GFD Program 2010
at
Woods Hole Oceanographic Institution

19th/January/2011　WTK オンラインセミナー

小布施　祈織（京都大学数理解析研究所）

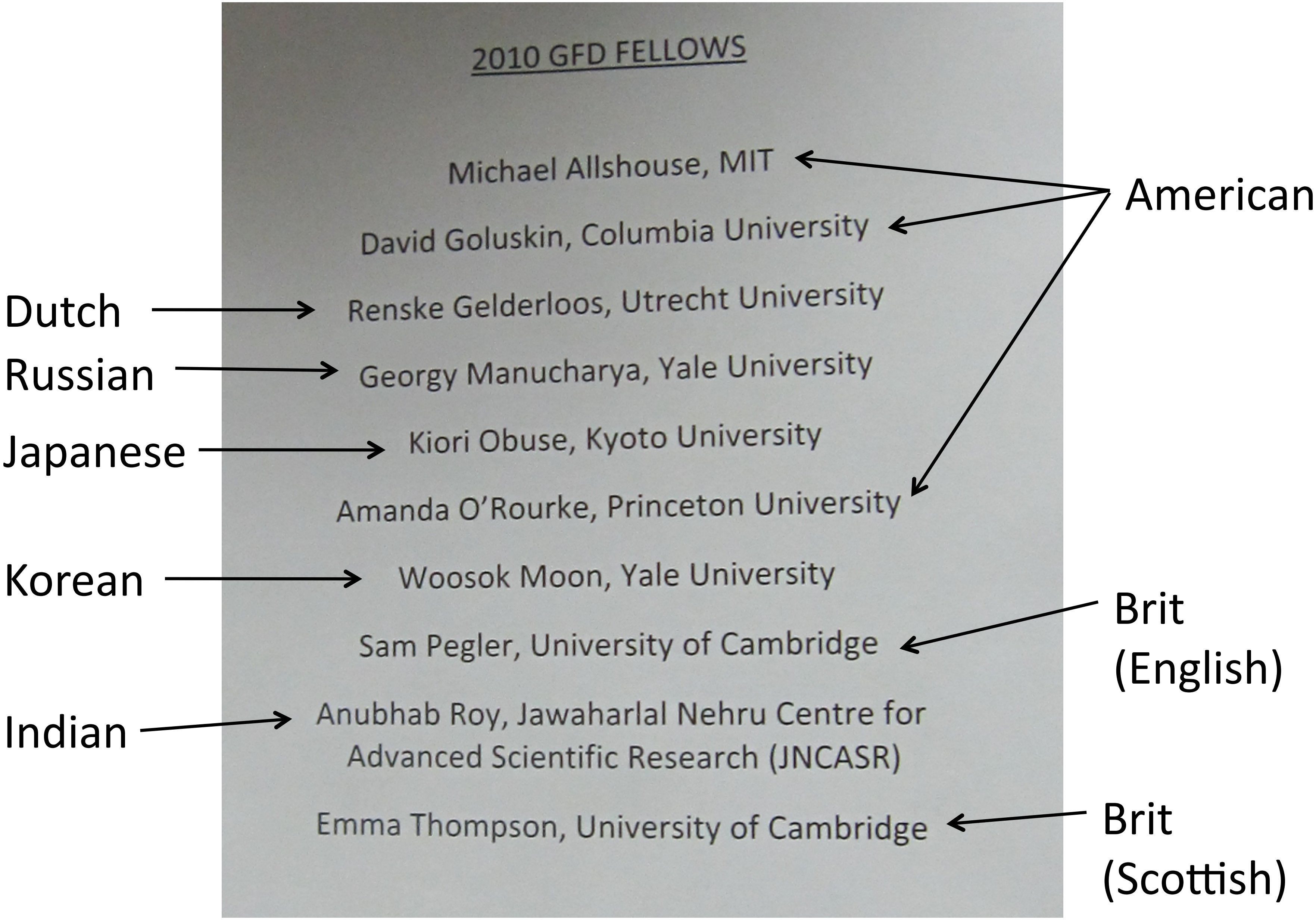
Fellowships in Geophysical Fluid Dynamics at Woods Hole Oceanographic Institution

June 21 to August 27, 2010

Since 1959 the GFD program has promoted an exchange of ideas among researchers in the many distinct fields that share a common interest in the nonlinear dynamics of fluid flows in oceanography, meteorology, geophysics, astrophysics, applied mathematics, engineering and physics. Each year, the program is organized around a ten-week course of study and research for a small group of competitively selected graduate-student fellows. The overall philosophy is to bring together researchers from a variety of backgrounds to provide a vigorous discussion of concepts that span different disciplines, and thereby to create an intense research experience. For the student fellows, the centerpiece of the program is a research project, pursued under the supervision of the staff. At the end of the program, each fellow presents a lecture and a written report for the GFD proceedings volume. Over its history, the GFD Program has produced numerous alumni, many of whom are prominent scientists at universities throughout the world. The interdisciplinary atmosphere of the Program is the ideal place for young scientists to learn the habits of broad inquiry, of speaking to others with very different backgrounds and viewpoints, and of seeking answers in unfamiliar places.

The Program commences with two weeks of Principal Lectures focusing on a particular theme in GFD. For 2010, the lectures will be entitled "Swirling and Swimming in Turbulence", and be delivered by Glenn Flierl (MIT), Antonello Provenzale (CNR, Italy) and Jean-Luc Thiffeault (U. Wisconsin). Lectures by staff and visitors will follow daily on a wide range of GFD and related topics.

Fellows



2010 GFD FELLOWS

Michael Allshouse, MIT

David Goluskin, Columbia University

Renske Gelderloos, Utrecht University

Georgy Manucharya, Yale University

Kiori Obuse, Kyoto University

Amanda O'Rourke, Princeton University

Woosok Moon, Yale University

Sam Pegler, University of Cambridge

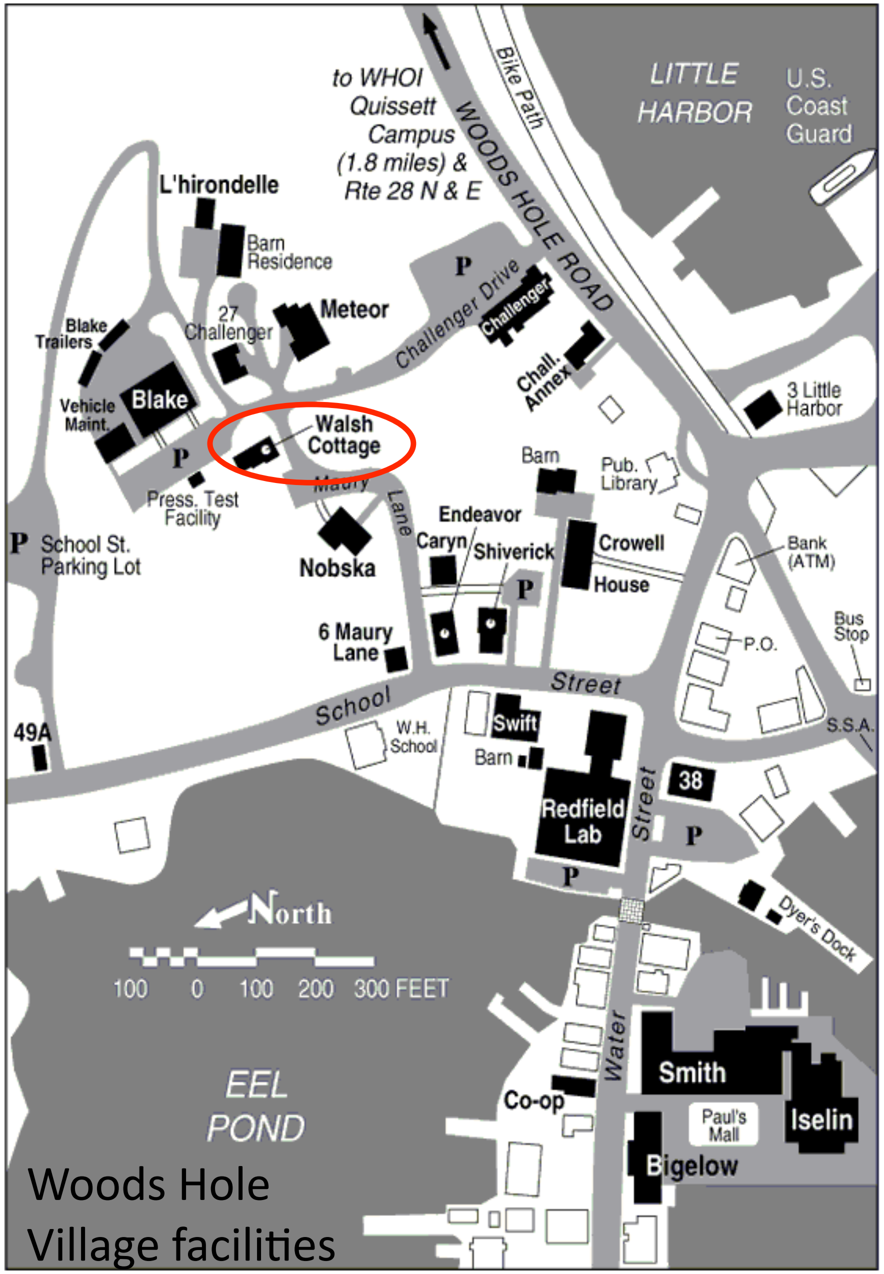
Anubhab Roy, Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR)

Emma Thompson, University of Cambridge

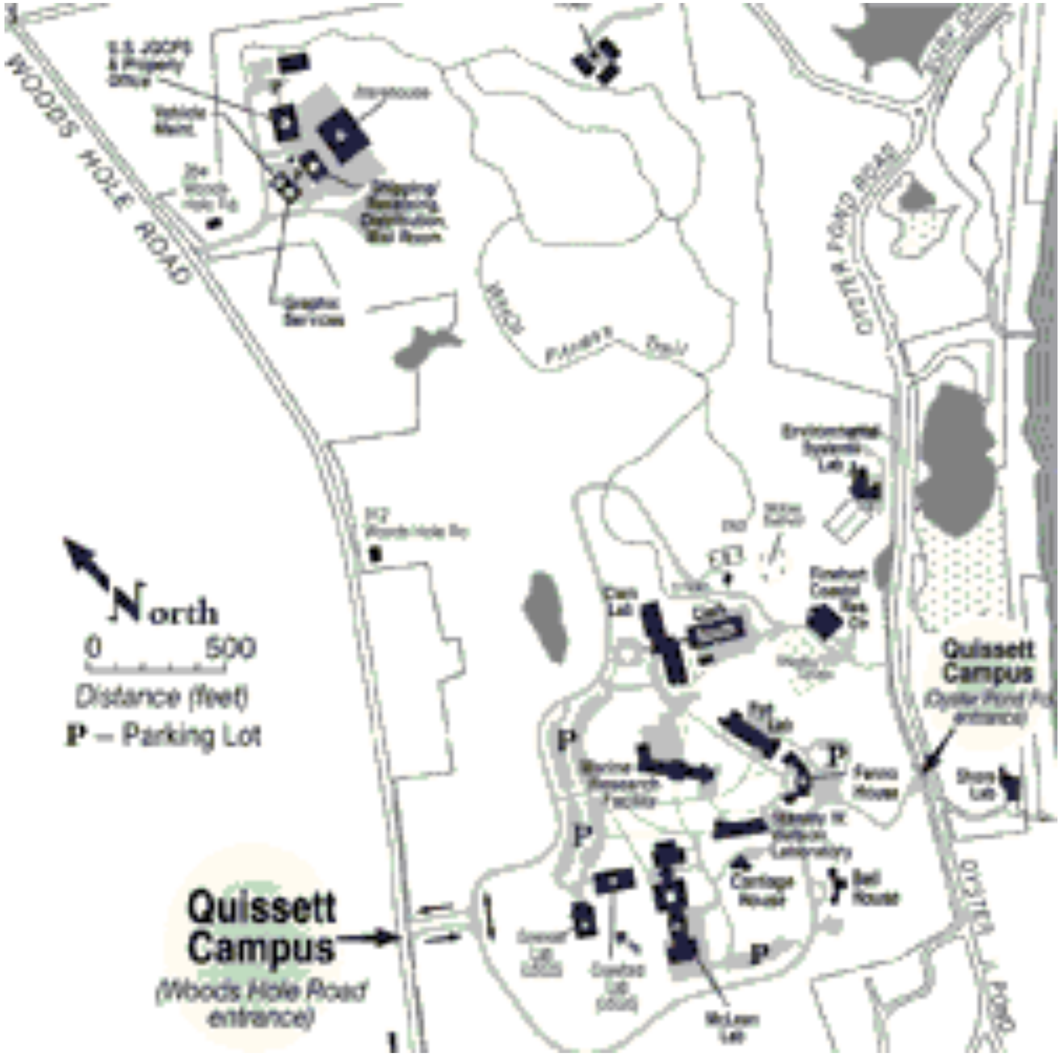
Woods Hole

THE LIFE AND THE LECTURES

WHOI



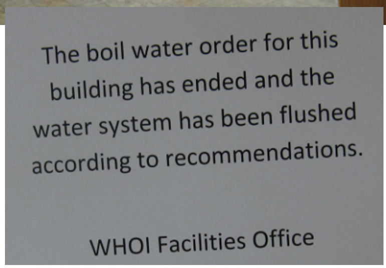
Woods Hole Village facilities



Woods Hole

Woods Hole

Accomodation



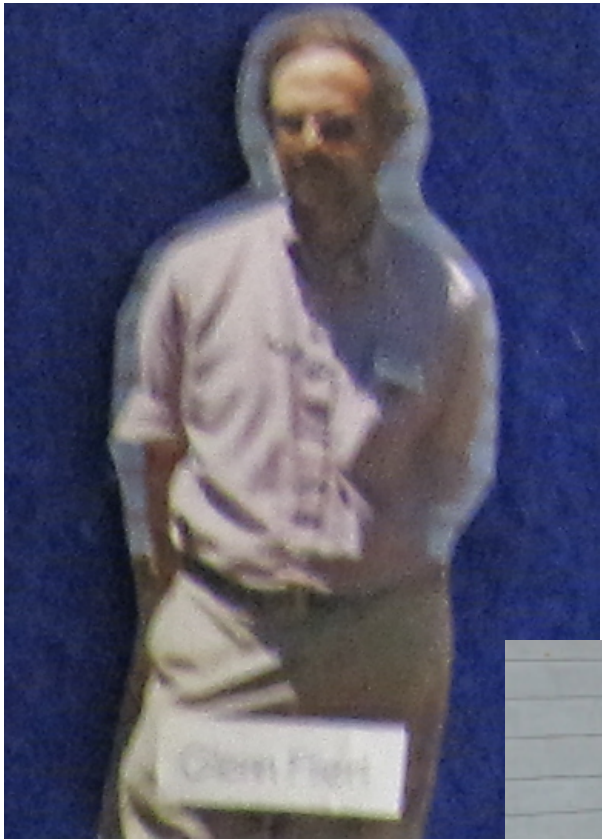
Walsh Cottage

Library


EXIT
METEORITES


Fluid Mechanics
turbulence
DAVIDSON
BATCHELOR
AN INTRODUCTION TO FLUID DYNAMICS
Dodd Eilbeck Gibbon Morris
Solitons and Nonlinear Wave Equations
NUMERICAL MODELING of WATER WAVES
LINEAR AND NONLINEAR WAVES
WHITHAM
Nonlinear Problems of Elasticity
THEORY OF PLATES AND SHELLS
SECOND EDITION
NAVIER STOKES EQUATIONS
CANUP RIGHTER
LAURETTA MCSWEEN
EARTH
LEVY LUNINE

Lecturers



Glenn Flierl, MIT

Jean-Luc Thiffeault, University of Wisconsin, Madison

+ Neil Balmforth, University of British Columbia

Antonello Provenzale, Istituto di Scienze Dell'Atmosfera

Lecturers

Lecture 1: Stirring and Mixing (Jean-Luc Thiffeault)

Lecture 2 : Introduction to Biological models (Glenn Flierl)

Lecture 3 : Effective Diffusivity and Swimming Organisms (Jean-Luc Thiffeault)

Lecture 4 : Local Stretching Theories (Jean-Luc Thiffeault)

Lecture 5 : Social Behaviour, Mixing, and the Evolution of Schooling (Glenn Flierl)

Lecture 6 : Mixing in the Presence of Sources and Sinks (Jean-Luc Thiffeault)

Lecture 7 : Examples at the Mesoscale (Antonello Provenzale)

Lecture 8: Dynamics of Heavy Impurities with Finite Size (Antonello Provenzale)

Lecture 9: Plankton Sinking and the Role of Turbulence (Antonello Provenzale)

Lecture 10: Evolutionary Models: Movement and Mixing in Trait and Physical Space Glenn Flierl)

Lecturer notes

Social Behavior. Mixing. and the Evolution of Schooling

Glenn Flierl

$_s$). This describes the behavior of the organisms that te
r neighbors.

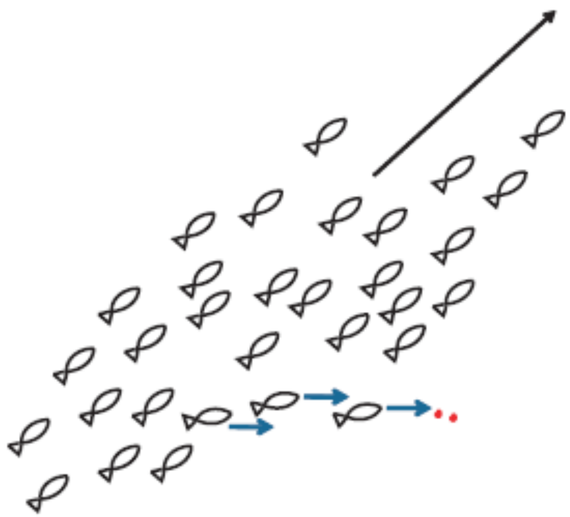


Figure 12: Representation of schooling.

direction of the swimming organisms results from a cc
ıment tendencies, and so schooling can be represented as

$$\mathbf{V} = V_0 \mathbf{V}_1 / |\mathbf{V}_1|,$$

$$\mathbf{V}_1 = \alpha \sum_{\mathbf{X}'} (\mathbf{X}' - \mathbf{X}) w(|\mathbf{X}' - \mathbf{X}|) + \sum_{\mathbf{X}'} \mathbf{U}' w(|\mathbf{X}' - \mathbf{X}|).$$

Dynamics of heavy impurities with finite size

Antonello Provenzale

ıtum relaxation time and δ determines the nature of buoyant part
es heavy and light particle respectively). Now if one considers fl
the Eulerian acceleration in the forces experienced by the partic

$$\frac{d\mathbf{V}}{dt} = \delta \frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_a}(\mathbf{V} - \mathbf{u}) - (1 - \delta) g \hat{\mathbf{z}}$$

ınalising,

$$\frac{d\mathbf{V}^*}{dt^*} = \delta \frac{D\mathbf{u}^*}{Dt^*} - \frac{1}{St}(\mathbf{V}^* - \mathbf{u}^*) - (1 - \delta) \frac{1}{Fr^2} \hat{\mathbf{z}}$$

ɜd to denote dimensionless variables. Henceforth we will be dr

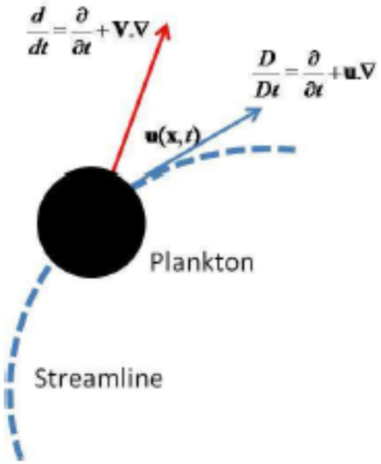


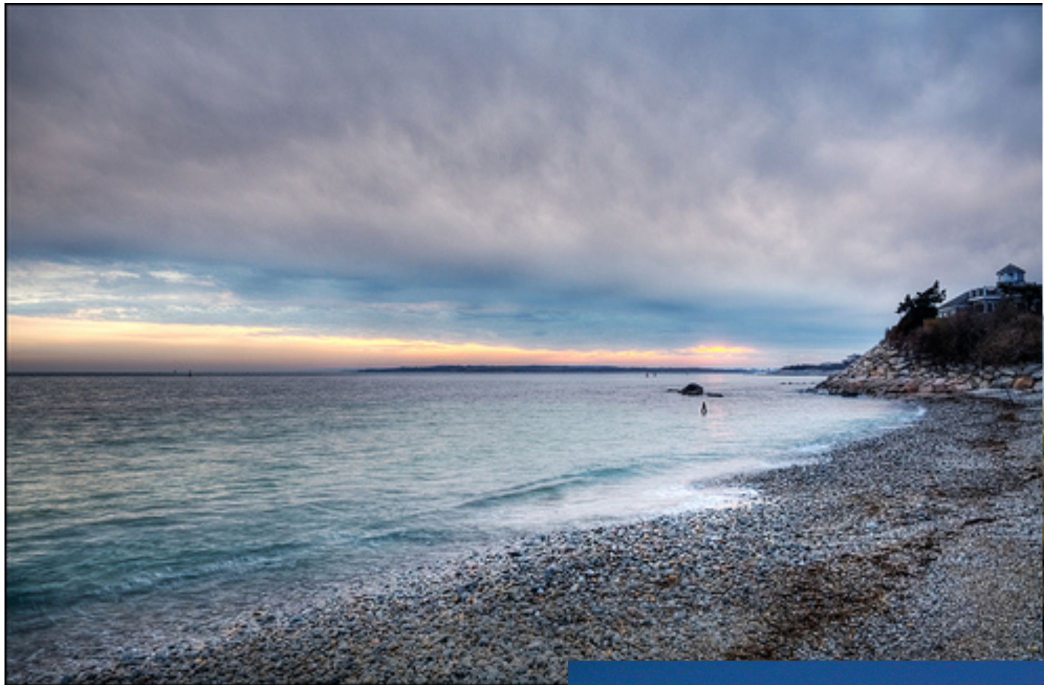
Figure 3: d/dt and D/Dt

note the non-dimensional variables by the same notation as the d

BBQ

Beer, beer, …, beer, vodka, vodka, wine, cocktail

Beach



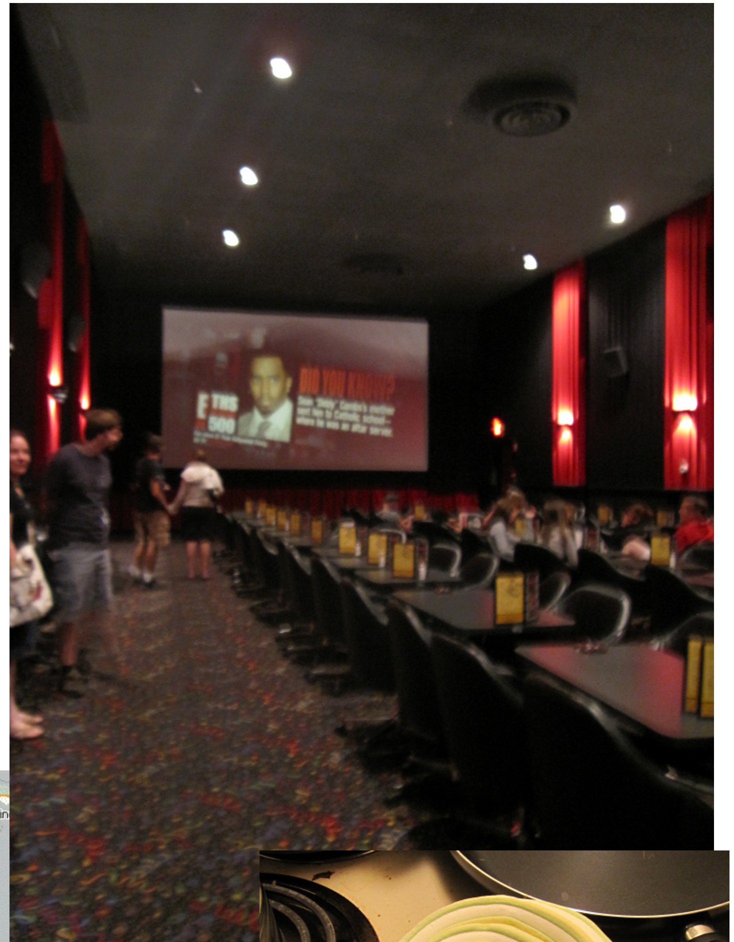
Softball

George Veronis,
Yale University

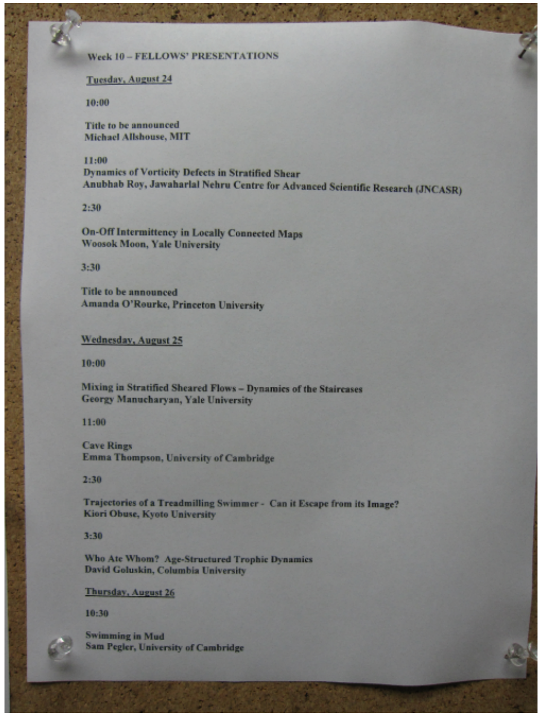
Softball

Japanese Party

Others




South Duxbury
Plymouth
Cape Cod Bay
North Eastham
Eastham
Salt Pond
Sagamore Beach
Sandwich
Orleans
Brewster
East Dennis
Dennis
Bourne
MASSACHUSETTS
Barnstable
Forestdale
South Dennis
East Harwich
Monument Beach
Pocasset
Marstons Mills
Hyannis
West Dennis
Harwich Port
West Chatham
North Falmouth
Mashpee
Centerville
West Yarmouth
BARNSTABLE
Osterville
Falmouth
Cotuit
East Falmouth
Teaticket



Week 10 – FELLOWS' PRESENTATIONS

Tuesday, August 24

10:00

Title to be announced
Michael Allshouse, MIT

11:00
Dynamics of Vorticity Defects in Stratified Shear
Anubhab Roy, Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR)

2:30

On-Off Intermittency in Locally Connected Maps
Woosok Moon, Yale University

3:30

Title to be announced
Amanda O'Rourke, Princeton University

Wednesday, August 25

10:00

Mixing in Stratified Sheared Flows – Dynamics of the Staircases
Georgy Manucharyan, Yale University

11:00

Cave Rings
Emma Thompson, University of Cambridge

2:30

Trajectories of a Treadmilling Swimmer - Can it Escape from its Image?
Kiori Obuse, Kyoto University

3:30

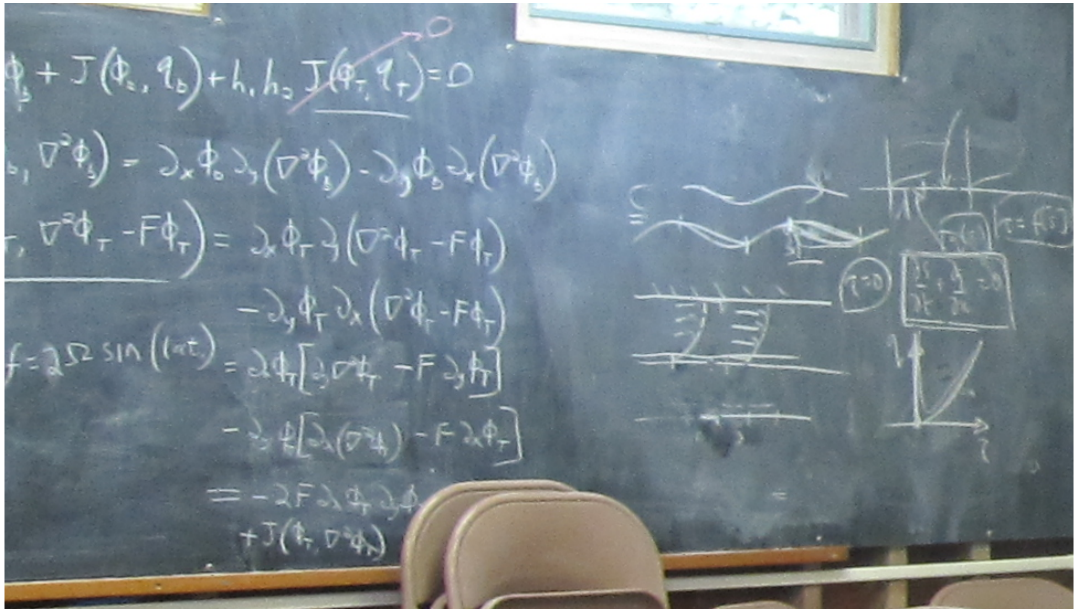
Who Ate Whom? Age-Structured Trophic Dynamics
David Goluskin, Columbia University

Thursday, August 26

10:30

Swimming in Mud
Sam Pegler, University of Cambridge

THE PROJECT



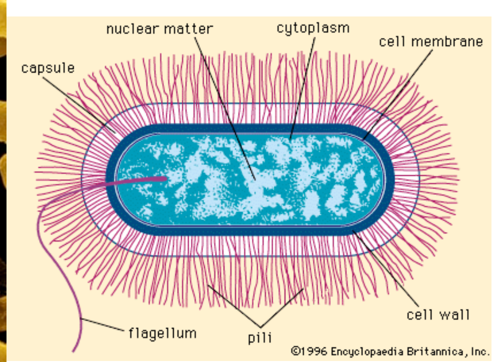
Trajectories of a treadmilling swimmer

—— Can it escape from its image? ——

Kiori Obuse (RIMS, Kyoto University)
Supervised by
Jean-Luc Thiffeault (University of Wisconsin)

Microorganism

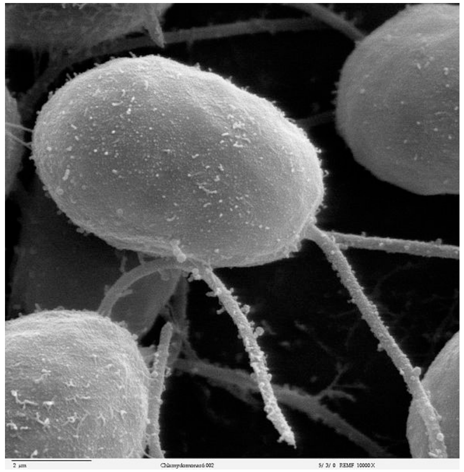
Bacillus



http://blackmonsbacillus.pbworks.com/

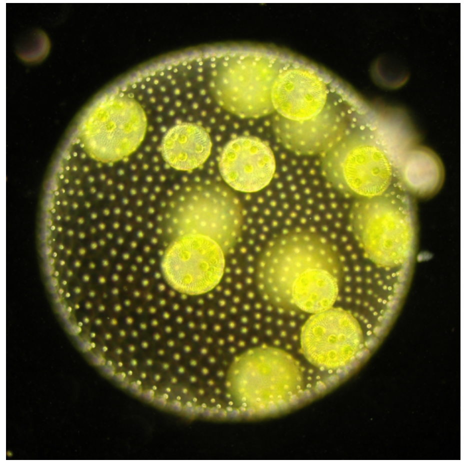
http://www.freewebs.com/jennalb03/

Chlamydomonas



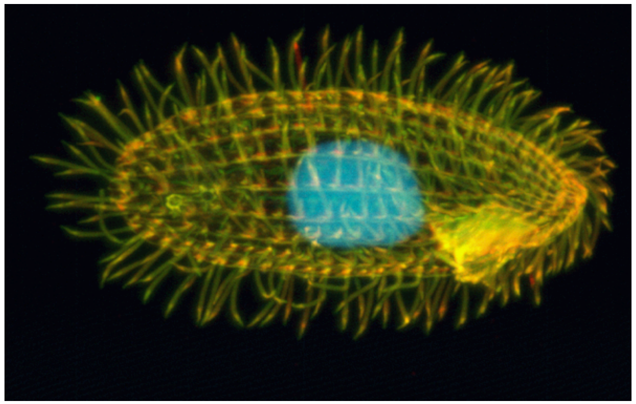
http://en.wikipedia.org/wiki/Chlamydomonas

Volvox



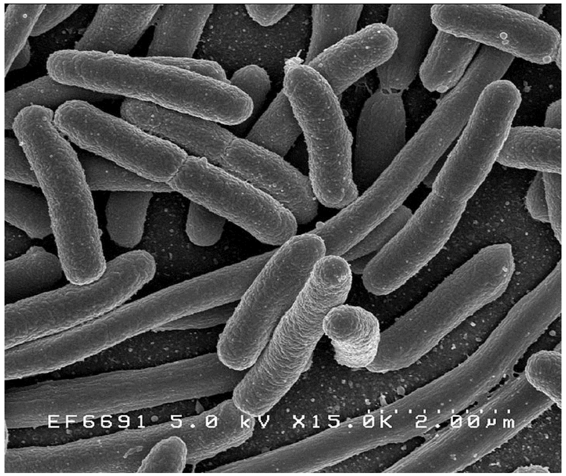
http://www.damtp.cam.ac.uk/user/gold/movies.html

Tetrahymena

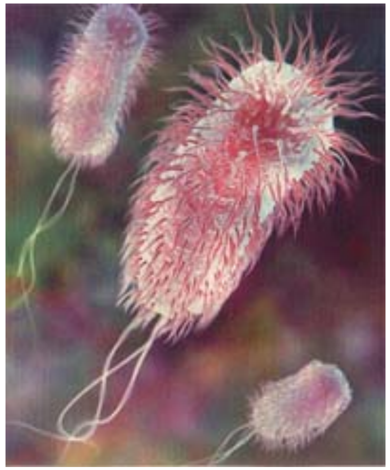


http://en.wikipedia.org/wiki/Tetrahymena

Escherichia coli

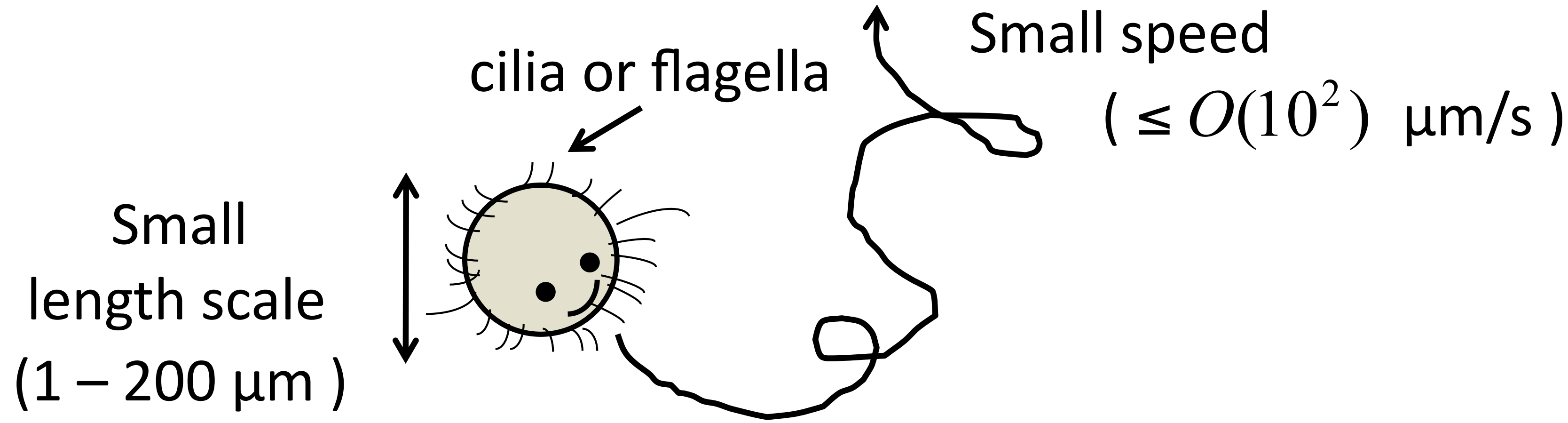


http://ja.wikipedia.org/wiki/%E5%A4%A7%E8%85%B8%E8%8F%8C



http://www.marlerblog.com/tags/e-coli/

Microorganism and the Stokes flow

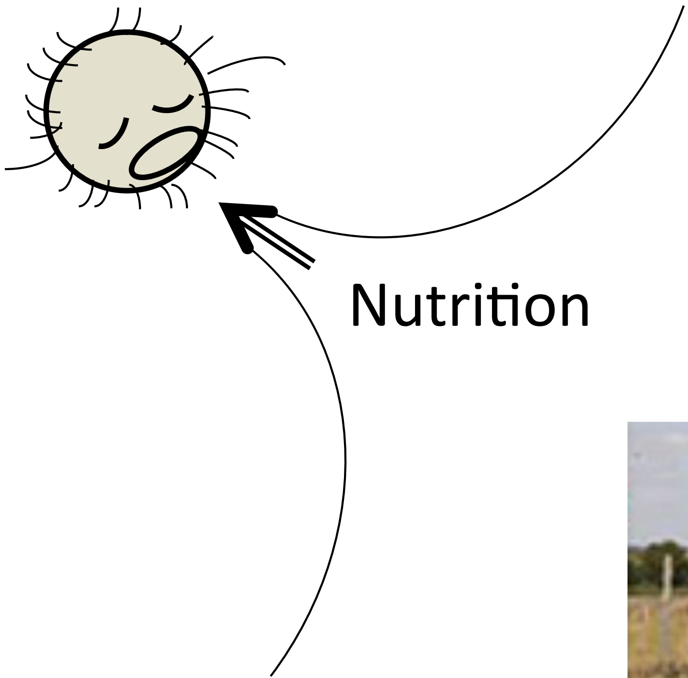


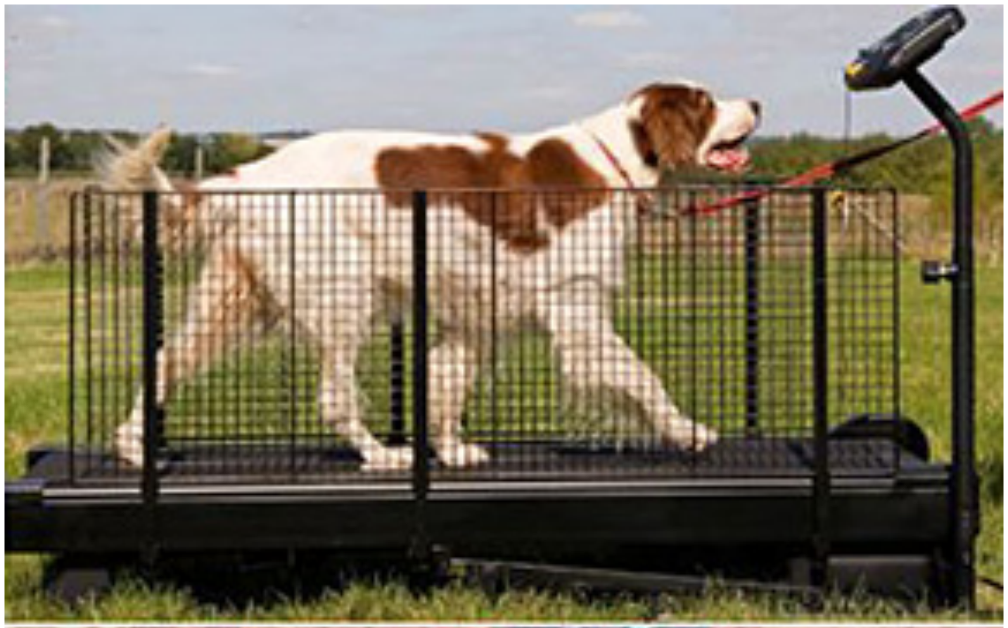
Low Reynolds number

($\leq O(10^2)$, ex. *Escherichia coli*: $O(10^{-4})$)

→ viscous effect becomes dominant

Treadmilling in a free space





http://gizmodo.com/338598/fit-fur-life-doggy-treadmill-walks-that-mutt-so-you-wont-have-to

Microorganisms near a boundary

E. Coli swimming in circles above
a flat glass surface, (Lauga et al. 2006)

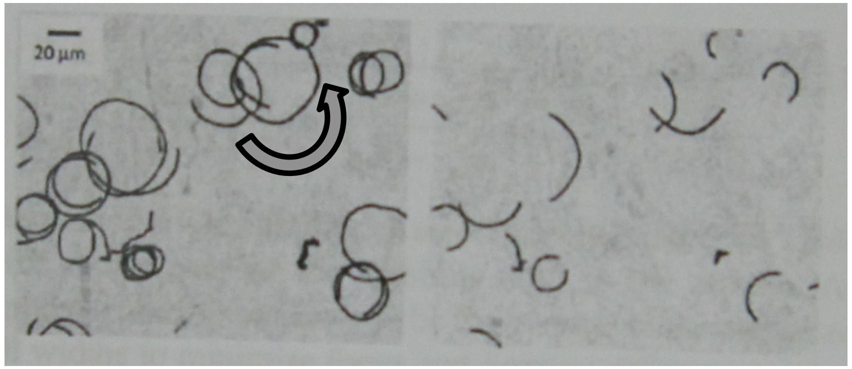
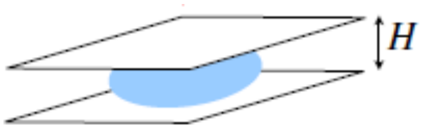
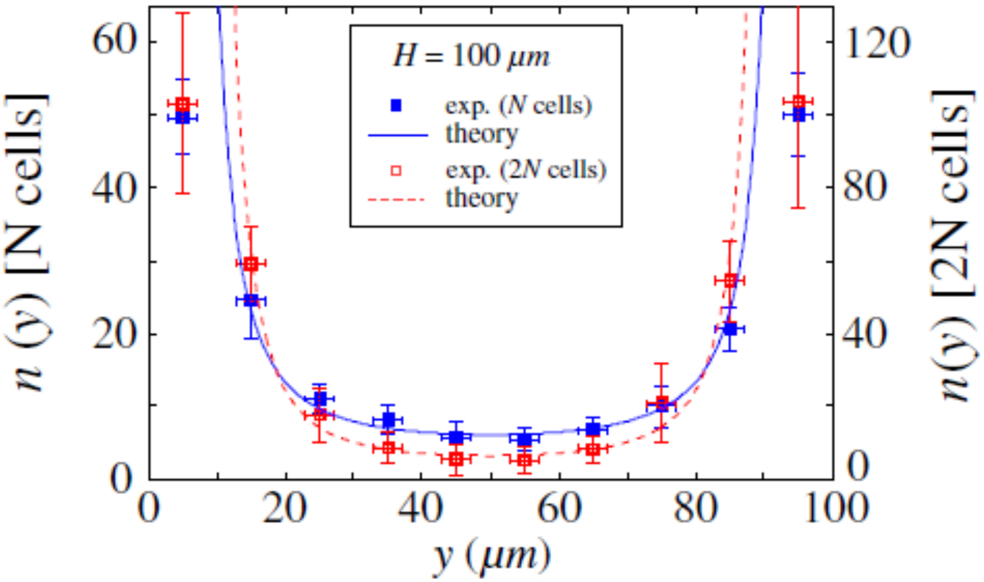
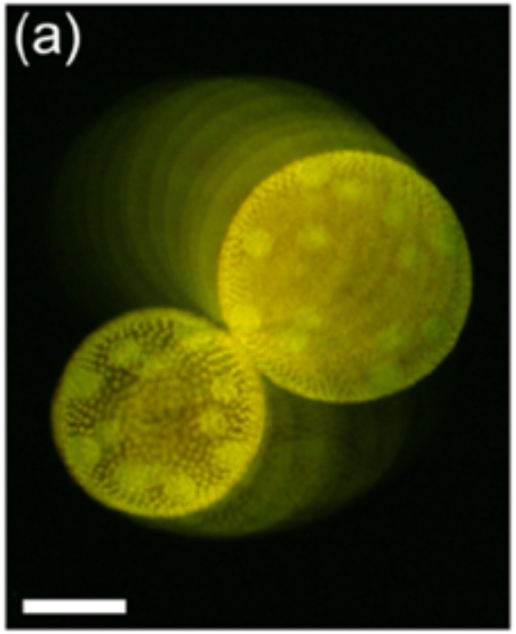


FIGURE
E. coli cell
(Left) Sup
superposit
length and
curvature





Accumulation of E. Coli near boundaries (measurement and force dipole singularity model), (Berke et al. 2008)

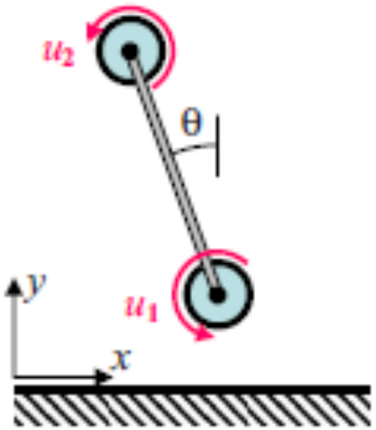


“waltzing” motion of pair of Volvox , (Drescher et al. 2009)

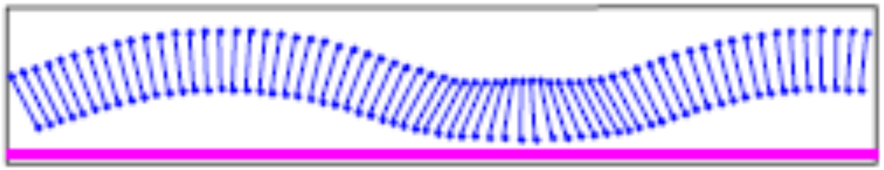
density-matched fluid. (b) The cell mixture is deposited between

FIG. 1 (color online). Waltzing of *V. carteri*. (a) Top view.

Preceding studies： bouncing above a no-slip wall

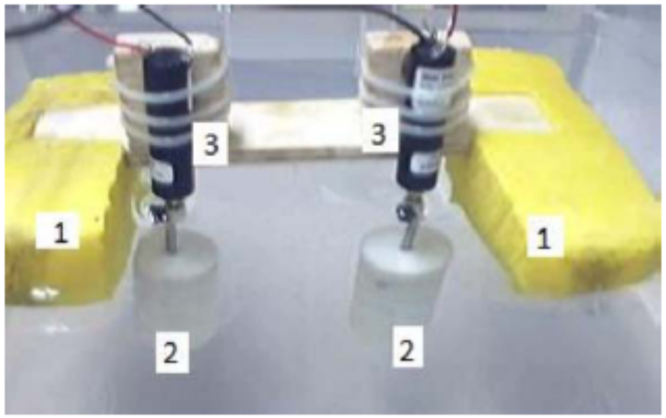


Two-rotating-sphere model

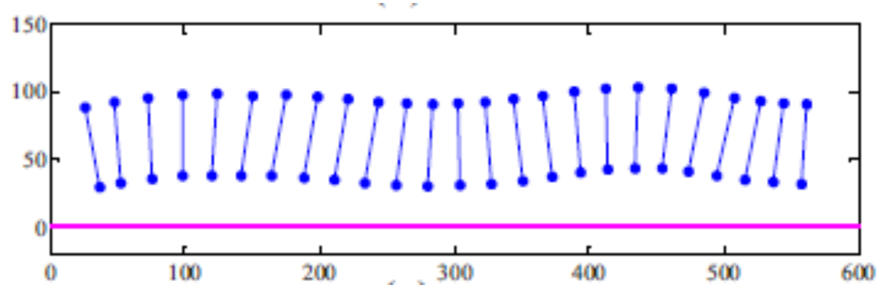


Trajectory of the two-sphere swimmer near wall

(Or and Murray, 2009)



Rotating two cylinders to generate macroscale robotic prototype swimming



Trajectory of the swimmer

(Zhang et al. , 2010)

assume
1 . . . m
fixed t
attache

were co
that in
$u_1 = -u_2$
axis du
[13]. H
and for
swims

We
swimn
that th
wall i
then d
lead to
showe
wall.

Stokeslet, stresslet, rotelet …

Stokes equation $\nabla \cdot \sigma = \nabla p - \eta \Delta u = 0,$ σ : Stress tensor

$$u(x) - u^{\infty}(x) = -\oint_{S_P} \left(\sigma(\xi)\cdot n\right)\cdot G(x-\xi)\,dS(\xi),$$ G : Dyadic Green's function

At $|x| \gg |\xi|$, take the Taylor series in ξ about $\xi = 0$ $G_{ij}(x) = \frac{1}{r}\delta_{ij} + \frac{1}{r^3}x_i x_j$

$$= -\frac{F_j}{8\pi\eta}G_{ij}(x) + \frac{D_{jk}}{8\pi\eta}G_{ij,k} + \cdots ,$$

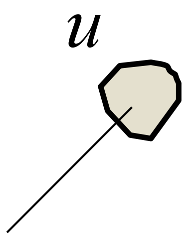
$$F_j = \oint_{S_P} (\sigma\cdot\hat{n})_j\,dS, \quad D_{jk} = \oint_{S_P} (\sigma\cdot\hat{n})_j\,\xi_k\,dS$$

Ambient flow field: u^{∞}

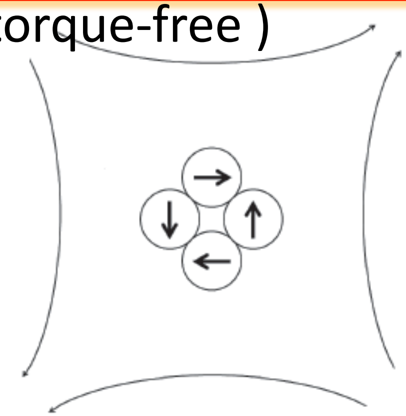
Stokeslet (point force)

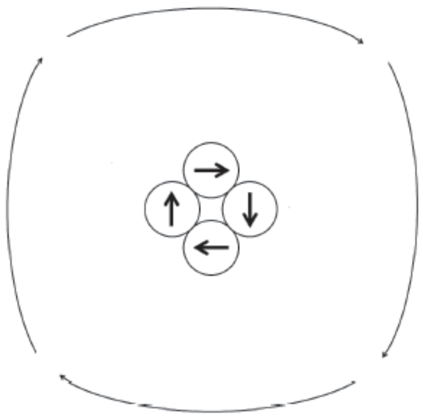
Symmetric part: Stresslet (force-free, torque-free)

Asymmetric part: rotlet (point torque)

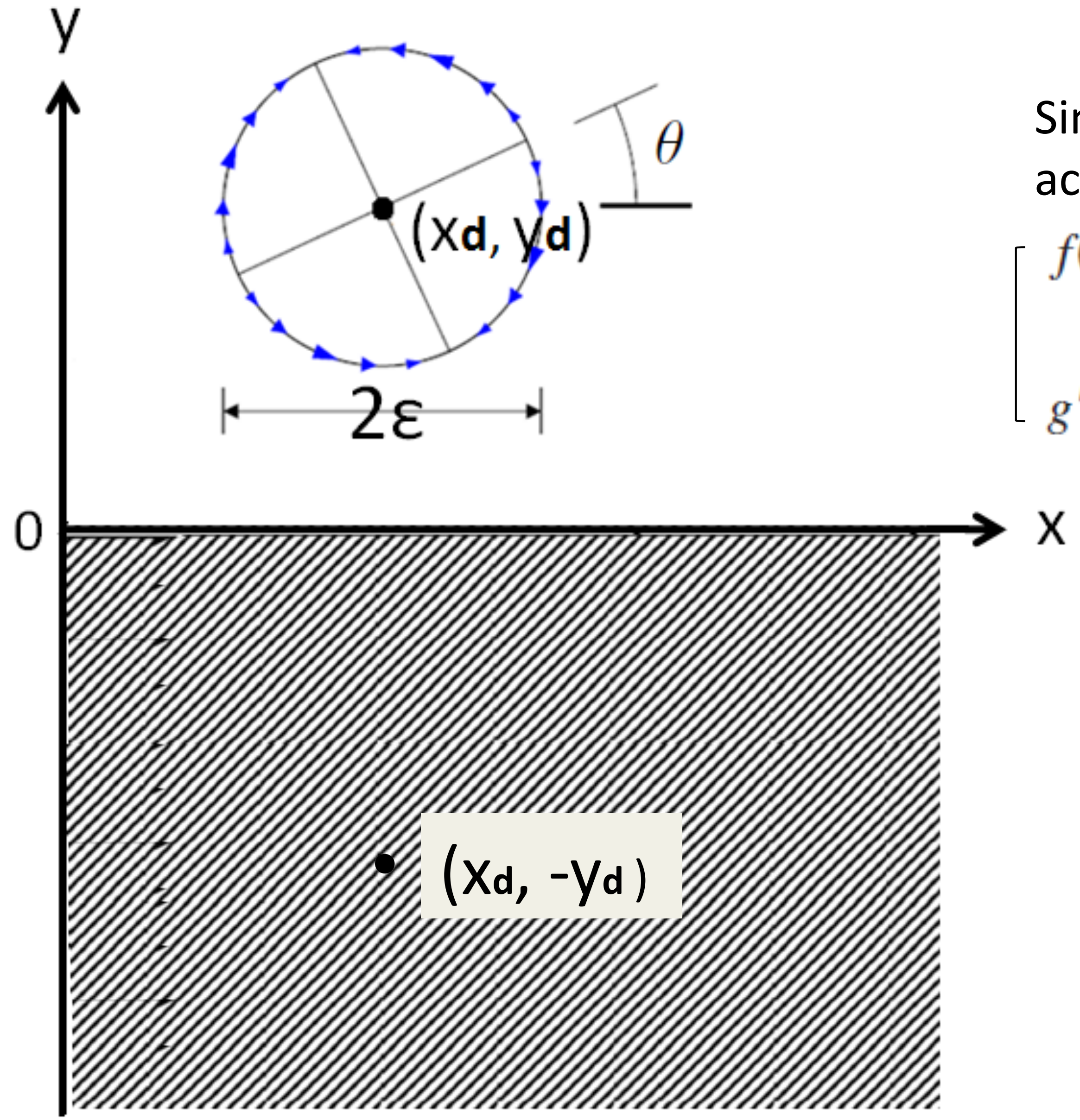


Rigid particle





Preceding studies(singularity model):
Near an infinite no-slip wall (Crowdy and Or , 2010)



Simplification to the synchronised action of cilia.

$$f(z) = \frac{\mu}{z - z_0} + f_0 + f_1(z - z_0) + \cdots,$$

$$g'(z) = \frac{\mu \overline{z_0}}{(z - z_0)^2} + \frac{2\mu\epsilon^2}{(z - z_0)^3} + g_0 + \cdots.$$

⇩

$$\dot{x} = -\frac{\sin(2\theta)}{y}\left(1 - \frac{\epsilon^2}{2y^2}\right),$$

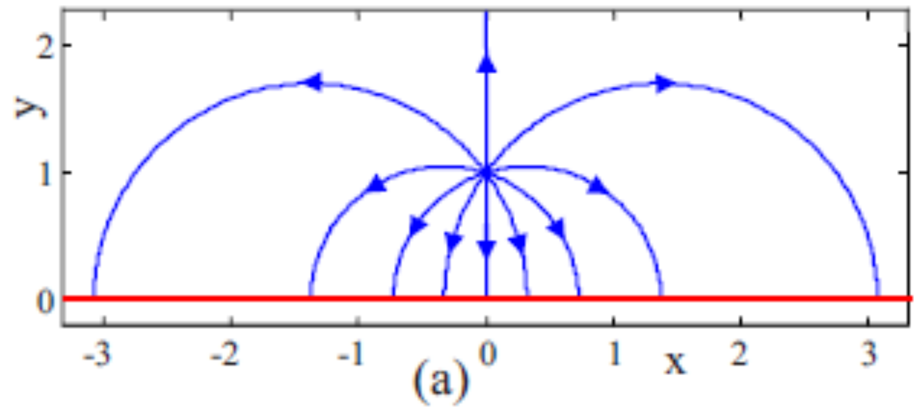
$$\dot{y} = \frac{\cos(2\theta)}{y}\left(1 - \frac{\epsilon^2}{y^2}\right),$$

$$\dot{\theta} = \frac{\sin(2\theta)}{2y^2}\left(1 - \frac{3\epsilon^2}{2y^2}\right).$$

Preceding studies(singularity model):
Near an infinite no-slip wall (Crowdy and Or , 2010)

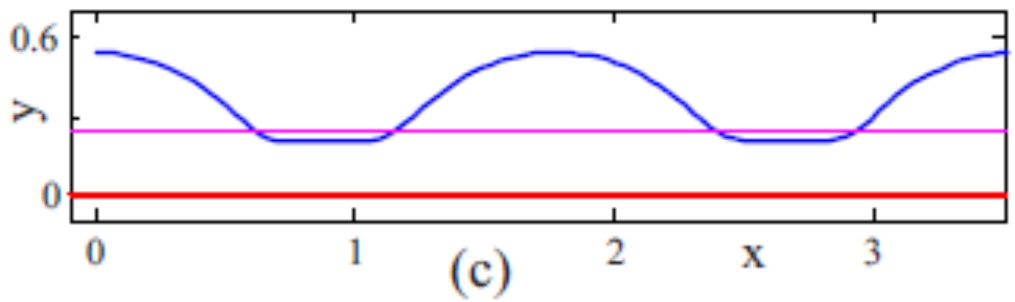
$\epsilon=0$

A point swimmer with $y(0)=1$ and different values of $\theta(0)$



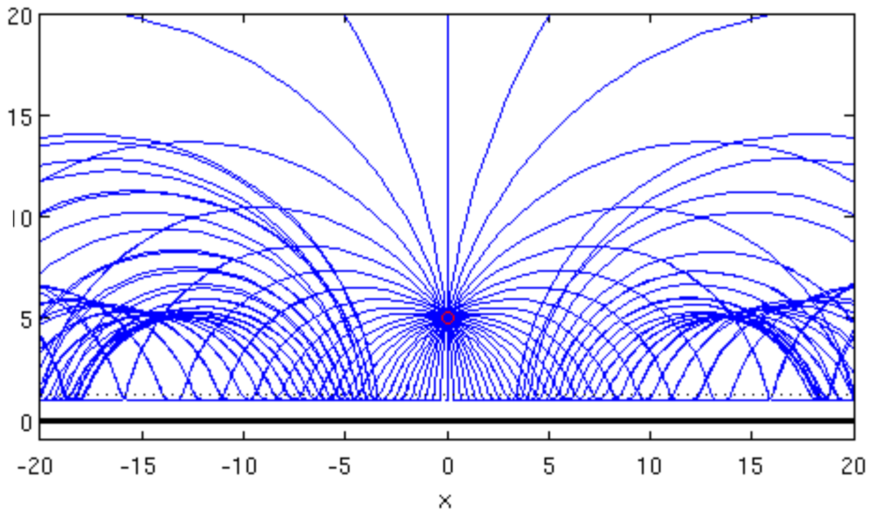
(a)

A swimmer with $\epsilon=0.2$ for $\theta(0)=-\pi/4$, $y(0)=y_e+0.3$.

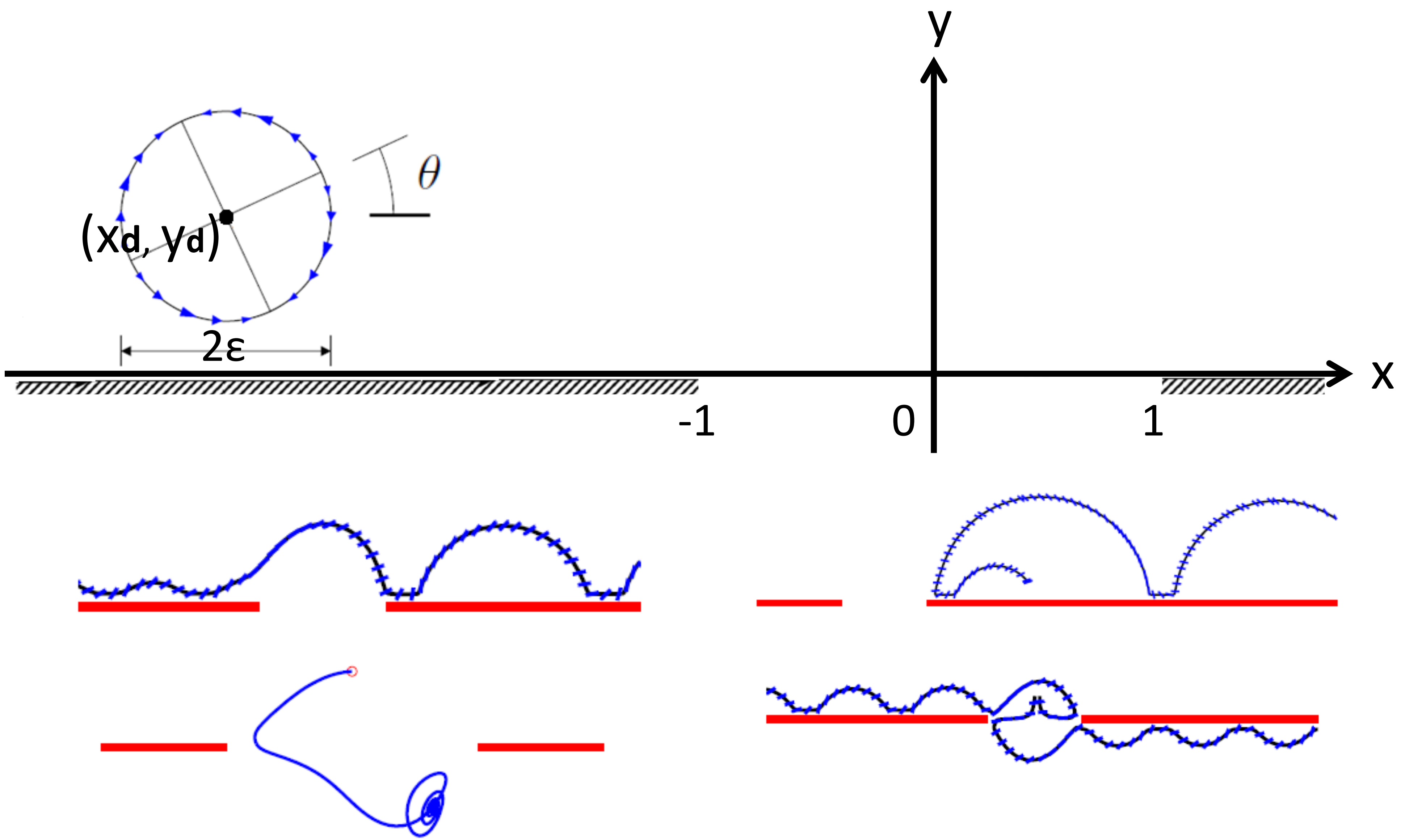


(c)

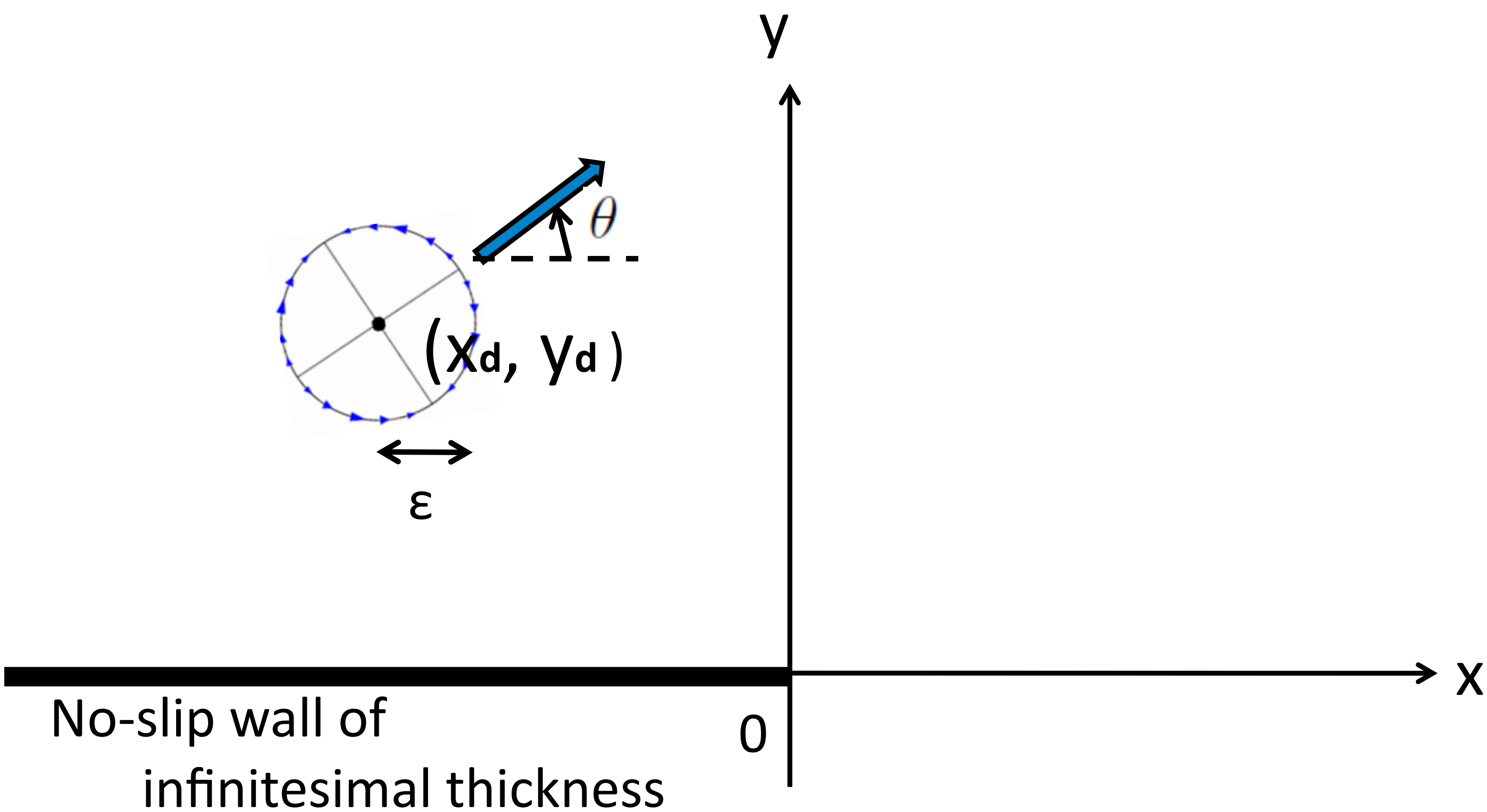
A swimmer with $\epsilon=0.2$ for $y(0)=y_e+0.3$



Preceding studies(singularity model):
Near an infinite no-slip wall with a gap (Crowdy and Ophir, 2010)



Project: Near an half-infinite no-slip wall ??



tangential velocity profile: $U(\phi, t) = 2V \sin(2(\phi - \theta))$

V :const. , sets the time scale for the treadmilling action by $V = \epsilon^{-1}$

Biharmonic equation and Goursat functions

Stokes equation in a complex plane ($z \equiv x + iy$):

$$\nabla p = \eta \Delta \mathbf{u}, \ \nabla \cdot \mathbf{u} = 0 \ \xrightarrow{\nabla \times} \ \Delta^2 \psi = 0.$$ (Biharmonic equation)

$$\boxed{\frac{\partial^4}{\partial z^2 \partial \overline{z}^2} \psi = 0.}$$

general solution for ψ:

$$\boxed{\mathrm{Im}[\overline{z} f(z) + g(z)],}$$

$f(z)$ and $g(z)$: Goursat functions (analytic functions)

complex velocity field:

$$\boxed{u_x + i u_y = -f(z) + z \overline{f'(z)} + \overline{g'(z)}}$$

Singularity model for 2D Stokes flow

Stokeslet at z_d. ($\mu \in \mathbf{C}$)

$$f(z) = \mu \log(z - z_d) + \text{analytic function},$$

$$g'(z) = -\frac{\mu \overline{z_d}}{\overline{z} - \overline{z_d}} - \overline{\mu} \log(z - z_d) + \text{analytic function}$$

stresslet at z_d ($\mu \in \mathbf{C}$)

$$f(z)\frac{\mu}{z - z_d} + \text{analytic function},$$

$$g'(z) = \frac{\mu \overline{z_d}}{(\overline{z} - \overline{z_d})^2} + \text{analytic function}$$

rotlet at z_d if $c \in \mathbf{C}$ (source/sink if $c \in \mathbf{R}$)

$$g(z) = c \log(z - z_d)$$

Treadmilling swimmer and a stresslet

Seek solutions for Goursat functions of the form;

$$f(z) = \frac{\mu}{z - z_d} + f_0 + f_1(z - z_d) + \mathcal{O}((z - z_d)^2),$$

stresslet of strength μ

$$g'(z) = \frac{2\mu\epsilon^2}{(z - z_d)^3} + \frac{\mu\overline{z_d}}{(z - z_d)^2} + g_0 + \mathcal{O}((z - z_d))$$

quadrupole of strength $2\epsilon^2\mu$

No Stokeslet: Force-free

No rotlet: Torque-free

Boundary condition on the wall:

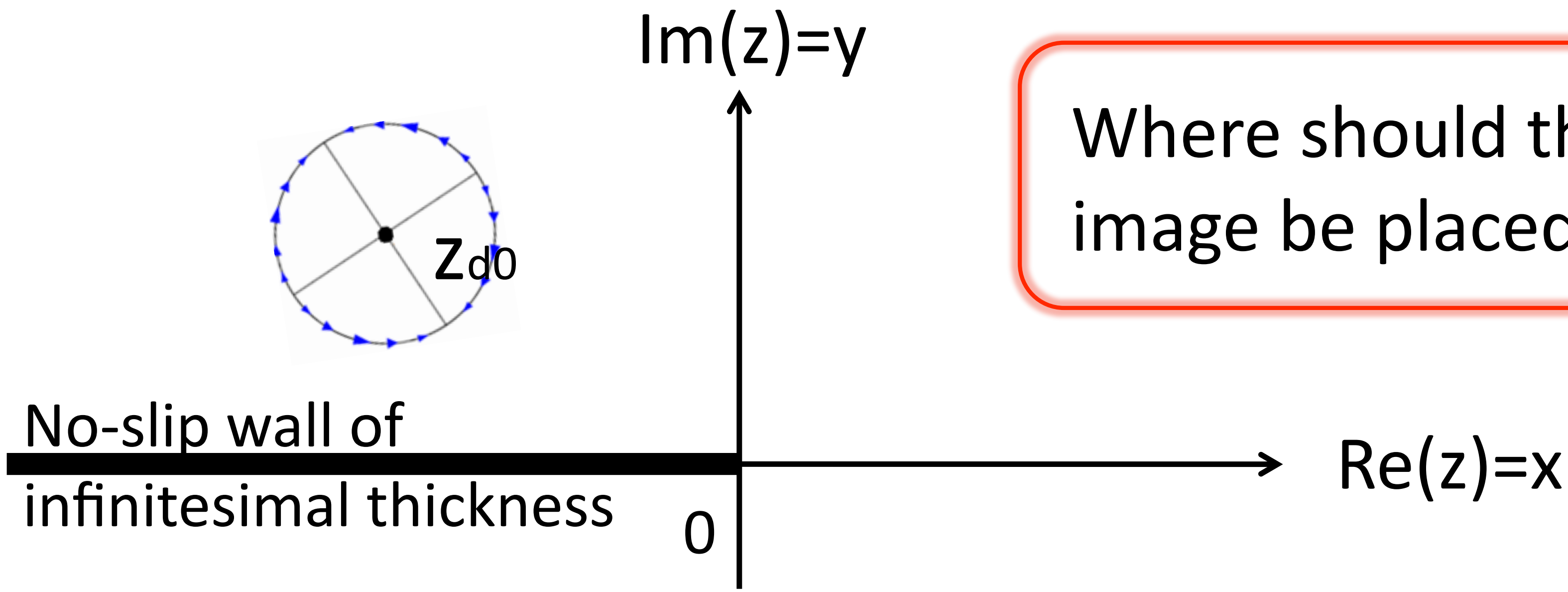
$$u_x + iu_y = -f(z) + z\overline{f'(z)} + \overline{g'(z)} = 0.$$

Image of the swimmer

Goursat functions

$$f(z) = \frac{\mu}{z - z_d} + f_0 + f_1(z - z_d) + \mathcal{O}((z - z_d)^2),$$

$$g'(z) = \frac{2\mu\epsilon^2}{(z - z_d)^3} + \frac{\mu\overline{z_d}}{(z - z_d)^2} + g_0 + \mathcal{O}((z - z_d))$$

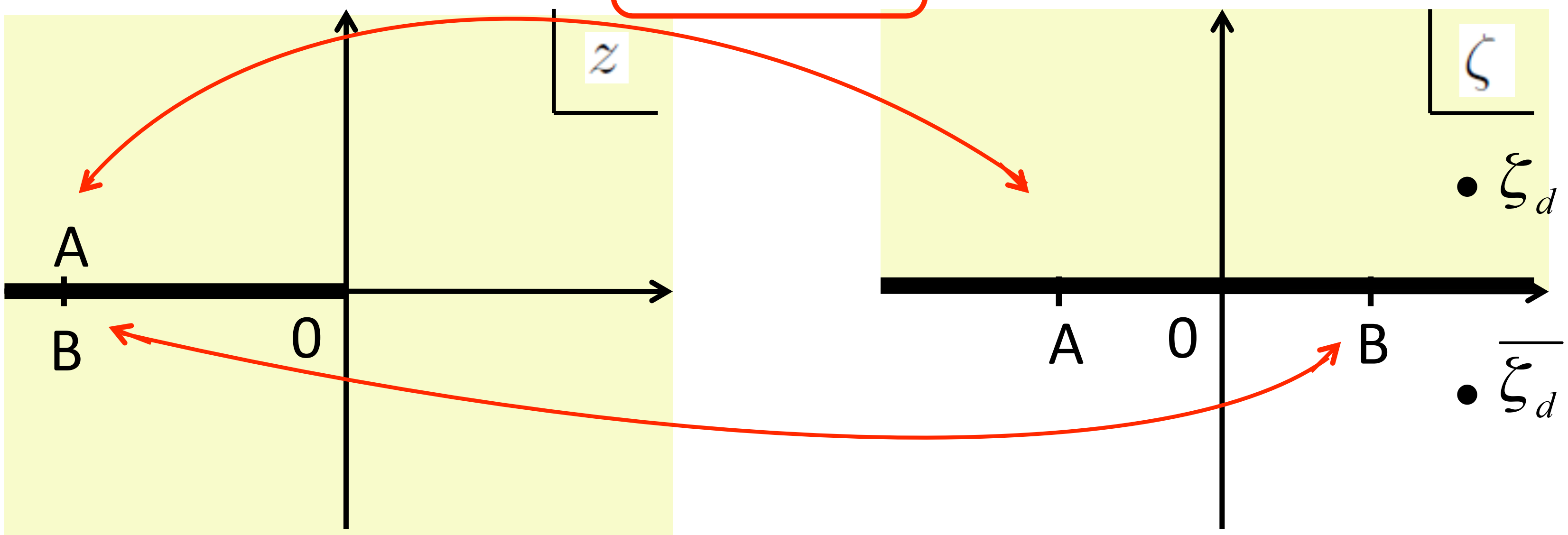


Where should the image be placed ?

Conformal mapping (change of variables)

$$\zeta = iz^{1/2}$$

$$\overline{\zeta} = \zeta \text{ on the wall}$$



$$z = z(\zeta) = -\zeta^2 \quad \overline{z_d} \equiv \overline{z(\zeta_d)} \equiv z(\overline{\zeta_d}).$$

$\Longrightarrow$

$$F(\zeta) \equiv f(z(\zeta)),$$
$$G(\zeta) \equiv g'(z(\zeta)),$$

Analytical single-valued functions

$F(\zeta)$ and $G(\zeta)$

Assume $F(\zeta)$ to have the form (image system method)

$$F(\zeta) = \frac{A}{\zeta-\zeta_d} + \frac{B}{\left(\zeta-\overline{\zeta_d}\right)^3} + \frac{C}{\left(\zeta-\overline{\zeta_d}\right)^2} + \frac{D}{\left(\zeta-\overline{\zeta_d}\right)} + E,$$

$E \to 0$

$G(\zeta)$ is determined by the boundary condition on the wall :

$$-f(z) + z\overline{f'(z)} + \overline{g'(z)} = 0.$$

$$\Longrightarrow G(\zeta) = \overline{f(z)} - \overline{z}f'(z) = \overline{F(\zeta)} - \overline{z}\frac{d\zeta}{dz}F'(\zeta) = \overline{F}(\zeta) - \frac{1}{2}\zeta F'(\zeta).$$

$$G(\zeta) = \frac{\overline{A}}{\zeta-\overline{\zeta_d}} + \frac{\overline{B}}{(\zeta-\zeta_d)^3} + \frac{\overline{C}}{(\zeta-\zeta_d)^2} + \frac{\overline{D}}{(\zeta-\zeta_d)} - \frac{1}{2}\zeta\left[\frac{-A}{(\zeta-\zeta_d)^2} + \frac{-3B}{\left(\zeta-\overline{\zeta_d}\right)^4} + \frac{-2C}{\left(\zeta-\overline{\zeta_d}\right)^3} + \frac{-D}{\left(\zeta-\overline{\zeta_d}\right)^2}\right].$$

$F(\zeta)$, $G(\zeta)$, $f(z)$, and $g(z)$

necessary boundary condition on the treadmiller's body :

Near z_d, $f(z(\zeta))$ and $g'(z(\zeta))$ have to have singularities written as

$$f(z) = \boxed{\frac{\mu}{z - z_d}} + f_0 + f_1(z - z_d) + \mathcal{O}((z - z_d)^2),$$

$$g'(z) = \boxed{\frac{2\mu\epsilon^2}{(z - z_d)^3}} + \boxed{\frac{\mu\overline{z_d}}{(z - z_d)^2}} + g_0 + \mathcal{O}((z - z_d))$$

$$\Downarrow$$

$$A = i\frac{1}{2}\,\mu\, z_d^{-1/2}$$

$$B = \frac{2i}{8}\,\epsilon^2\,\overline{\mu}\,\overline{z_d}^{-3/2},$$

$$C = \frac{1}{2}\overline{\mu}\left(\frac{3}{4}\epsilon^2 - \frac{1}{2}z_d\overline{z_d} + \frac{1}{2}\overline{z_d}^2\right)\overline{z_d}^{-2},$$

$$D = -\frac{i}{2}\overline{\mu}\left(\frac{3}{4}\epsilon^2 - \frac{1}{2}z_d\overline{z_d} - \frac{1}{2}\overline{z_d}^2\right)\overline{z_d}^{-5/2}.$$

$\Longrightarrow$ f_0 , f_1 , and g_0

Governing equations

$\frac{dz_d}{dt} = \dot{x}_d + i\dot{y}_d$ = velocity at $z = z_d$ induced by an image (regular part of the velocity

$$u_x + iu_y = -f(z) + z\overline{f'(z)} + \overline{g'(z)}$$

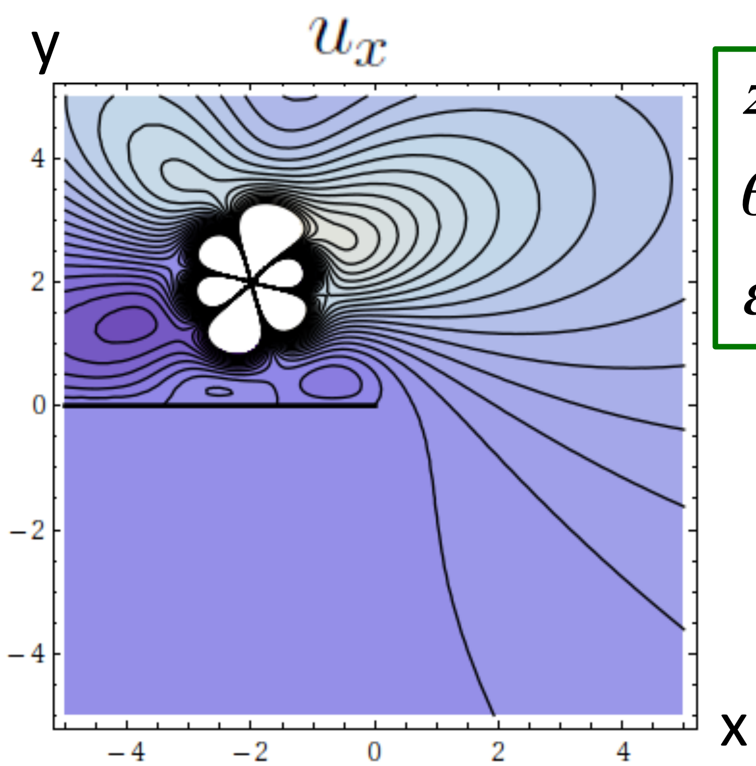
at $z = z_d$)

$$\boxed{\frac{dz_d}{dt} = -f_0 + z_d\overline{f_1} + \overline{g_0},}$$

$\frac{d\theta}{dt}$ = ½ vorticity at $z = z_d$ induced by an image (regular part of the vorticity $\omega = -4\ \mathrm{Im}[f'(z)]$ at $z = z_d$)

$$\boxed{\frac{d\theta}{dt} = -\mathrm{Im}[2f_1].}$$

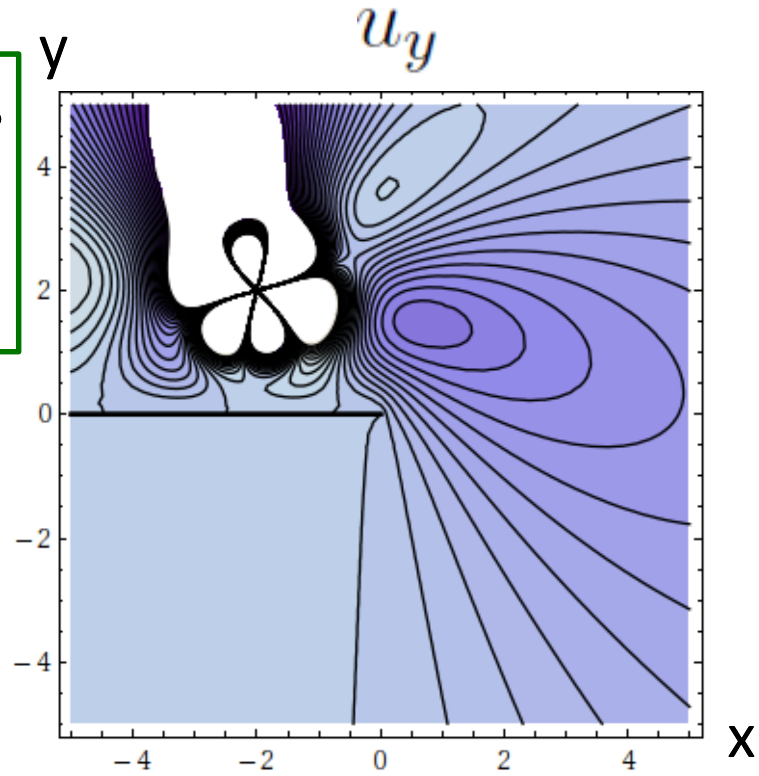
Example of the velocity field: u_x and u_y

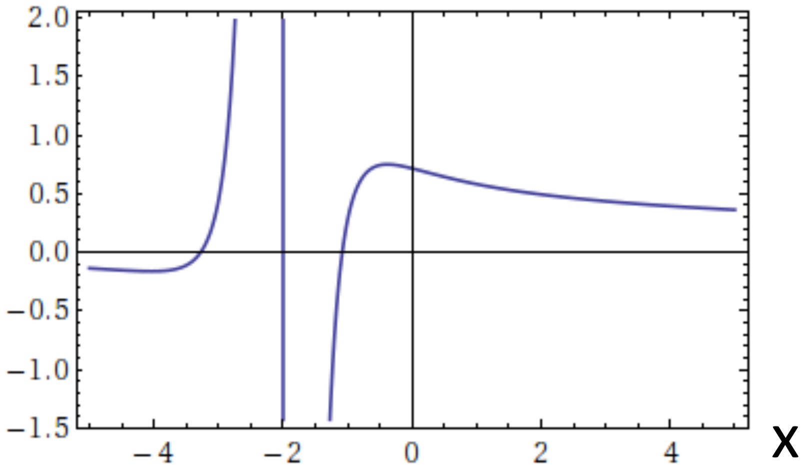


$z_d = -2 + 2i,$

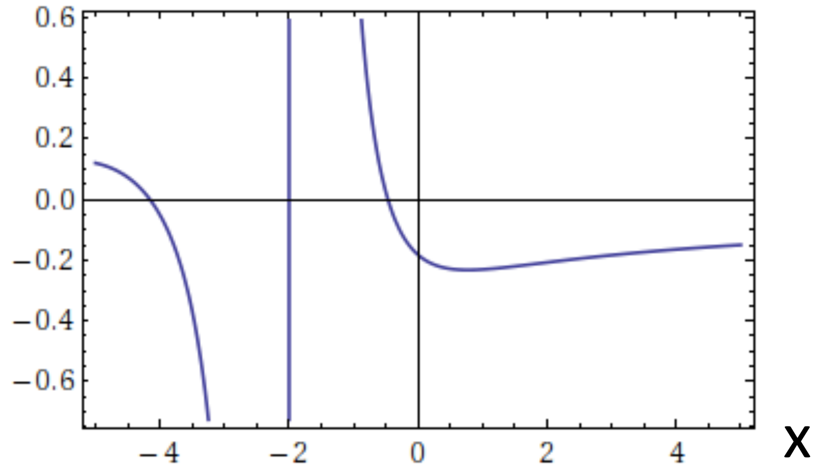
$\theta = 5\pi / 4,$

$\varepsilon = 1.$

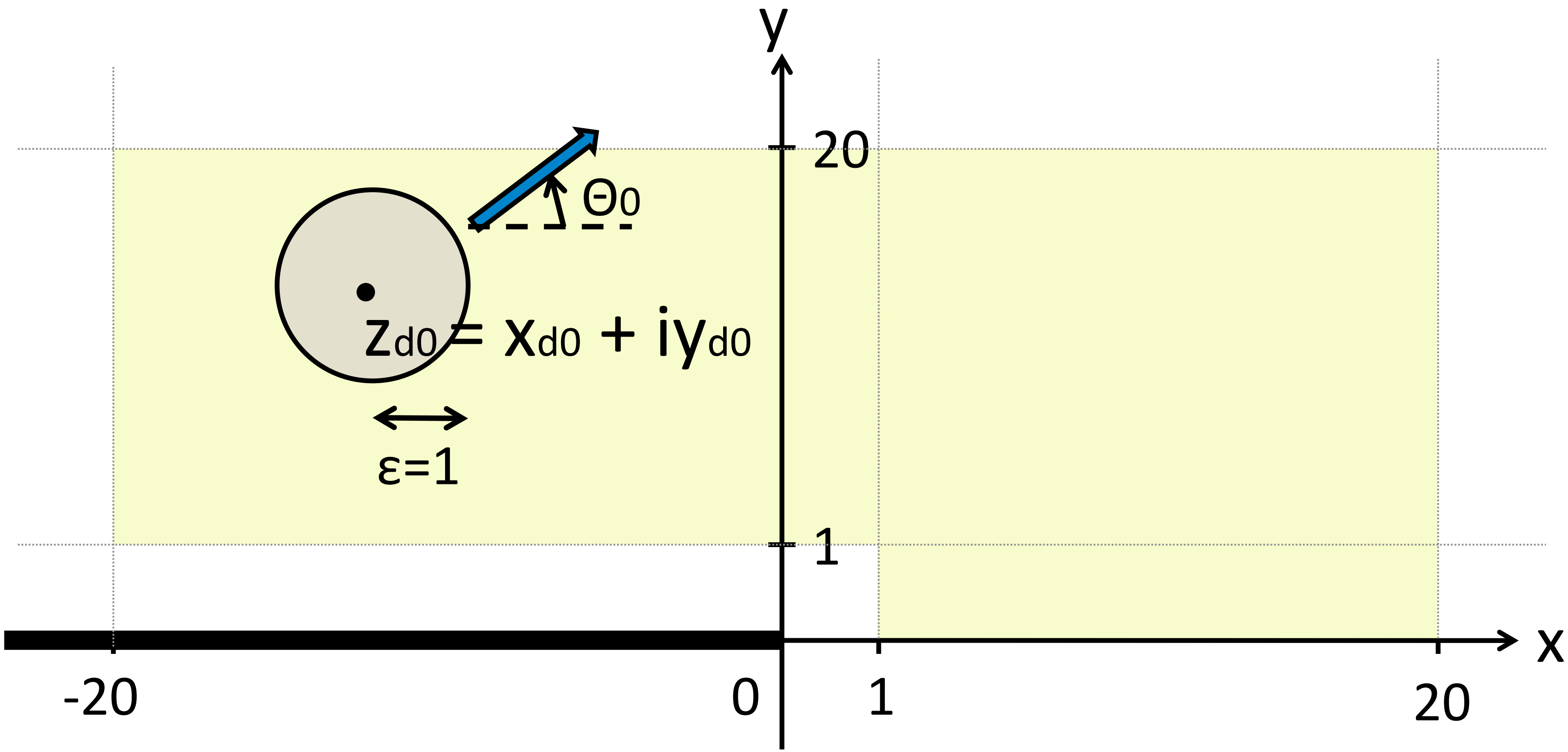




y = 2.0



Initial conditions and parameters



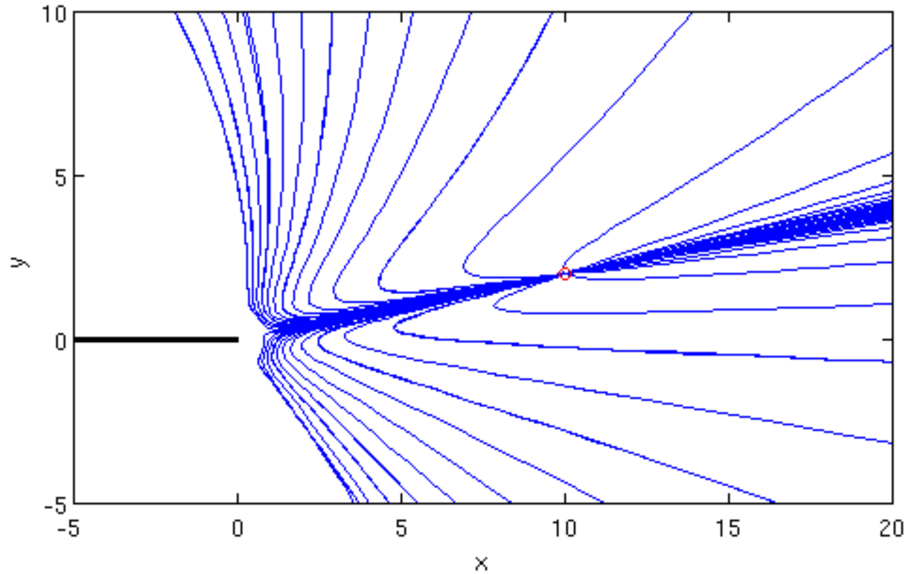
Time integration: ode45 solver,

dx = 0.5, dy = 0.5, $d\Theta_0 = \pi/100$, Maximum time: tmax = 1500,

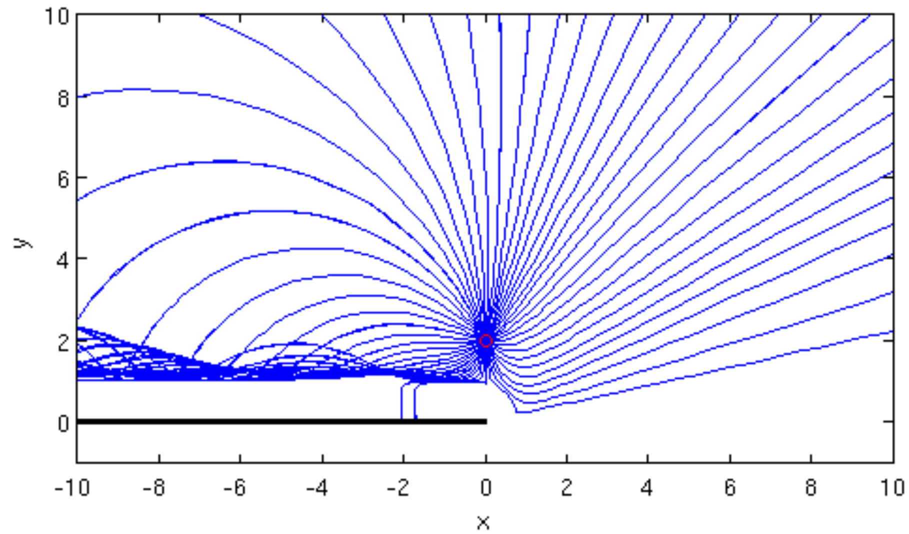
$\varepsilon = 1$, $\mu = \exp(2i\Theta(t))$

Examples of trajectories

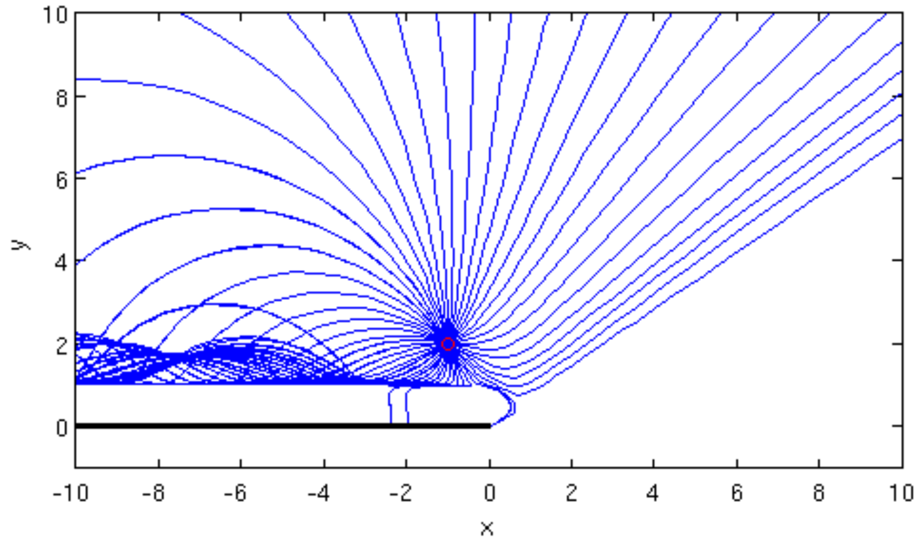
x d0 = 10, yd0 = 2



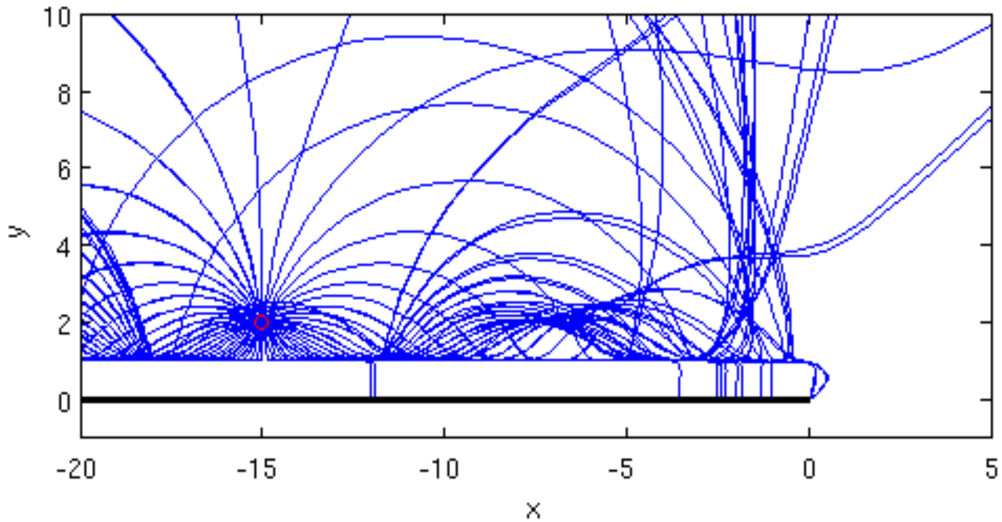
x d0 = 0, yd0 = 2



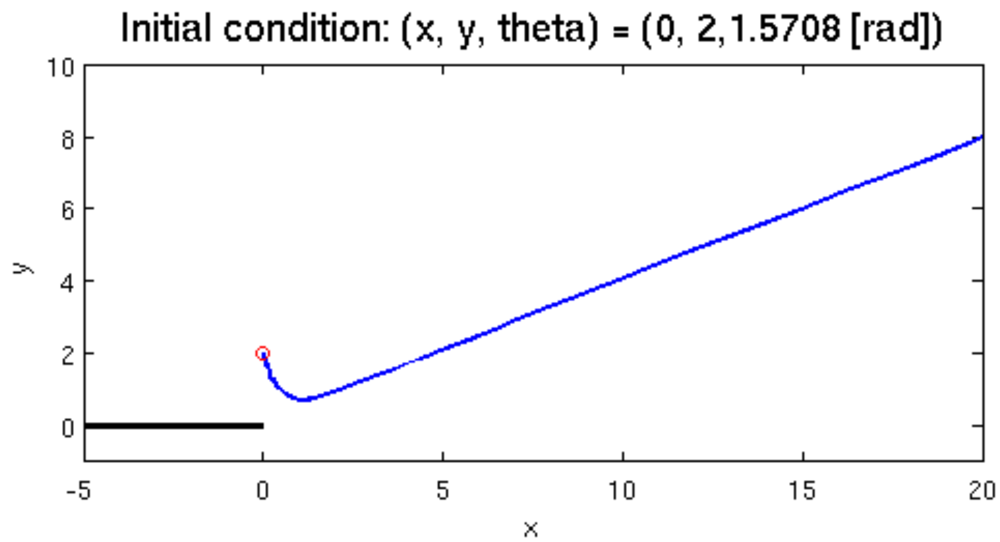
x d0 = -1, y d0 = 2

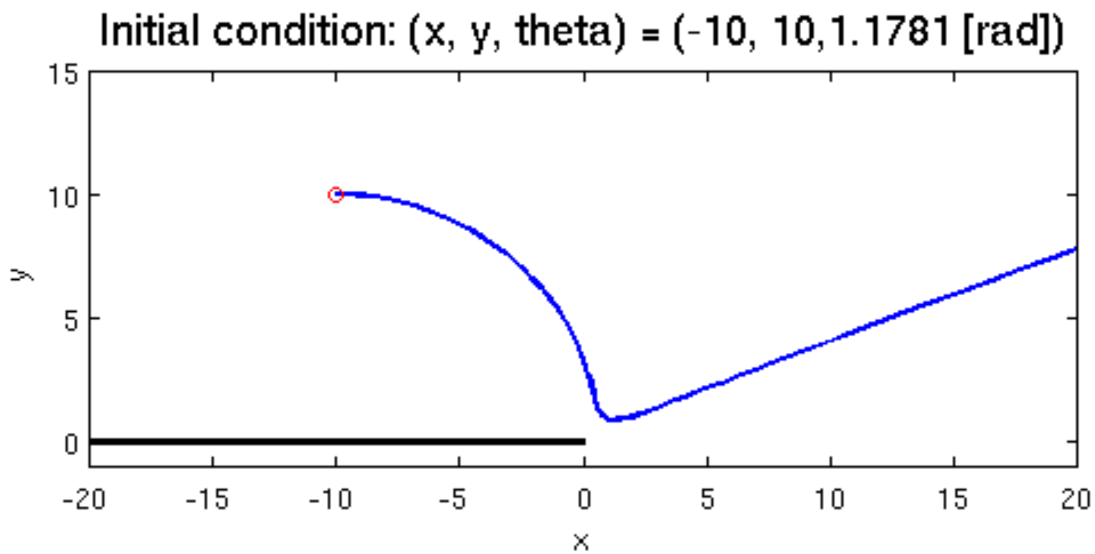


x d0 = -15, yd0 = 2

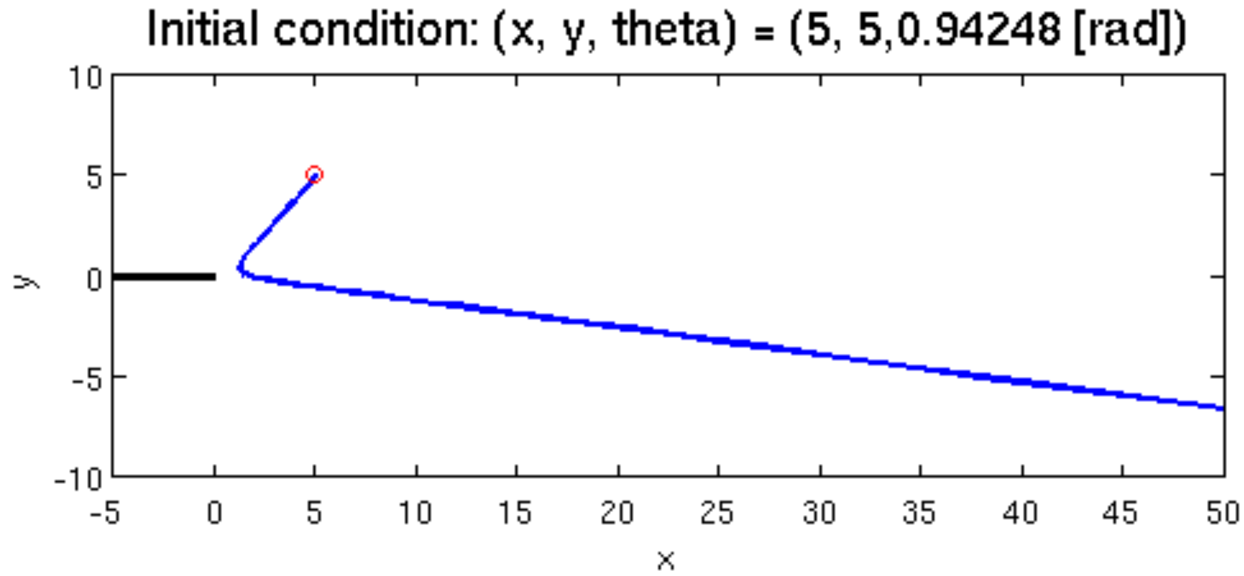


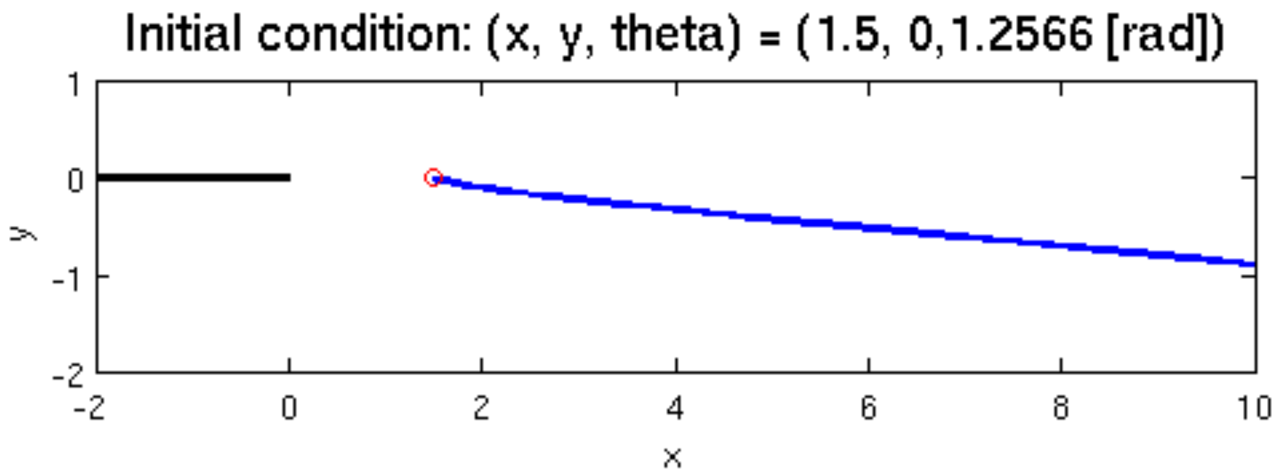
Examples of the trajectories: Escaping from the wall



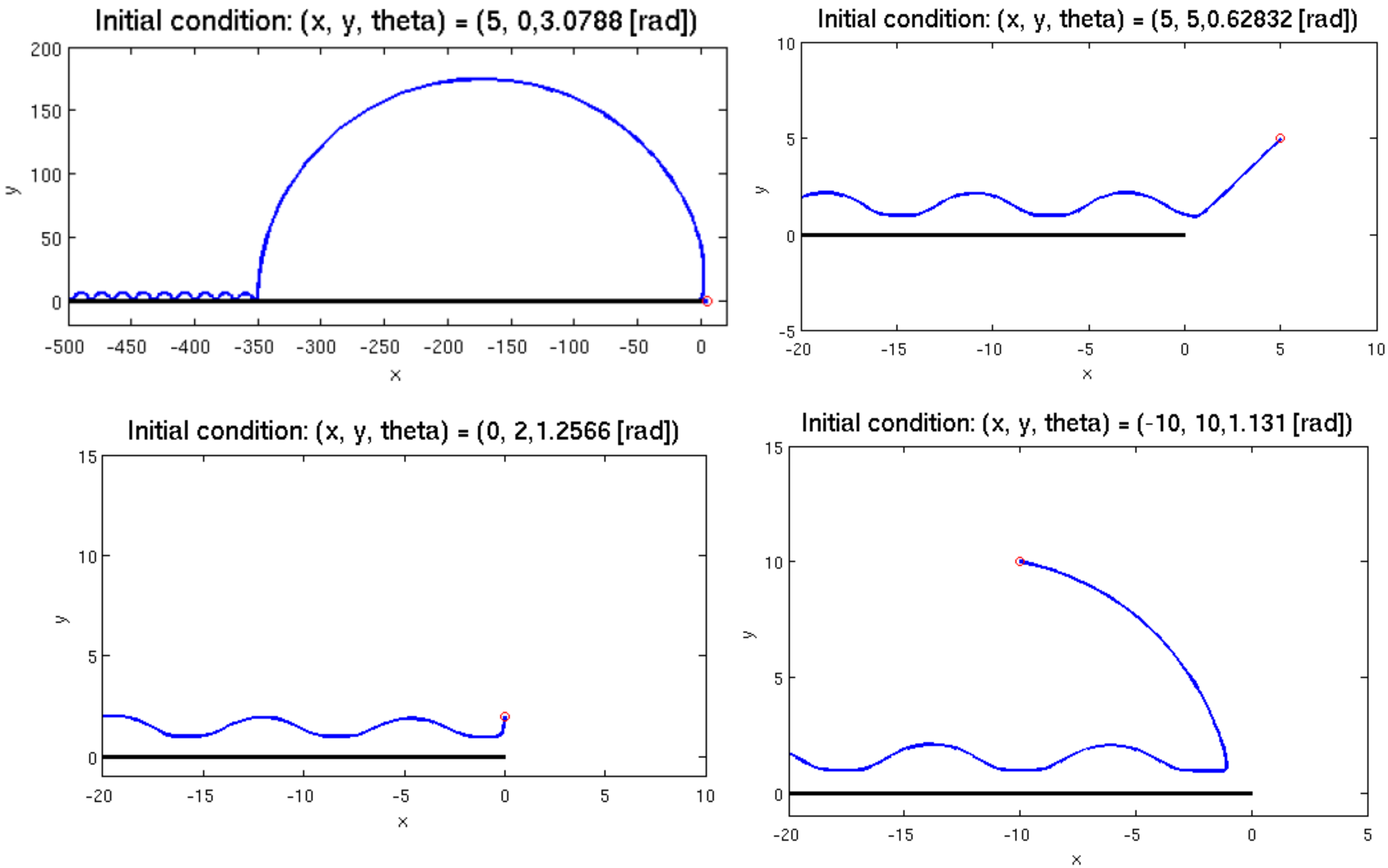


Examples of the trajectories: Escaping from the wall

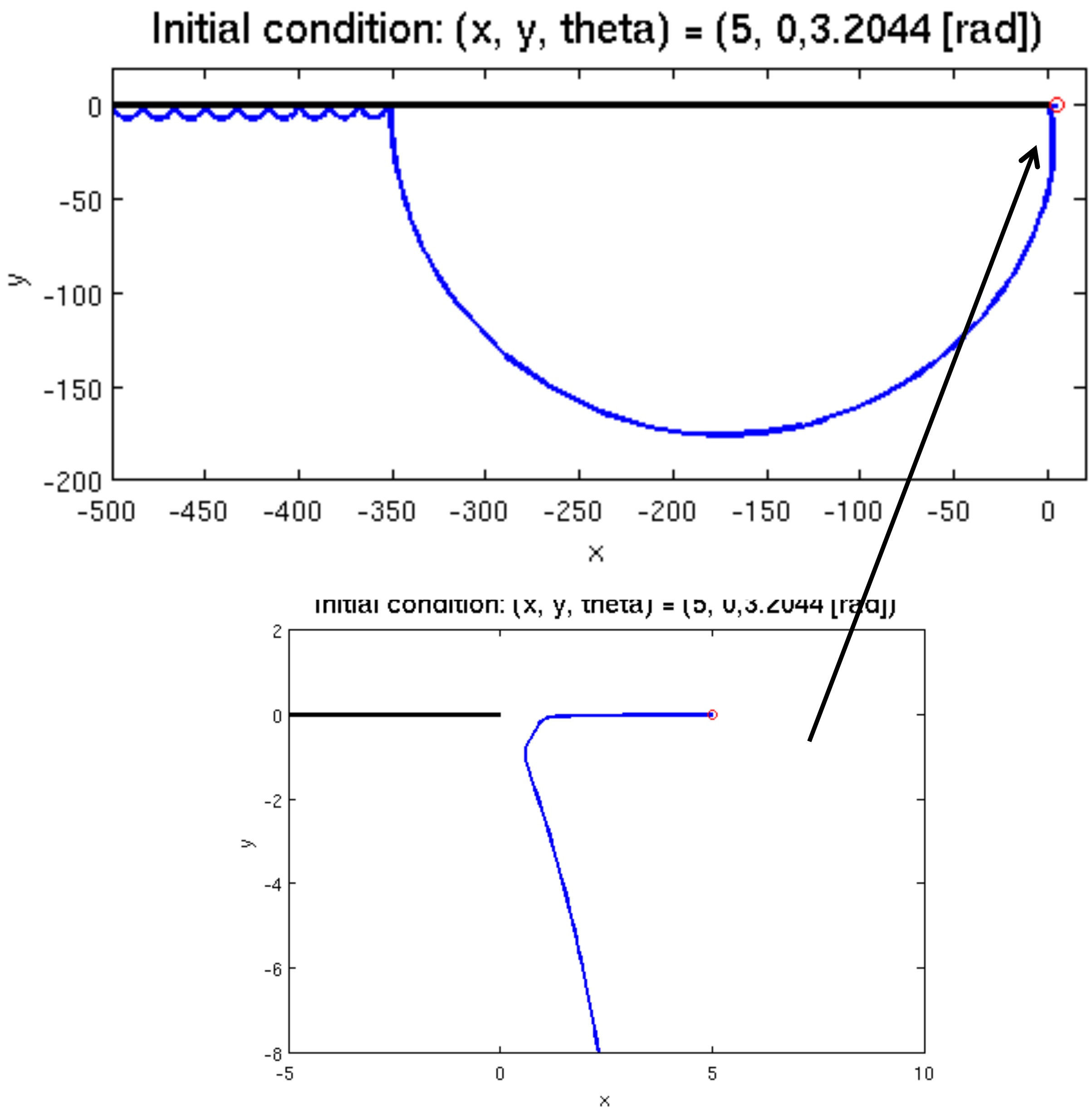




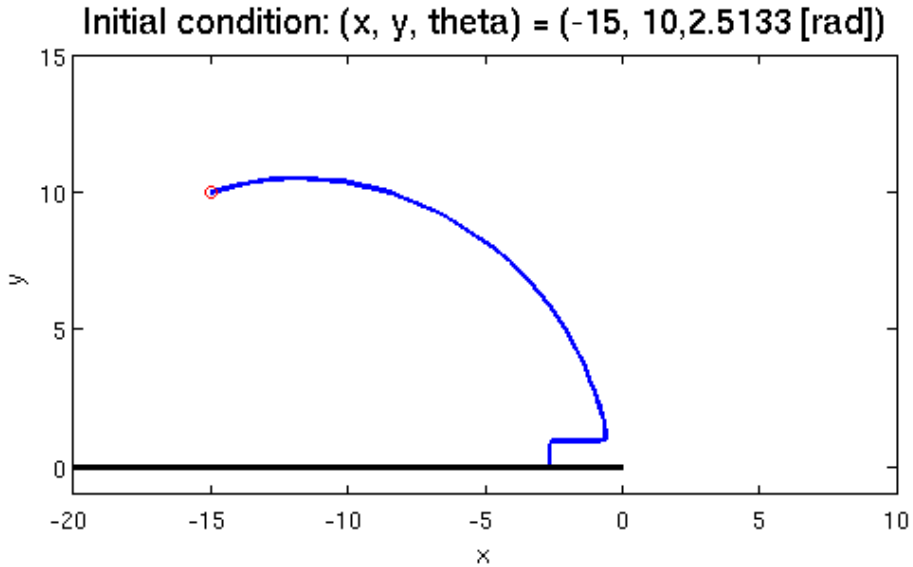
Examples of the trajectories: Being above the wall

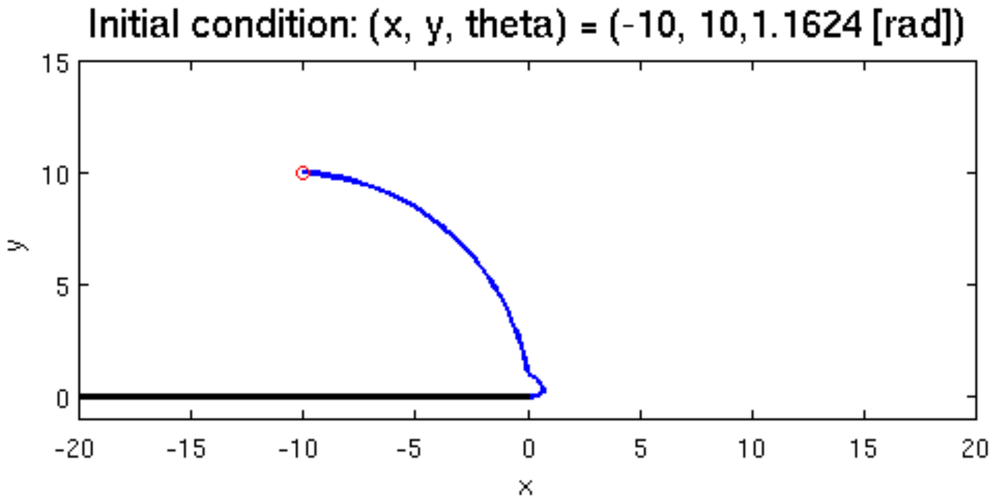


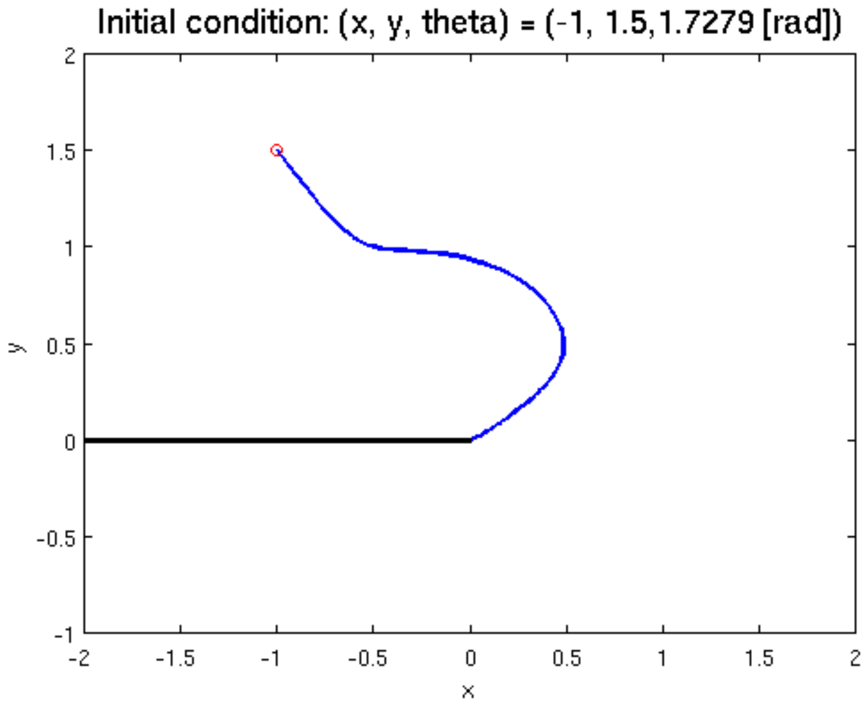
Examples of the trajectories: Going beneath the wall

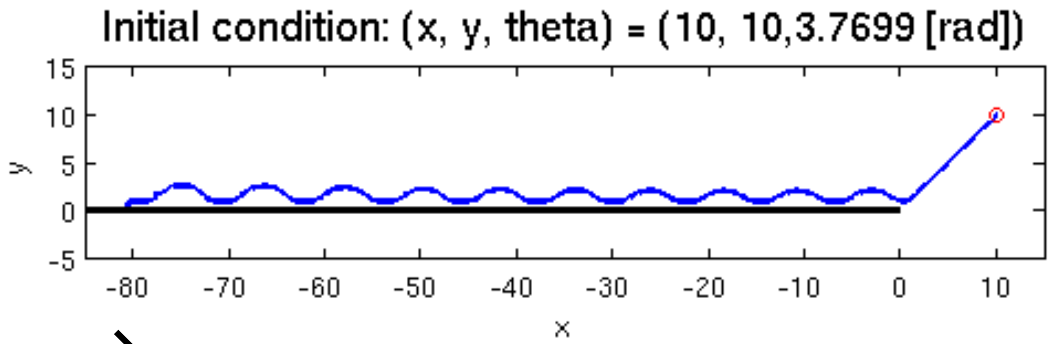


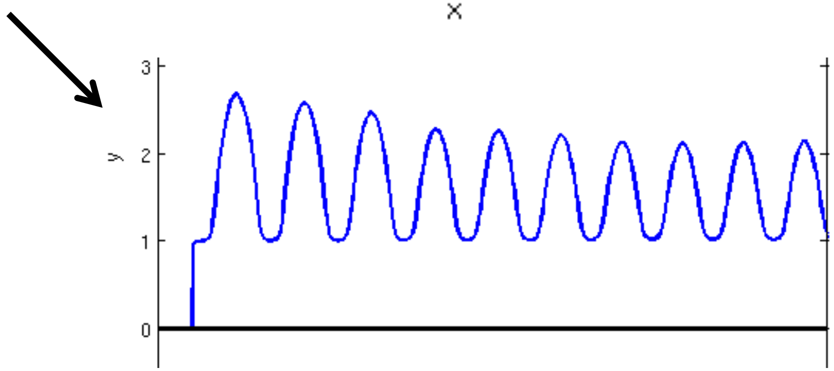
Examples of the trajectories: Crashing into the wall



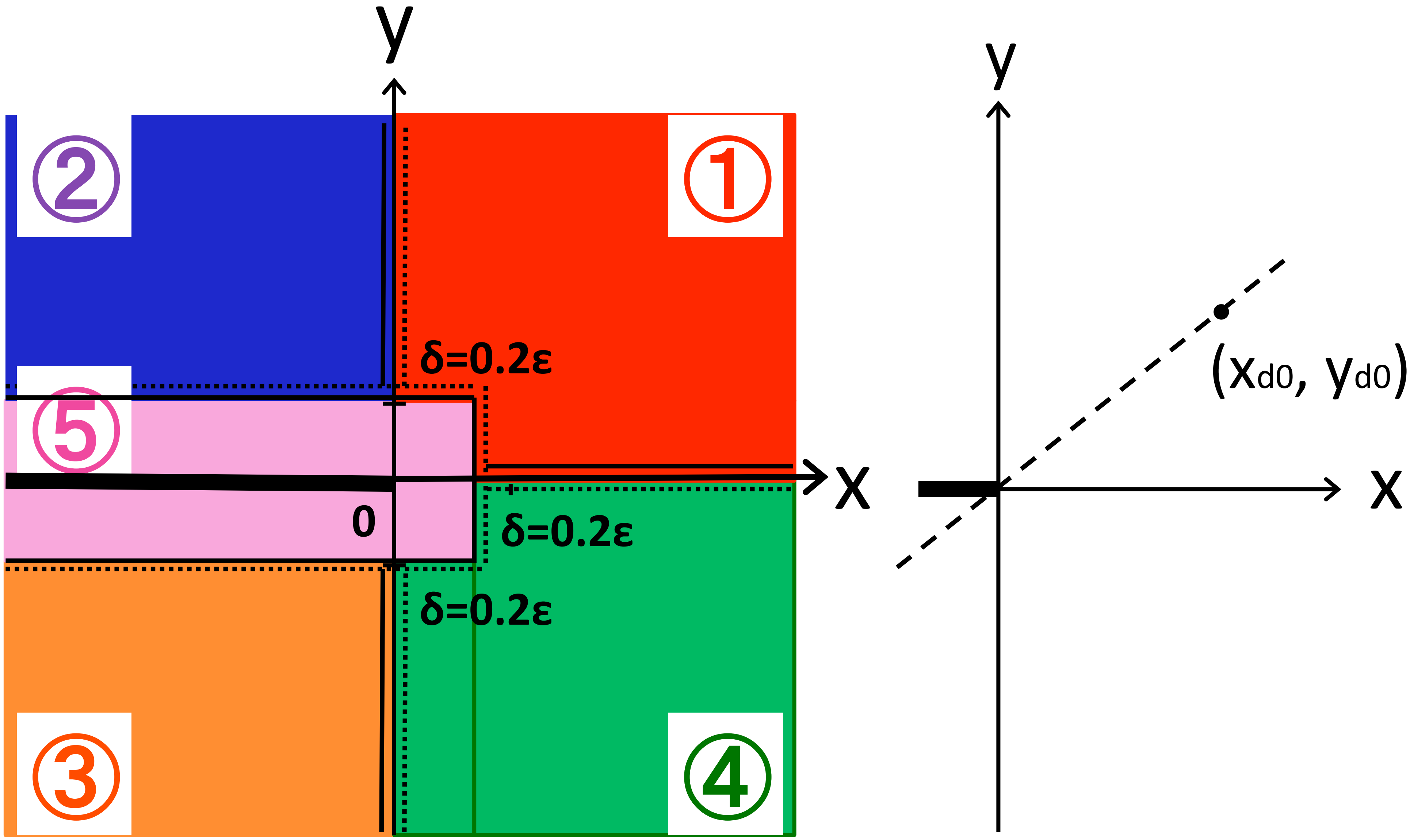






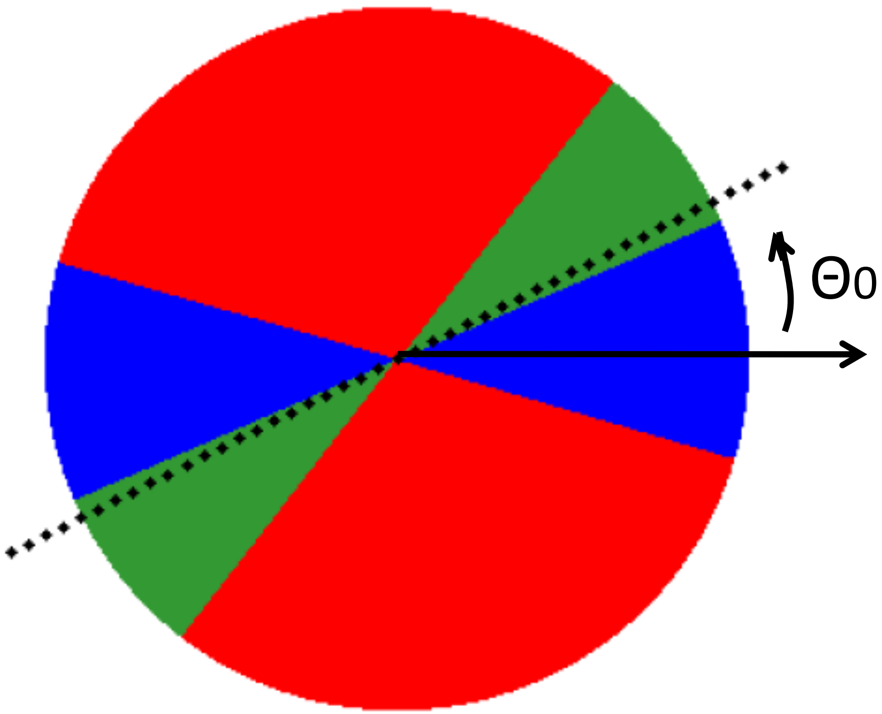


The position at a sufficiently large time

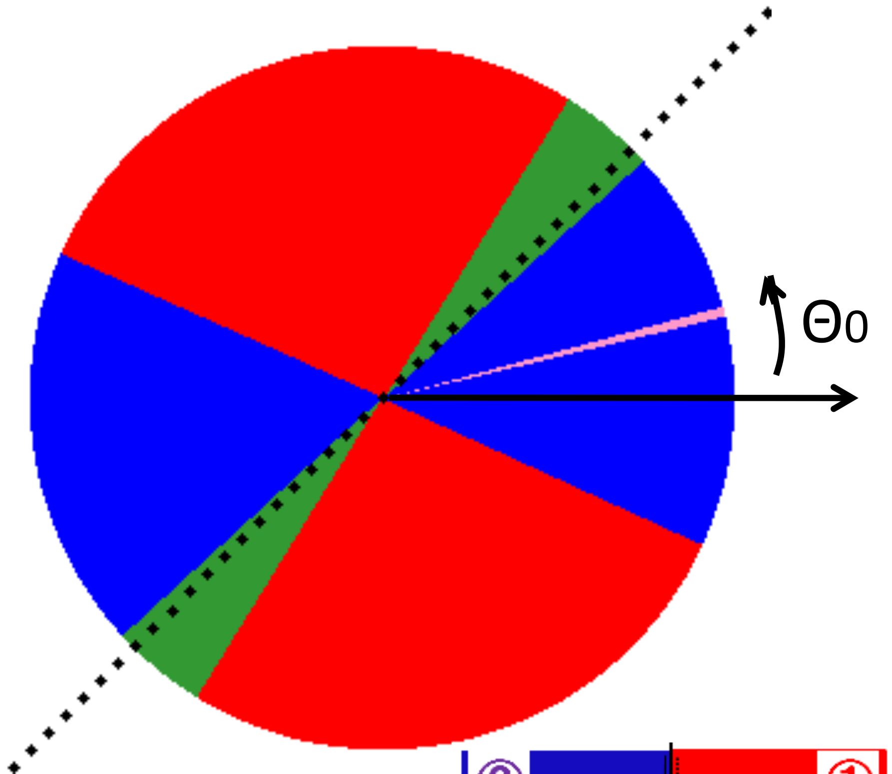


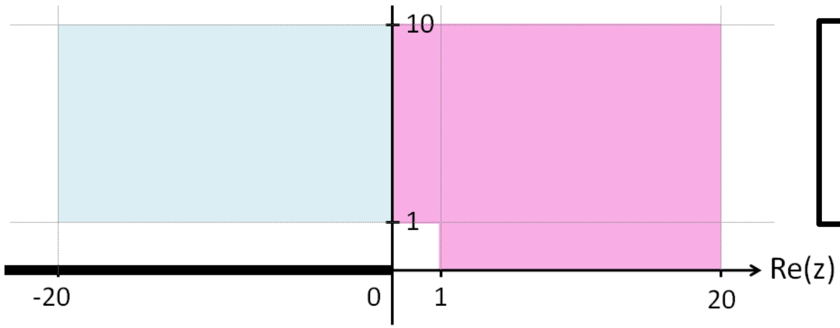
From an initial point $x_{d0} > 0$, $y_{d0} > 0$

Initial position (x, y) = (20,10)

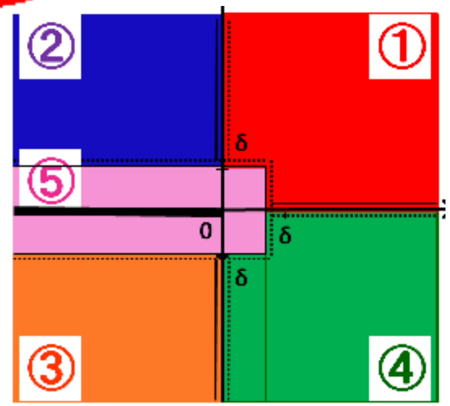


Initial position (x, y) = (2,2)

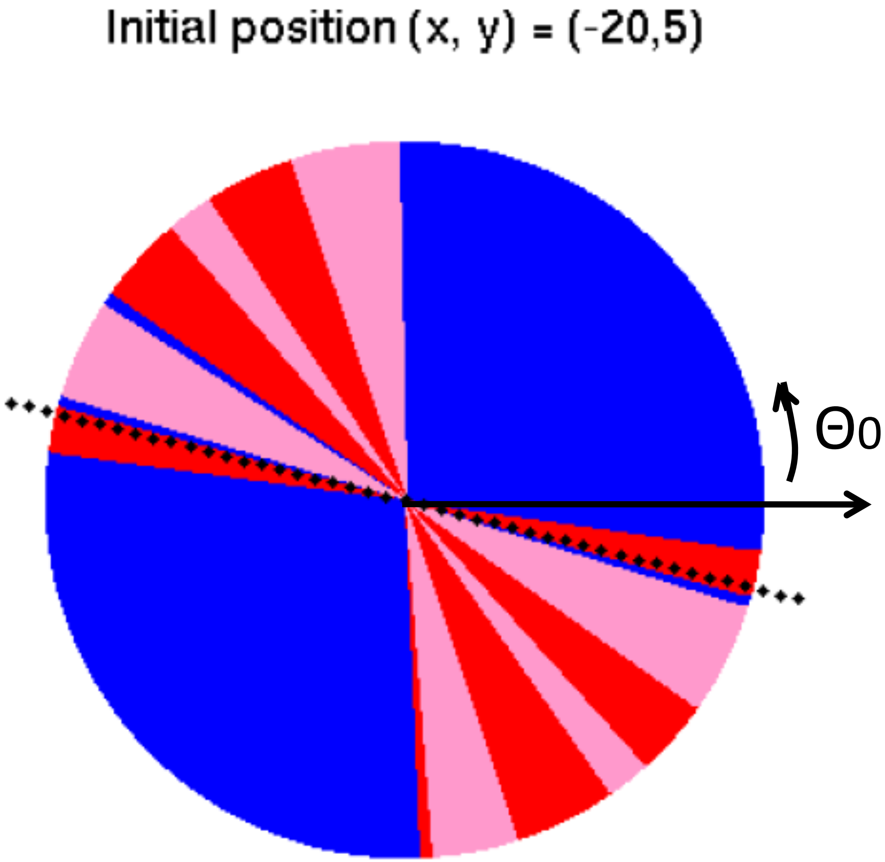


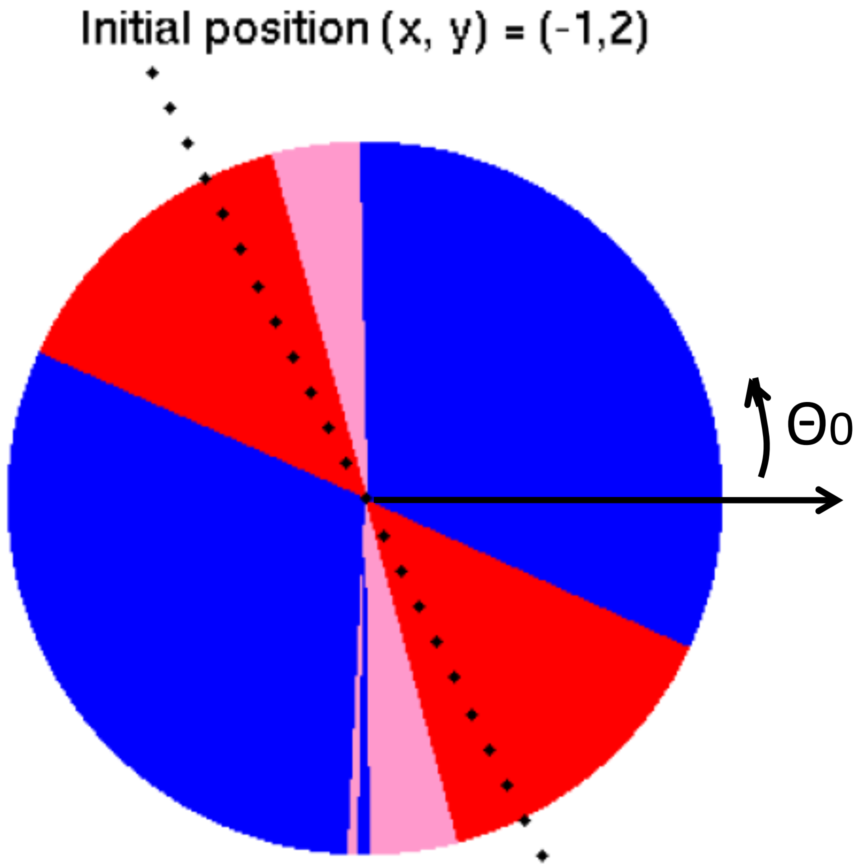


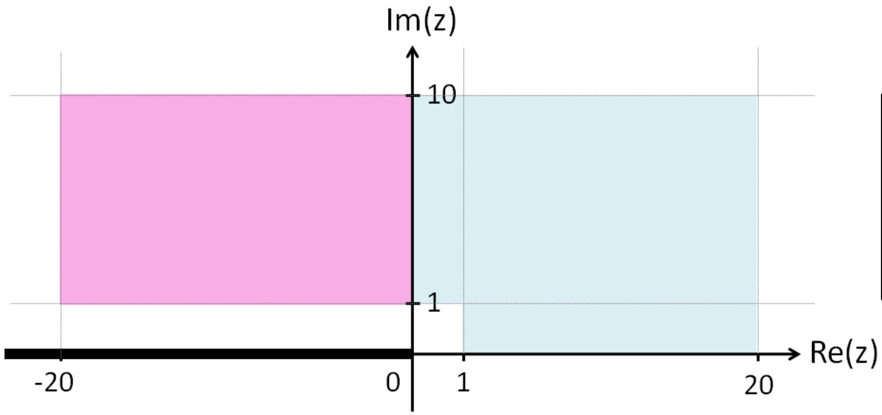
① ② ④ ⑤



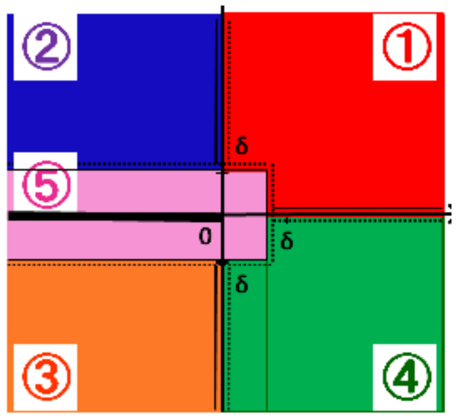
From an initial point $x_{d0} < 0$, $y_{d0} > 1$



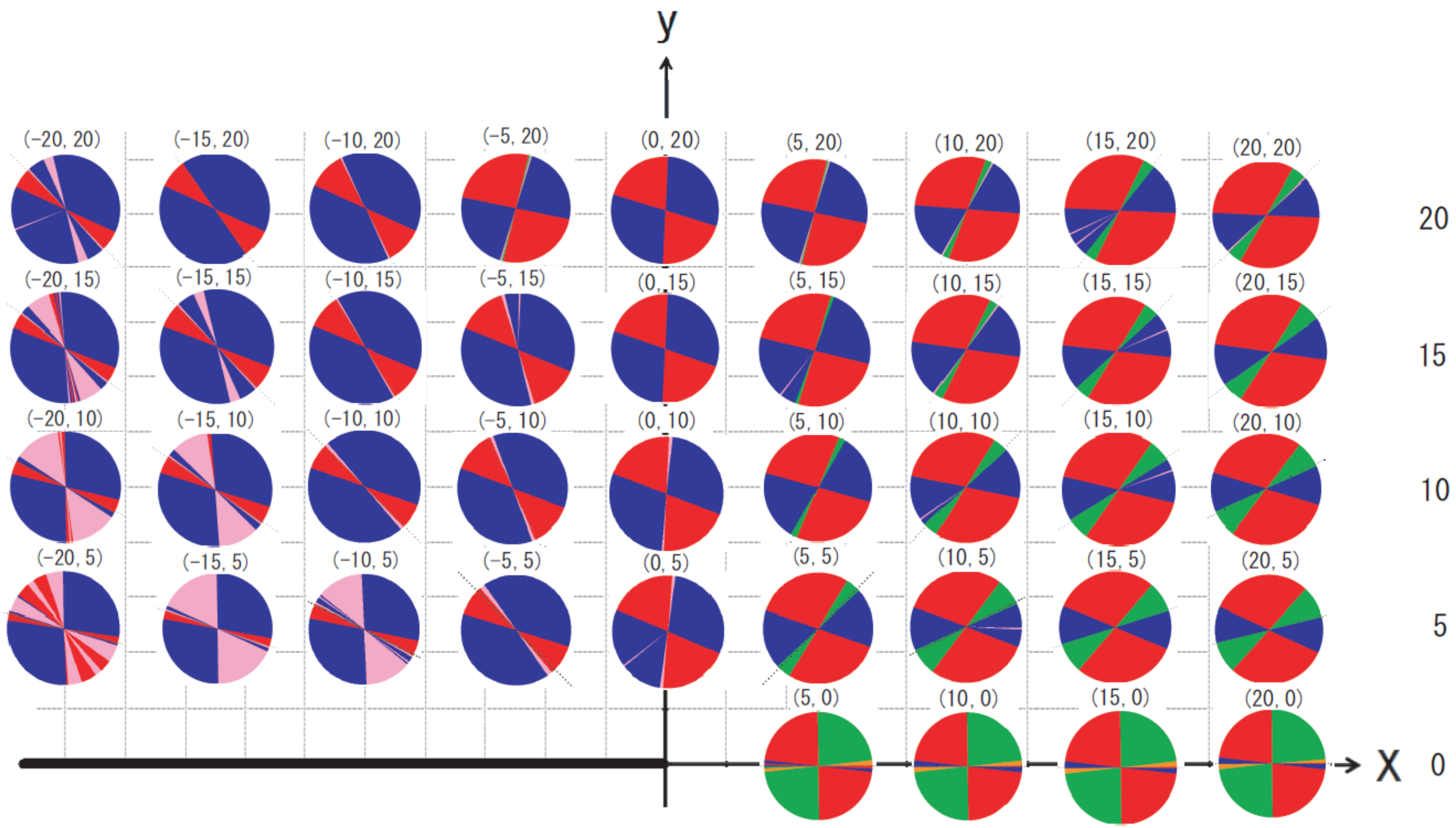




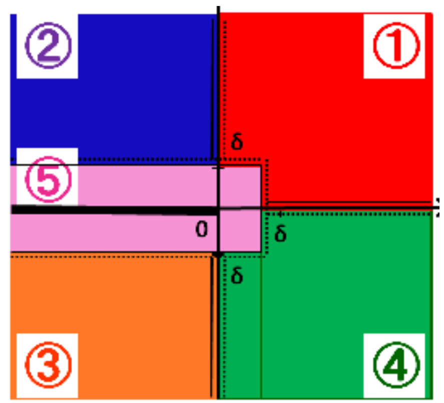
① ② ⑤

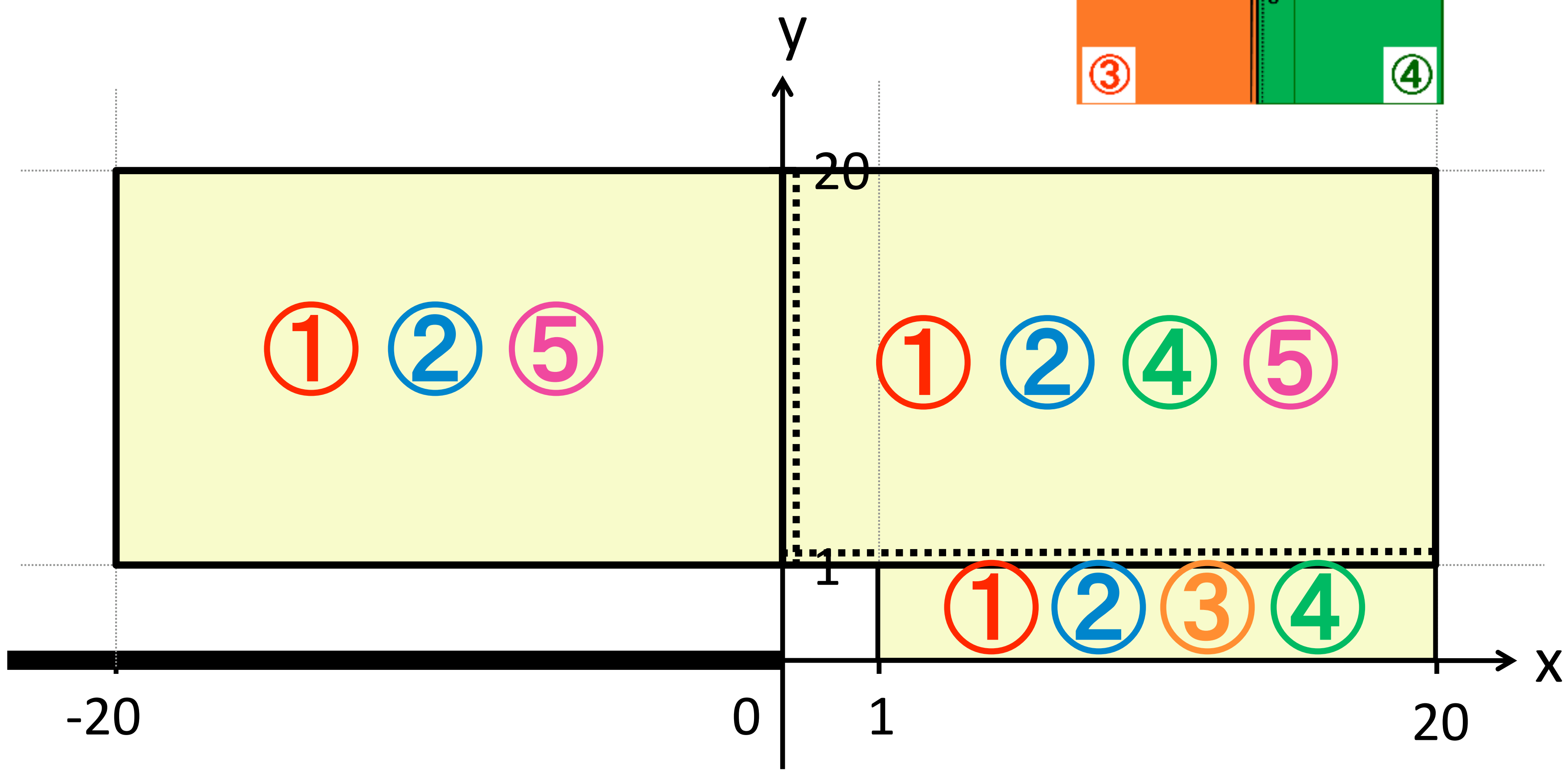


From different initial points

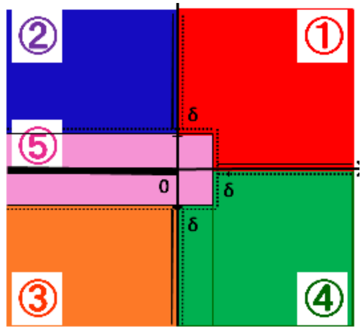


From different initial points: Several rules

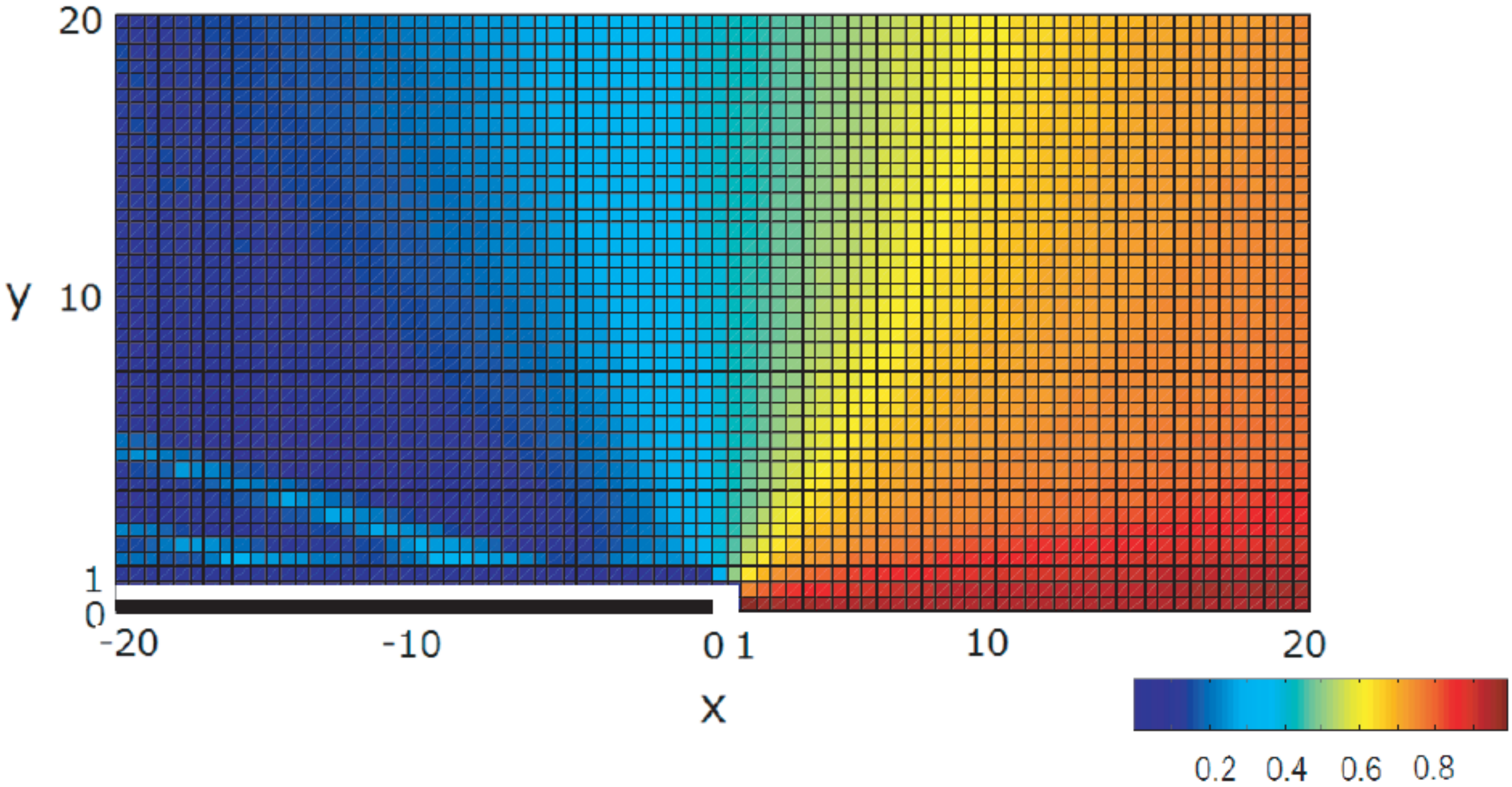




Escaping probability P_E for each initial point



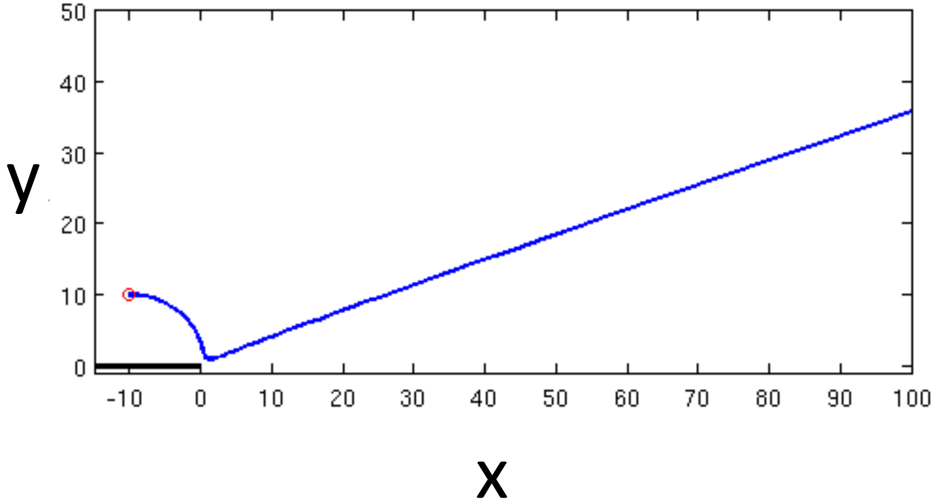
$$P_E \equiv \frac{\{\#\theta_0 \mid z_d \in \text{ regions 1 or 4 } \text{ at } t = t_l\}}{\#\theta_0}, \; t_l = 1500$$

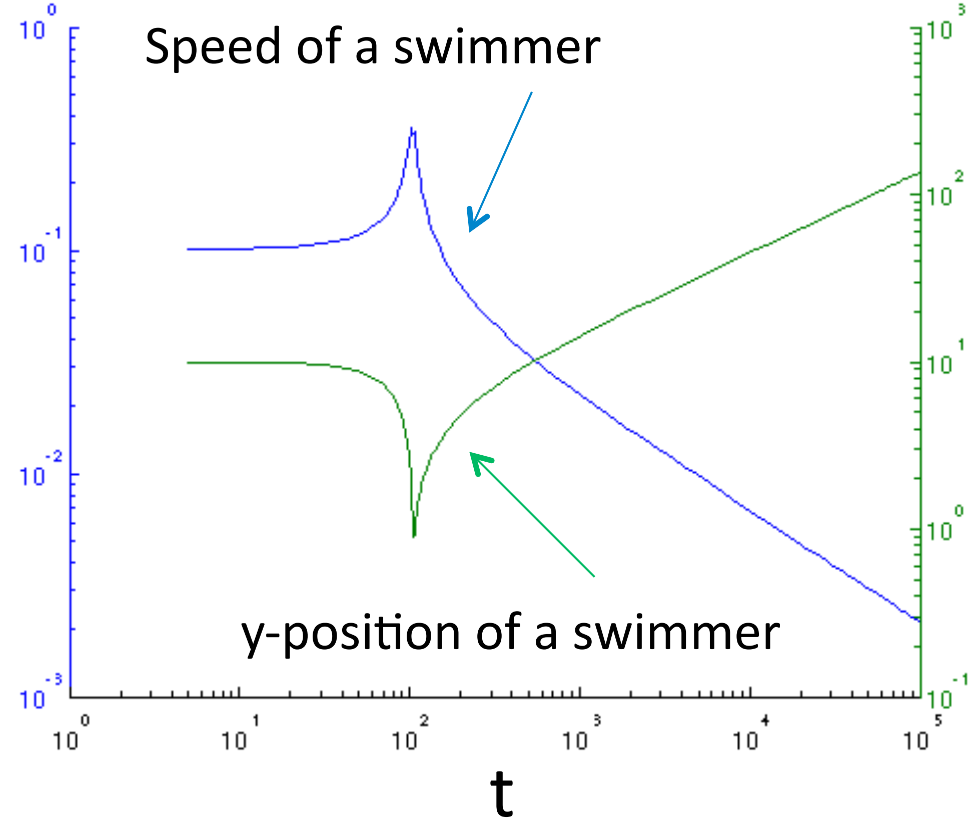


Temporal variation of the speed: Escaping from the wall

(x0, y0, theta0) = (-10, 10,2.3562 [rad])

Trajectory of a swimmer





Conclusions

- Treadmilling swimmer feels the presence of the wall
  $\longrightarrow$ escaping probability < 1

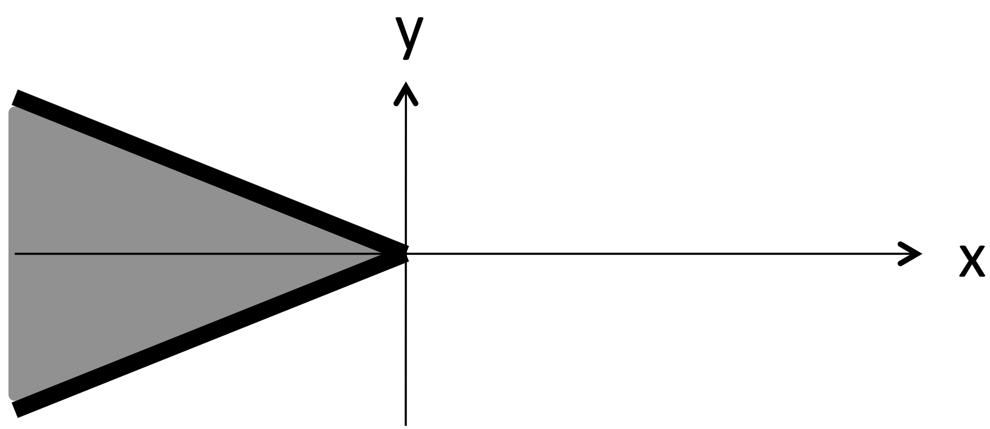
- The speed of a swimmer slows down as the swimmer goes further from the wall since the image should remain on the wall

Further works

- half a wall $\overset{??}{\sim}$ size of the swimmer << width of the gap

The original problem

- near a corner



Acknowledgements




林先生をはじめ, 北大・神戸大
GCOE/CPS関係者のみなさまに
深く感謝致します.

Many thanks to everyone
Jean-Luc, Matt, and the fellows





Appendixes

Lecturer note 1

Social Behavior, Mixing, and the Evolution of Schooling — Glenn Flierl

Dynamics of swimming organism (position $\mathbf{X}$, velocity $\mathbf{U}$):

$$\begin{cases} dX_i = U_i dt, \\ dU_i = -r(U_i - u_i - V_i)dt + \beta_{ij} dW_j. \end{cases}$$

- u_i: Water velocity
- V_i: Preferred swimming velocity
- $\beta_{ij} dW_j$: Random acceleration

- **taxis** (describes a large-scale preferred velocity that the group tends to.):

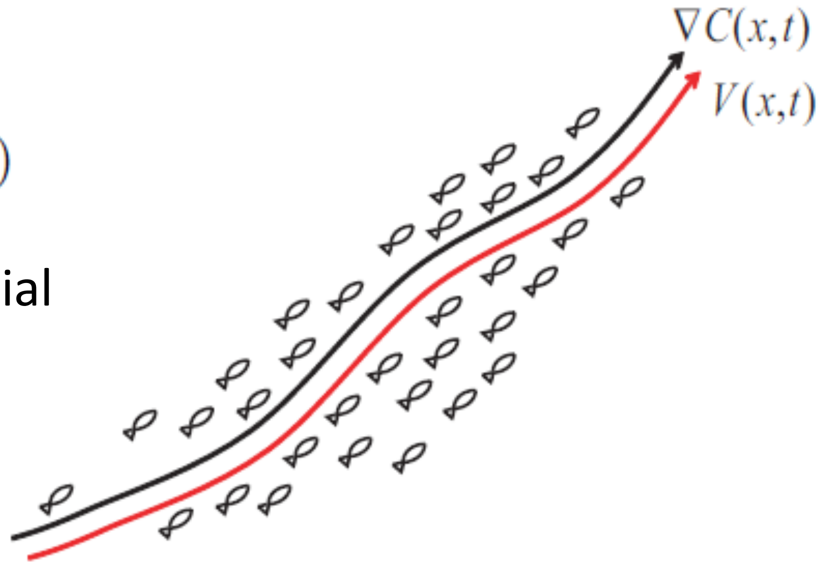
$\mathbf{V}$ depends on gradient of cue field $\nabla C(\mathbf{x}, t)$

cue $C(\mathbf{x}, t)$ *may be*

environmental (food, light, depth, etc.) or social (positions of neighbors, etc.).

Here, we define taxis as a preference for moving up the gradient of the cue field,

$$\mathbf{V} = \alpha \nabla C(\mathbf{x})$$

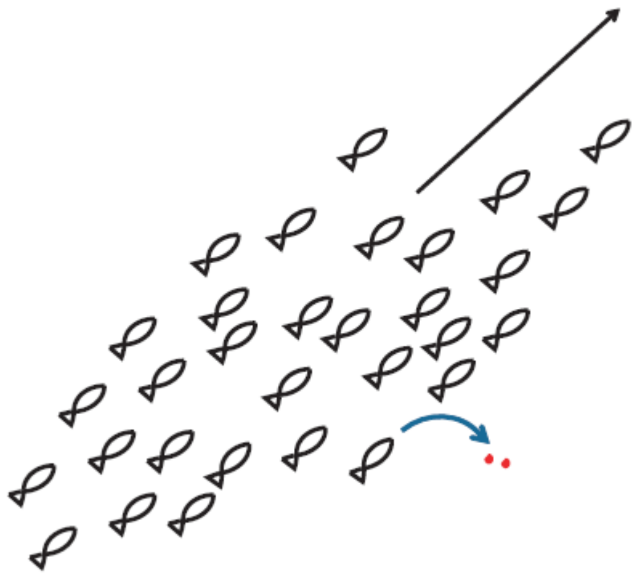


Lecturer note 1

Social Behavior, Mixing, and the Evolution of Schooling —— Glenn Flierl

・ **kinesis** (describes an individual's tendency to move randomly):

β *depends on the cue field*: $\beta = \beta(C)$.



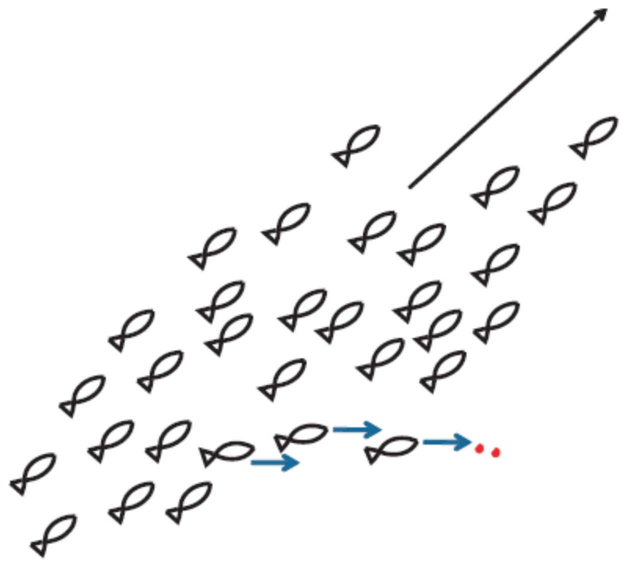
・**schooling** (describes the behavior of the organisms that tend to swim similarly to their neighbours.):

$\mathbf{V}$ depends on neighbor's $\mathbf{U}$: $\mathbf{V} = \mathbf{V}(\mathbf{U}_{\text{neighbors}})$.

The preferred direction of the swimming organisms results from a combination of attraction and alignment tendencies, and so schooling can be represented as

$$\mathbf{V} = V_0 \mathbf{V}_1 / |\mathbf{V}_1|,$$

$$\mathbf{V}_1 = \alpha \sum_{\mathbf{X}'} (\mathbf{X}' - \mathbf{X}) w(|\mathbf{X}' - \mathbf{X}|) + \sum_{\mathbf{X}'} \mathbf{U}' w(|\mathbf{X}' - \mathbf{X}|).$$



Singularities in Goursat functions

Example: Stokeslet at z_d.

strength of the singularity

Assume $f(z)$ of the form $f(z) = \mu \log(z - z_d),$ $(\mu \in \mathbf{C})$

Then the complex velocity field is

$$u_x + iu_y = -\mu \log(z - z_d) + \frac{\overline{\mu} z}{\overline{z} - \overline{z_d}} + \overline{g}'(\overline{z})$$

$$= -\mu \log(z - z_d) + \frac{\overline{z}(z - z_d)}{\overline{z} - \overline{z_d}} + \frac{\overline{\mu} z_d}{\overline{z} - \overline{z_d}} + \overline{g}'(\overline{z})$$

Insist that the velocity filed should be both

single-valued and, at least, logarithmically singular

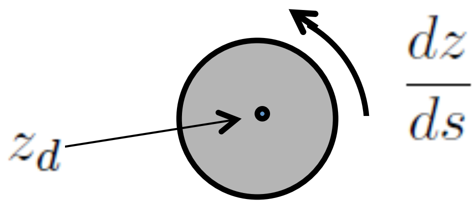
$$g'(z,t) = -\frac{\mu \overline{z_d}}{\overline{z} - \overline{z_d}} - \overline{\mu} \log(z - z_d).$$

$g(z)$ should have concomitant singularities to those in $f(z)$, but not but not conversely ($g(z)$ can have singularities which is independent of those of $f(z)$.).

necessary boundary condition on the treadmiller's body

$$f(z,t) = \frac{\mu}{z - z_d(t)} + f_0 + f_1(z - z_d(t)) + \cdots ,$$

$$g'(z,t) = \frac{b}{(z - z_d(t))^3} + \frac{a}{(z - z_d(t))^2} + \cdots .$$



On the surface of the treadmilling organism, where $|z - z_d| = \epsilon$,

$$z - z_d = \frac{\epsilon^2}{\overline{z} - \overline{z_d}}, \quad \frac{dz}{ds} = i\frac{z - z_d}{\epsilon}, \quad \frac{dz}{ds} : \text{ complex unit tangent to the boundary.}$$

$$u_x + iu_y = \dot{x}_d(t) + i\dot{y}_d(t) + [\epsilon\Omega + U(\phi, \theta(t))]\frac{dz}{ds},$$

$\Longrightarrow$ $\mu = -i\epsilon\overline{c}, \; a = \mu\overline{z_d}, \; b = \mu\epsilon - i\overline{c}\epsilon^3 = 2\mu\epsilon^2$ ($c(t) \equiv -iV\exp(-2i\;\theta(t))$)

Near z_d, $f(z(\zeta))$ and $g'(z(\zeta))$ have to have singularities written as

$$f(z) = \frac{\mu}{z - z_d} + f_0 + f_1(z - z_d) + \mathcal{O}((z - z_d)^2),$$

$$g'(z) = \frac{2\mu\epsilon^2}{(z - z_d)^3} + \frac{\mu\overline{z_d}}{(z - z_d)^2} + g_0 + \mathcal{O}((z - z_d))$$

Goursat functions $f(z)$, and $g(z)$

$$f_0 = \frac{\mu}{4z_d} - \frac{\overline{\epsilon}^2\overline{\mu}}{4\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^3 \overline{z_d}^{3/2}} + \frac{\left(2z_d\overline{z_d} - 2\overline{z_d}^2 - 3\overline{\epsilon}^2\right)\overline{\mu}}{8\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^2 \overline{z_d}^2} - \frac{\left(-2z_d\overline{z_d} - 2\overline{z_d}^2 + 3\overline{\epsilon}^2\right)\overline{\mu}}{8\left(z_d^{1/2} + \overline{z_d}^{1/2}\right) \overline{z_d}^{5/2}},$$

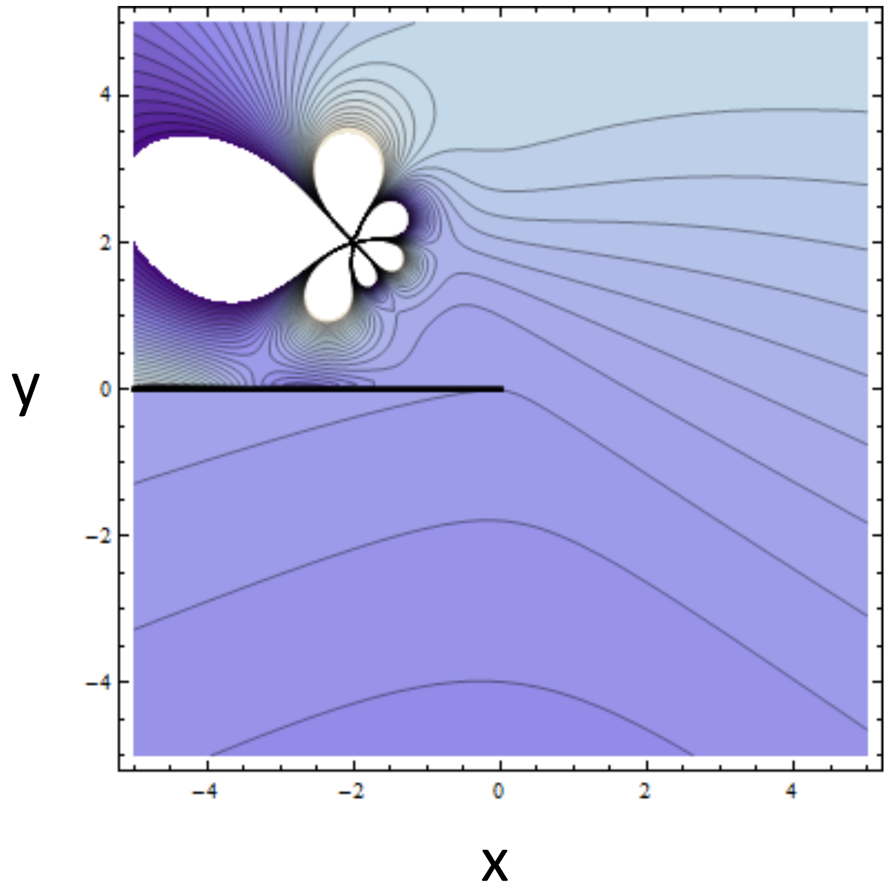
$$f_1 = \frac{1}{12z_d^2}\left(-\frac{3\mu}{4} + \frac{9z_d^{3/2}\overline{\epsilon}^2\overline{\mu}}{2\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^4 \overline{z_d}^{3/2}} - \frac{3z_d^{3/2}\left(2z_d\overline{z_d} - 2\overline{z_d}^2 - 3\overline{\epsilon}^2\right)\overline{\mu}}{2\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^3 \overline{z_d}^2} + \frac{3z_d^{3/2}\left(-2z_d\overline{z_d} - 2\overline{z_d}^2 + 3\overline{\epsilon}^2\right)\overline{\mu}}{4\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^2 \overline{z_d}^{5/2}}\right).$$

$$g_0 = -\frac{3\mu\overline{z_d}}{16z_d^2} + \frac{10\epsilon^2\mu}{32z_d^3} + \frac{3\epsilon^2\overline{\mu}}{8\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^4 \overline{z_d}} - \frac{z_d - \overline{z_d}}{4\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^3 z_d^{1/2}} - \frac{\left(-2z_d\overline{z_d} + 6\overline{z_d}^2 + 3\overline{\epsilon}^2\right)\overline{\mu}}{16\left(z_d^{1/2} + \overline{z_d}^{1/2}\right)^2 \overline{z_d}^2} - \frac{\left(-2z_d\overline{z_d} + 6\overline{z_d}^2 + 3\overline{\epsilon}^2\right)\overline{\mu}}{16\left(z_d^{1/2} + \overline{z_d}^{1/2}\right) \overline{z_d}^{5/2}}$$

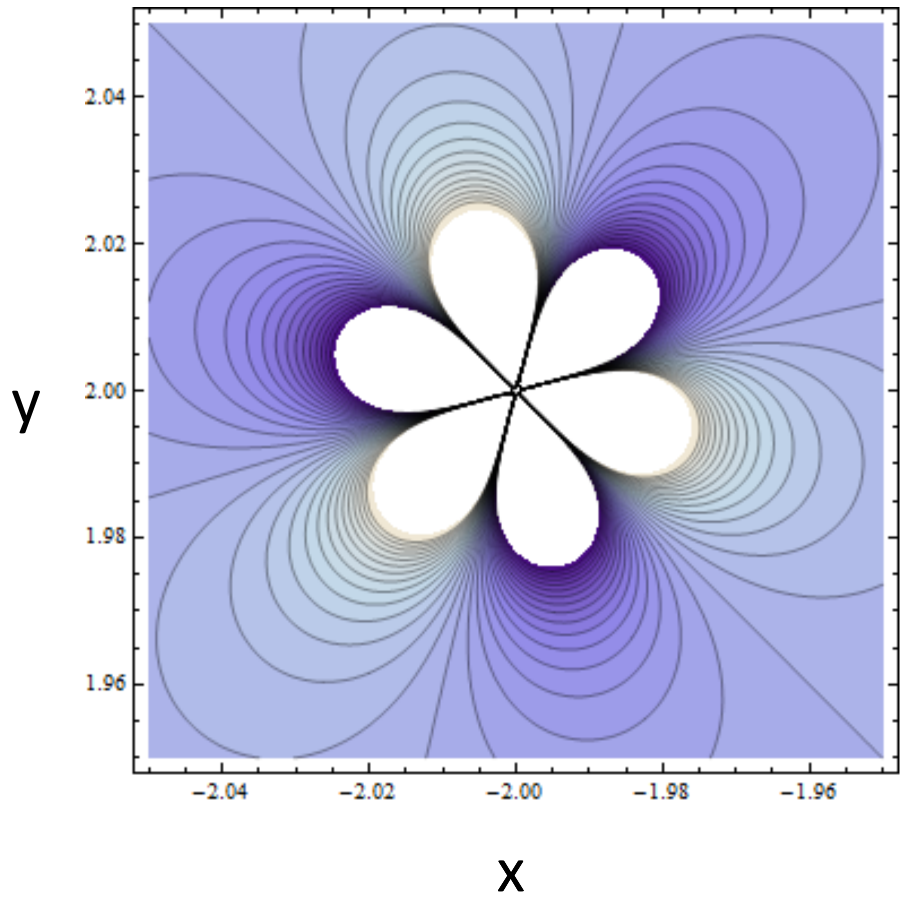
Example of the velocity stream function:

$$z_d = -2 + 2i, \quad \theta = 5\pi/4, \quad \varepsilon = 1.$$

$\psi = \mathrm{Im}[\bar{z} f(z) + g(z)]$



$\psi = \mathrm{Im}[\bar{z} f(z) + g(z)]$



From different initial points