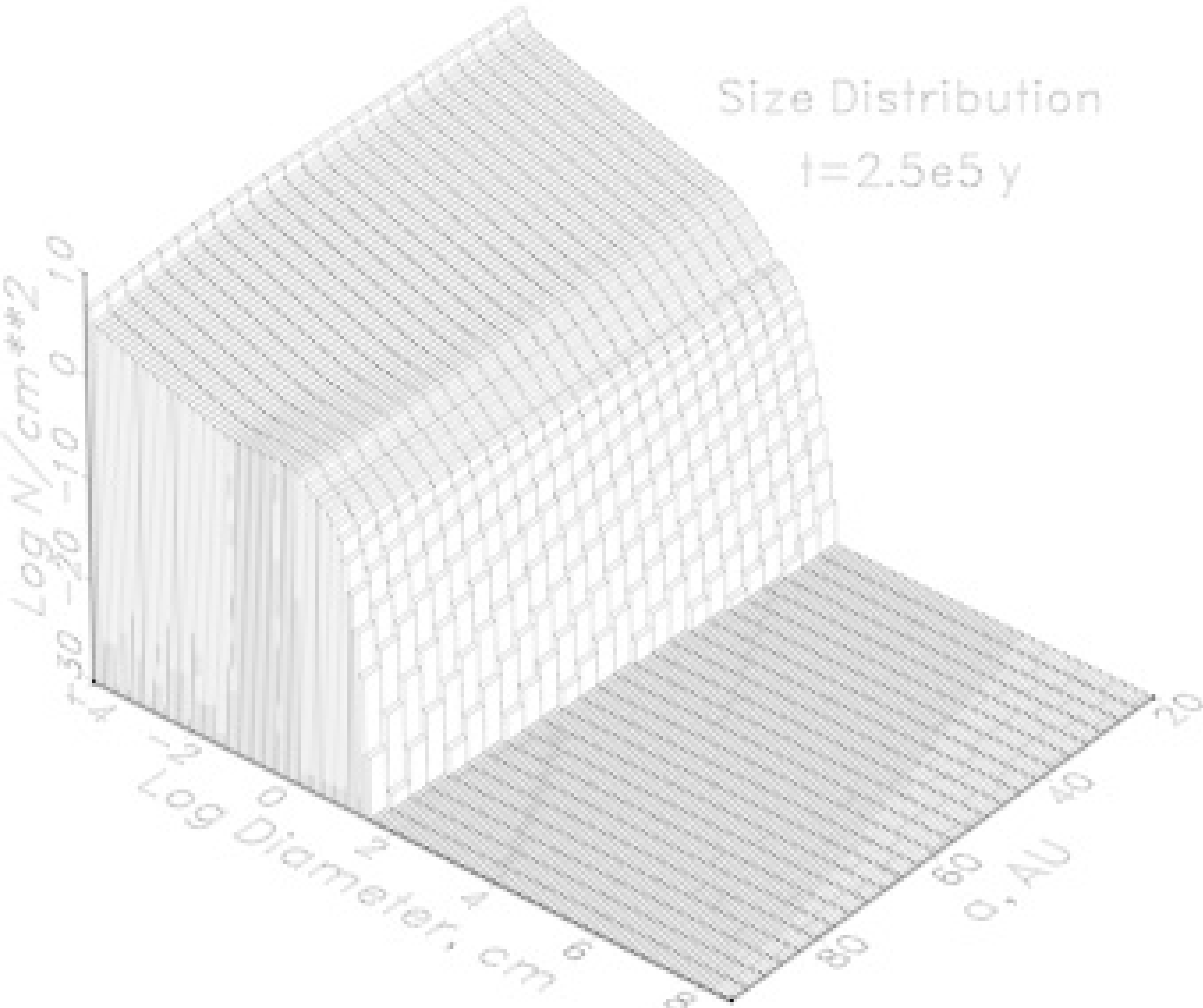
Size Distribution
t=5e4 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU

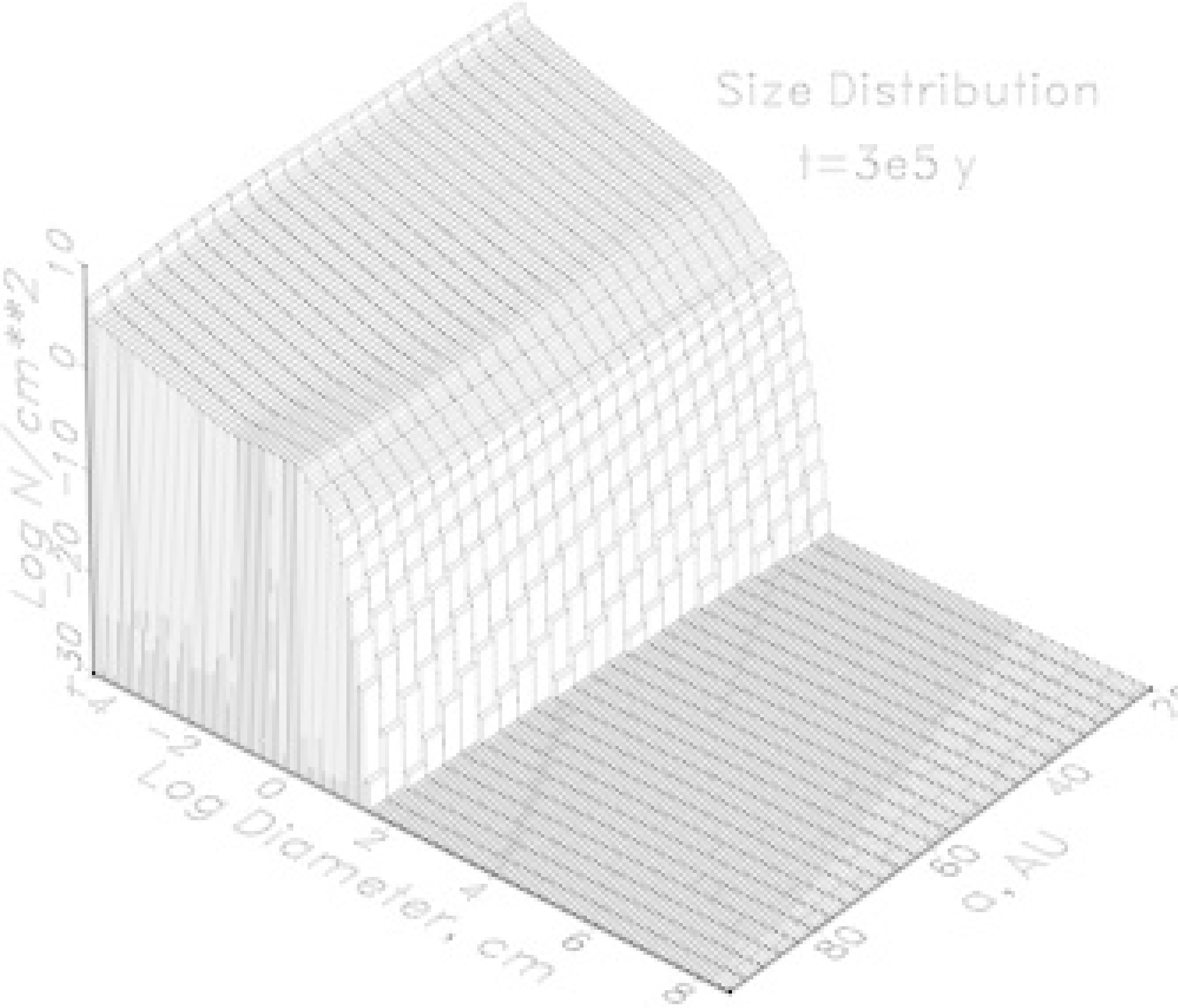


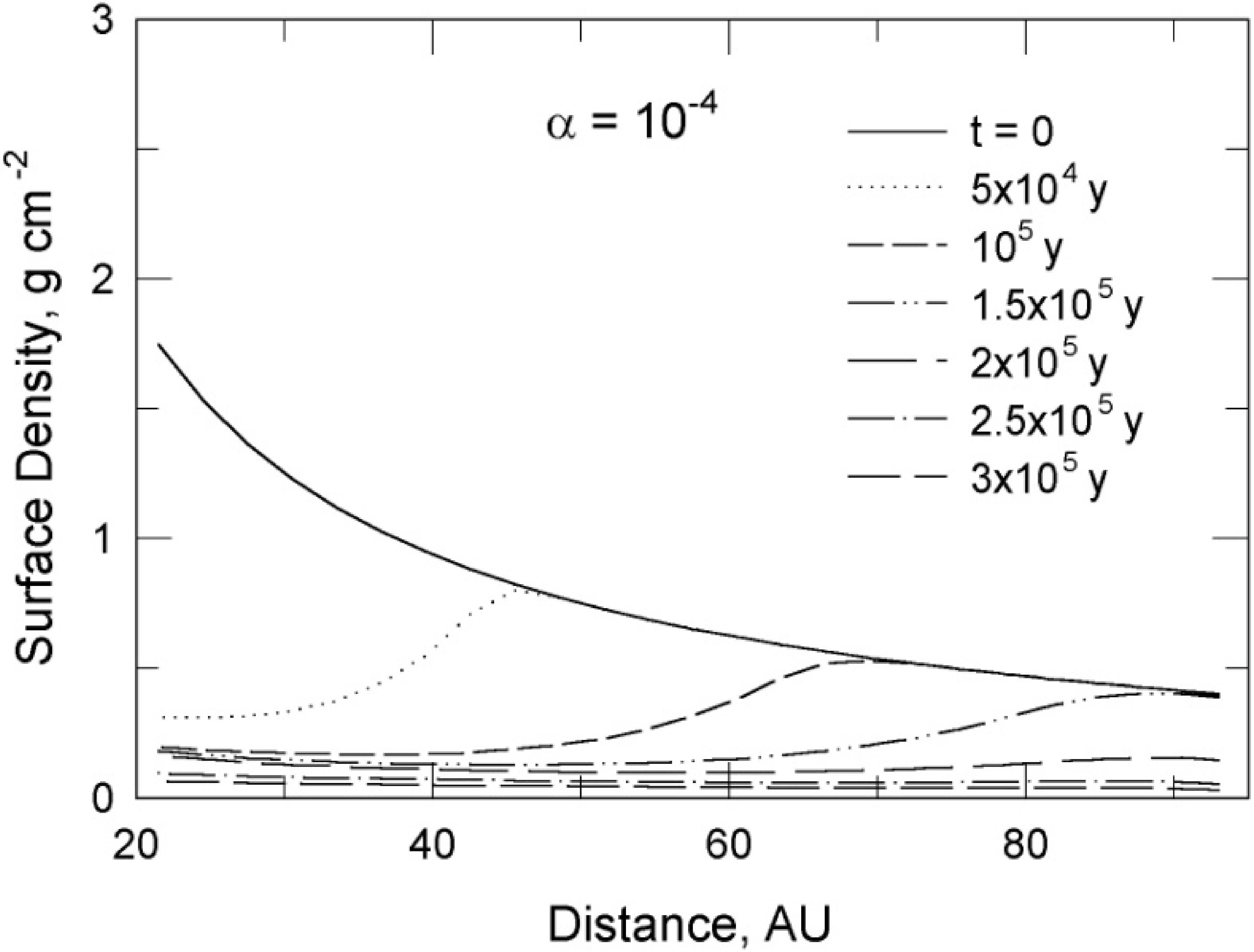
Size Distribution
t=1e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU


Size Distribution
t=1.5e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU


Size Distribution
t=2e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU


Size Distribution
t=2.5e5 y
Log N/cm**2
-30
-20
-10
0
10
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU


Size Distribution
t=3e5 y
Log N/cm**2
10
0
-10
-20
-30
-4
-2
0
2
4
6
8
Log Diameter, cm
20
40
60
80
a, AU



α = 10^-4
t = 0
5x10^4 y
10^5 y
1.5x10^5 y
2x10^5 y
2.5x10^5 y
3x10^5 y
Surface Density, g cm^-2
Distance, AU
0
1
2
3
20
40
60
80

Turbulent concentration?

Cuzzi et al. (2001) have suggested an alternative mechanism for planetesimal formation in a highly turbulent nebula. In this model, particles are sorted in
small eddies that result from a Kolmogorov cascade from the largest eddies to the inner scale of viscous dissipation. Such eddies can concentrate chondrule-sized particles, producing localized regions of higher particle density. For sufficiently energetic turbulence, such regions can exceed δ_{crit}.

However, the free-fall collapse time of these concentrations Is much longer than the eddy lifetime at this scale. The actual collapse time is likely to be much longer due to gas pressure. Thus, such condensations are likely to dissipate, rather than become planetesimals.

Summary and Conclusions

- The gas of the solar nebula does not rotate at the Kepler velocity due to pressure support. This deviation has strong consequences for the behavior of solid particles and planetesimal formation.
- The settling of particles toward the midplane of the nebula is limited by turbulence generated by shear between the particle-rich layer and pressure-supported gas.
- The maximum density of the particle layer is ~100 times normal solar abundance, if the particles are small enough to be well coupled to the gas (~<cm). Higher densities are possible only if the particles are too large to be in chemical equilibrium with the gas.

- Formation of planetesimals from small particles by gravitational instability is difficult. The particle layer must reach a density ~100 times that of the gas. Any general turbulence in the nebula will prevent this. Even in a laminar nebula, shear-induced turbulence limits the density to values comparable to that of the gas.
- Enhancement of the abundance of solids by about an order of magnitude can allow the layer to reach the critical density. Such enhancement is unlikely, either by inward migration of solids or localized drag instability mechanism.
- Depletion of gas is less effective than enhancement of solids for raising the density of the layer, due to the increase of response time at lower gas density.

- Attainment of the critical density is necessary for gravitational instability, but not sufficient. If the particles are coupled to the gas, collapse is inhibited by gas pressure. Settling of particles within a condensation takes much longer than the free-fall collapse timescale.
- A layer of large (m-sized) particles can attain the critical density, and is not affected by gas pressure. However, instability is inhibited by the dispersion of drag-induced velocities expected with a plausible size distribution of non-identical particles.

- Collisional growth of planetesimals is possible if the mechanical properties of particles are suitable.
- Aggregates of small grains should be compressible and dissipate energy in collisions. The required level of impact strength is not clear.
- Sticking mechanisms are unknown, and may vary with composition (ice vs. silicates) and location.
- Growth may be favored if relative velocities are driven by differential gas drag, as most such collisions will involve bodies of very different sizes; the projectile may become embedded in the target.
- Turbulence is a problem for collisional formation of planetesimals, as modest values of α ($\sim 10^{-4}$) may result in depletion of solids in the nebula by inward migration.
- Either way, it seems necessary to conclude that the solar nebula was quiescent when planetesimals formed.

Suggested Reading

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