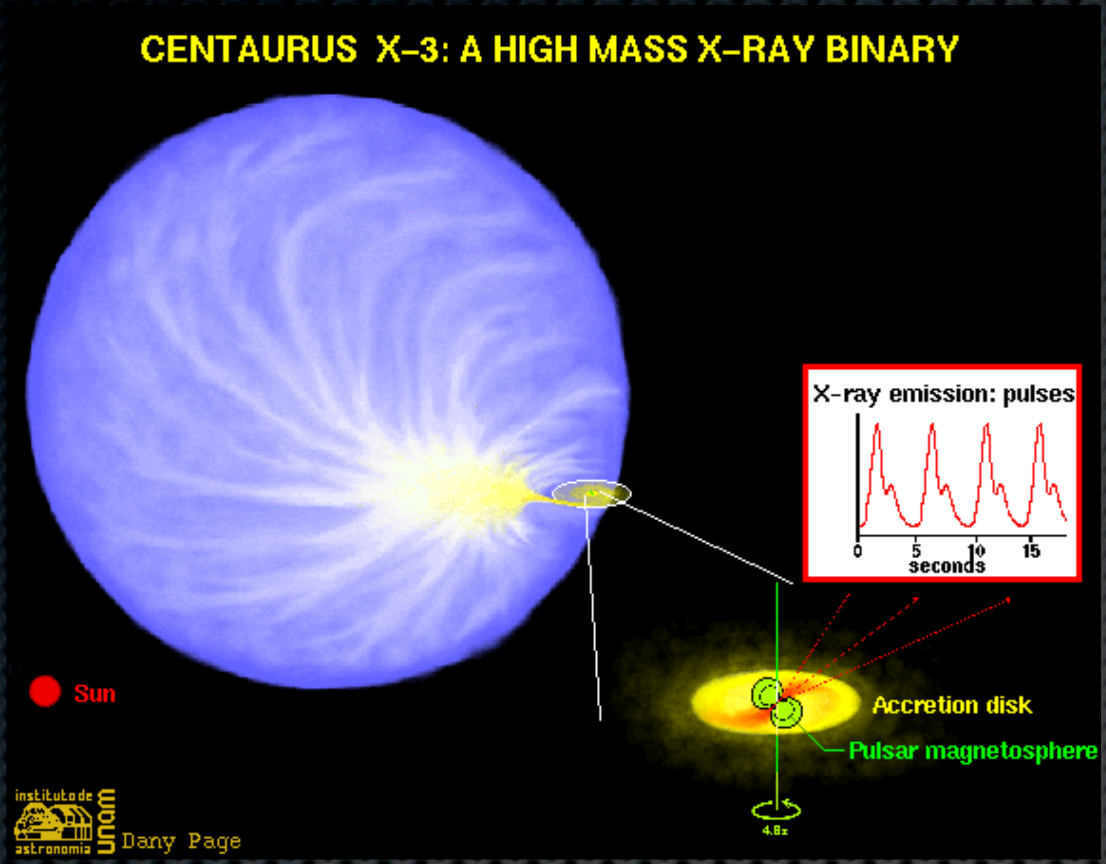
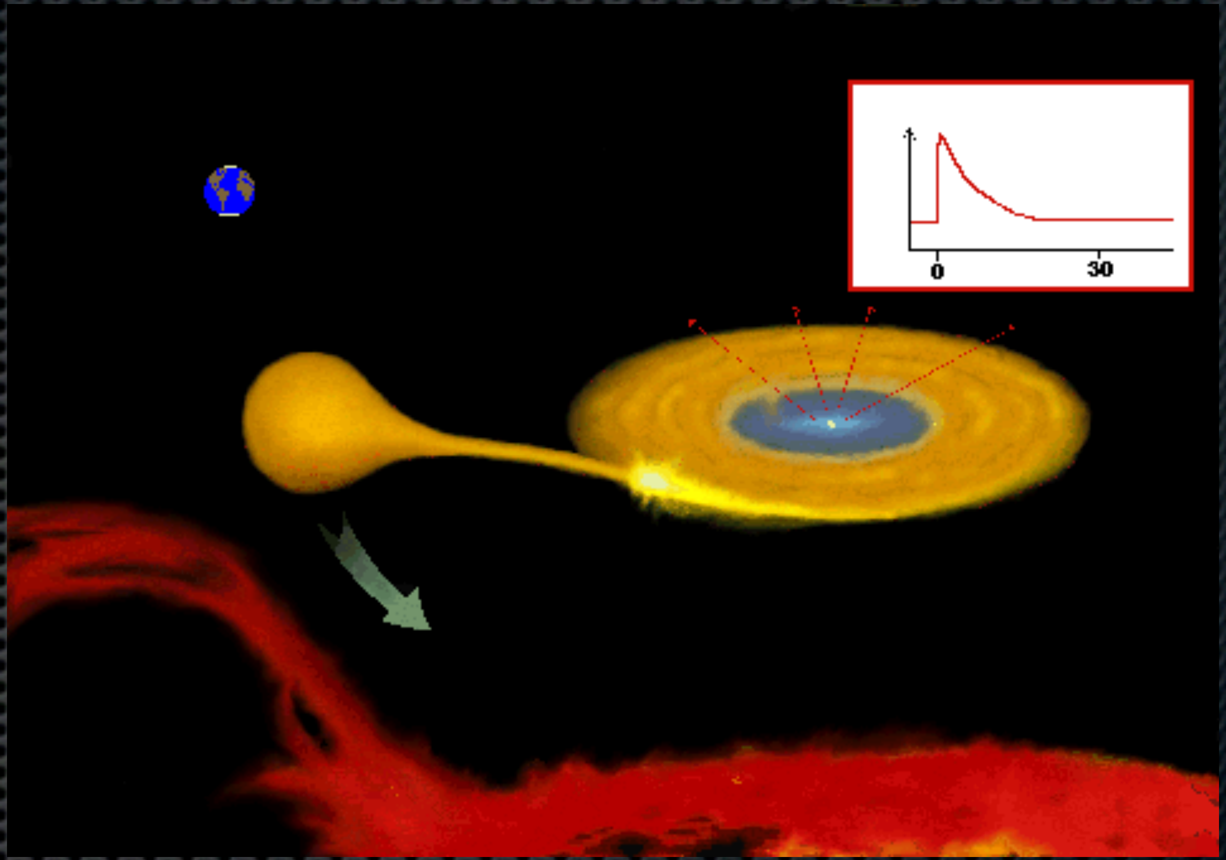
Time-Frequency Analysis of Superorbital Modulation of X-ray Binary SMC X-1 by Hilbert-Huang Transform

Chin-Ping Hu @ CPS seminar

Oct. 4. 2011

SMC X-1: High-Mass X-ray Binary





Credit: Dany P. Page

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- X-ray Binary: consist with a compact object (black hole / neutron star) and a mass donor star (main-sequence / white dwarf).
- High-Mass X-ray Binaries (HMXB): the companion is a massive star.
- Low-Mass X-ray Binaries (LMXB): the companion is a low mass star or white dwarf.

X-ray Emission

- The mass transfered by either Roche-lobe filling, or steller wind, will form an accretion disk around the compact object.
- The temperature of inner accretion disk is so high that can emit X-ray photons.
- The surface of neutron star/pulsar will also provide X-ray emission.
- For nuclear reaction,

$$E = 0.007mc^2$$

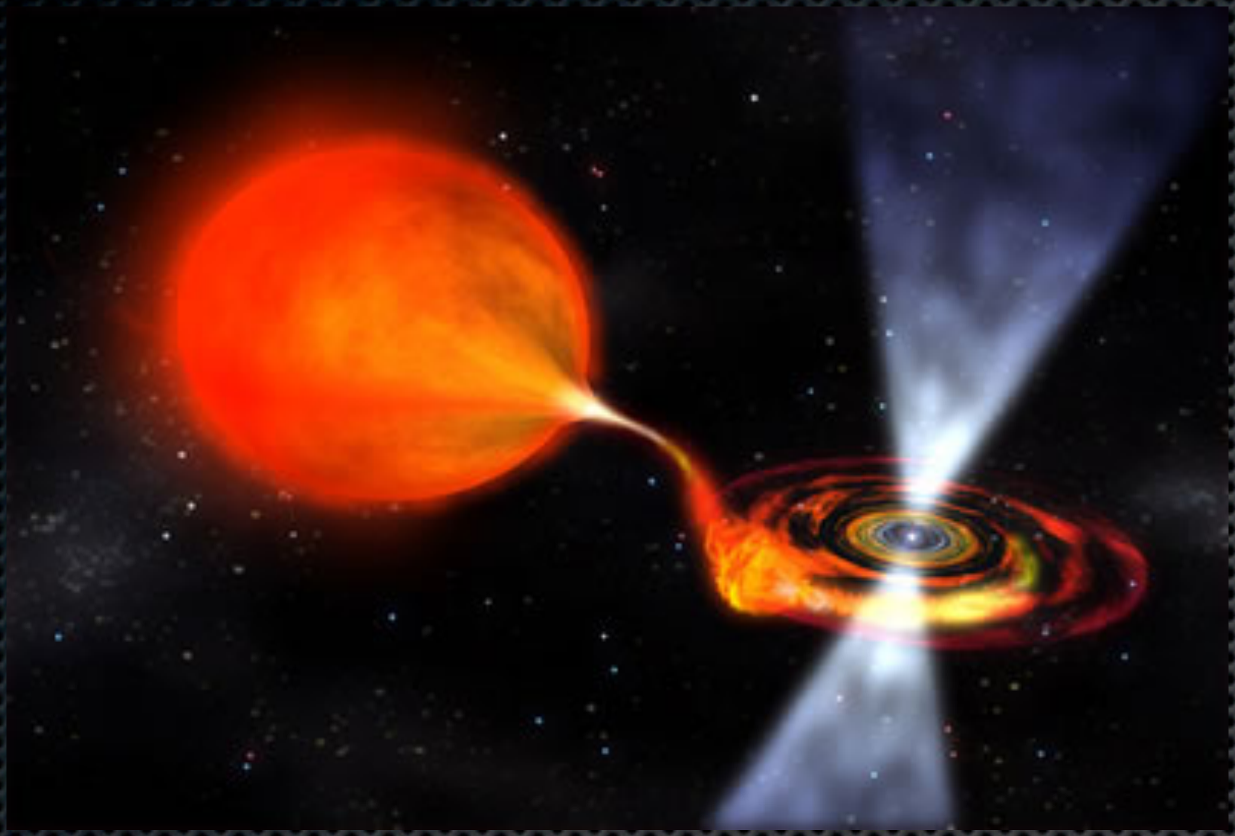
- However, for accretion on to the neutron star or black hole, the efficiency is much higher than the nuclear reaction:

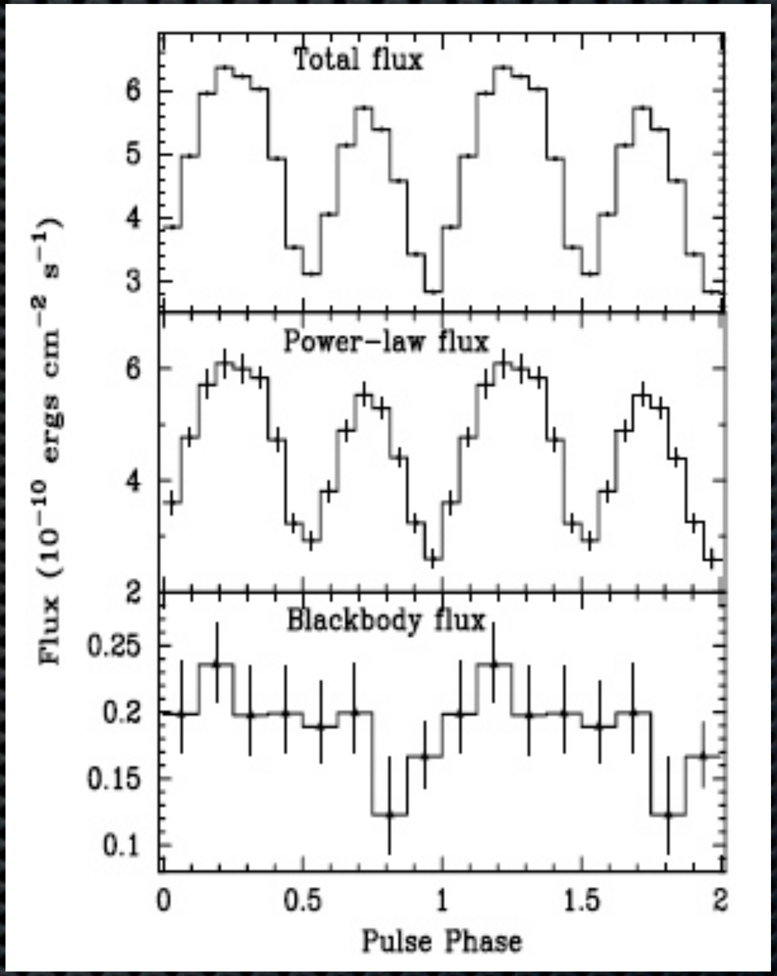
$$E = 0.00025mc^2$$ for white dwarf

$$E = 0.15mc^2$$ for neutron star

$$E = 0.1 - 0.4mc^2$$ for black hole

Timing Properties: Pulsar

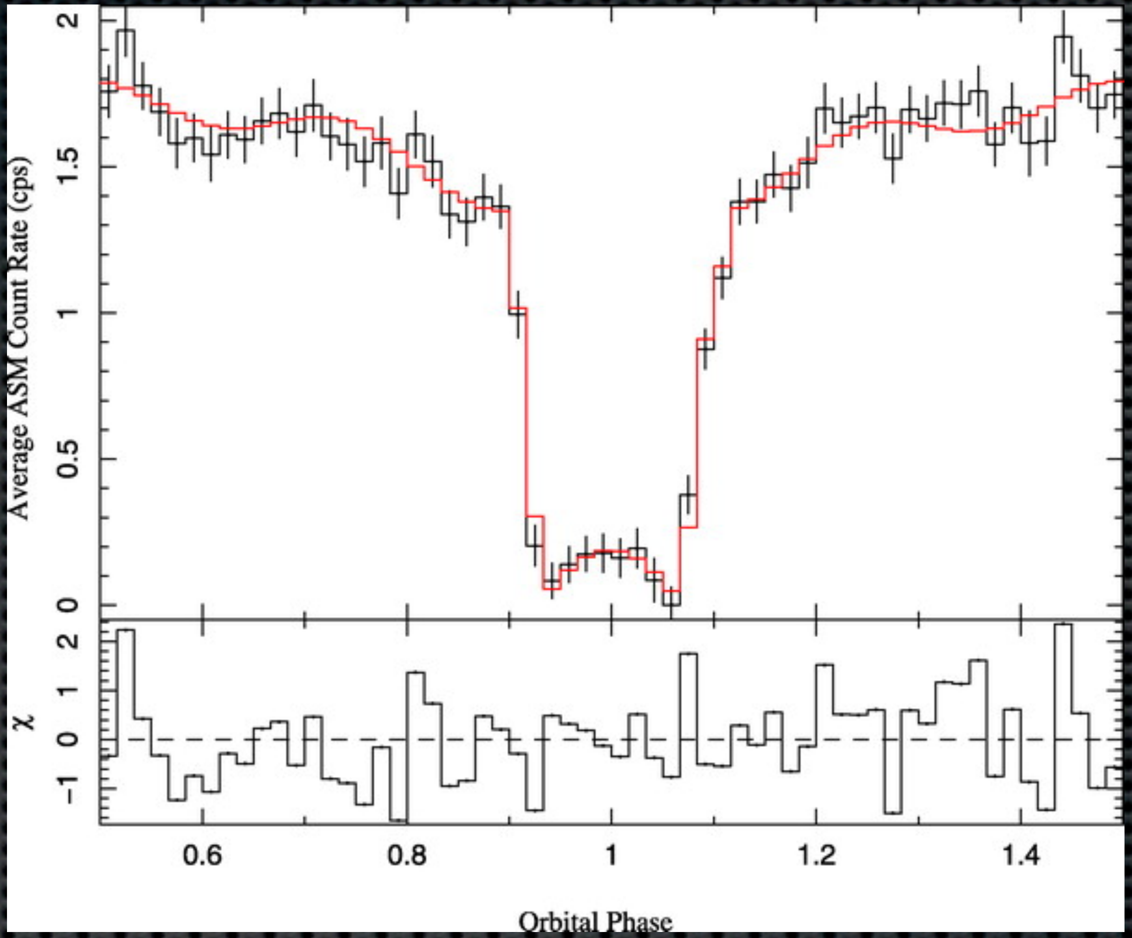




Naik & Paul (2004)

- SMC X-1 is an X-ray Pulsar with period of 0.71 s
- The spin period of neutron star

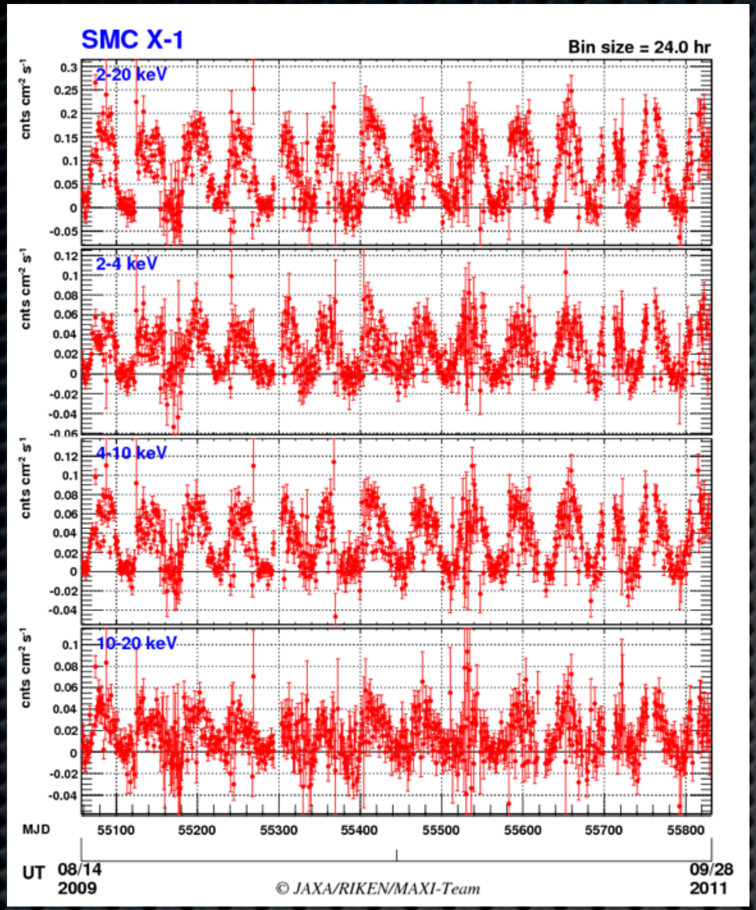
Timing Properties: Eclipse

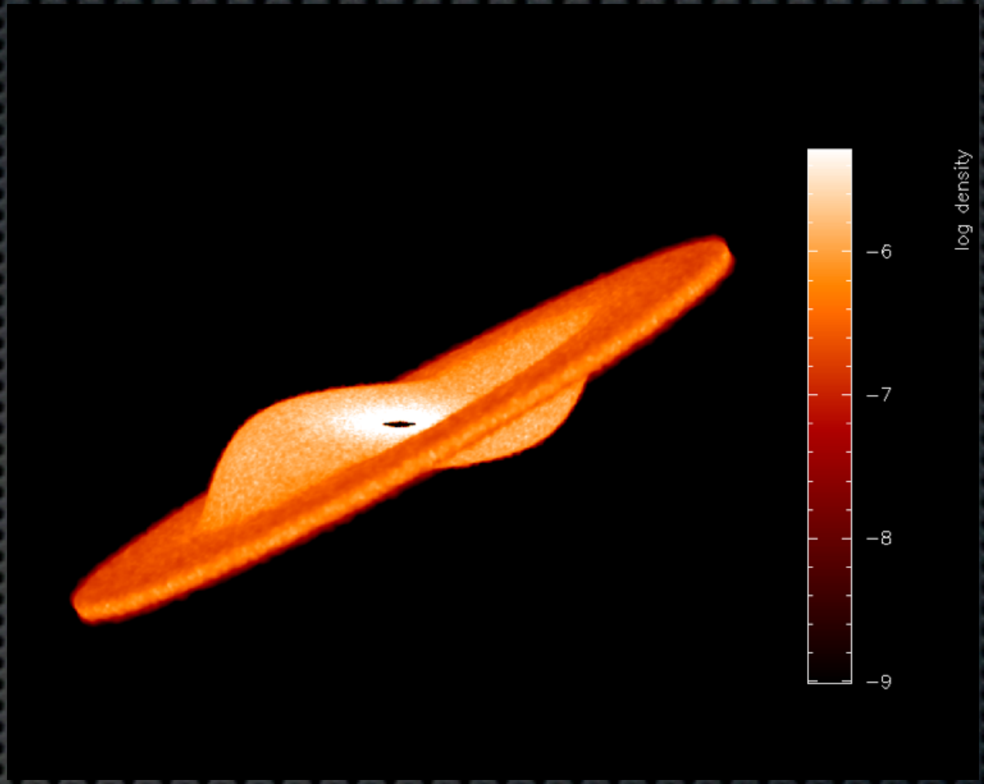


Trowbridge et al. (2007)

- The recurrent time of eclipse is 3.89 days, which denotes the binary orbital period of this system.

Timing Properties: Superorbital Modulation

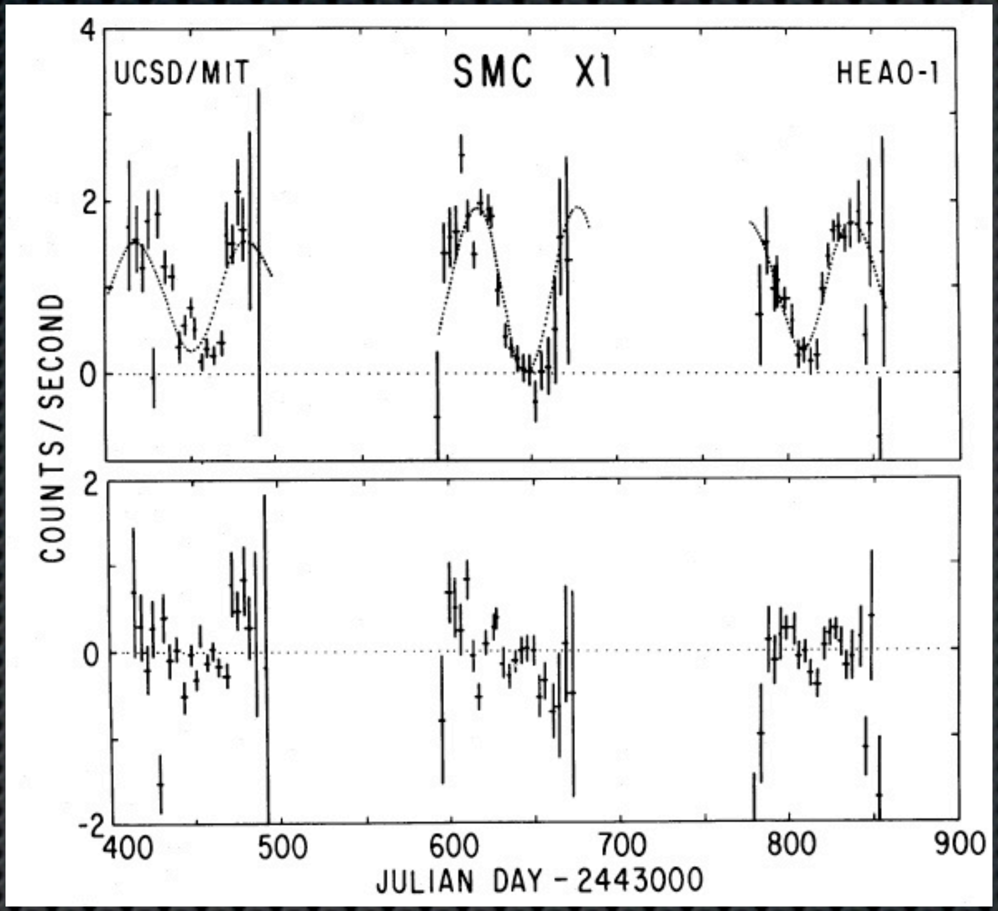




MAXI light curve

- SMC X-1 exhibits a flux change with period of ~40 to ~60 days.
- This kind of modulation is named as superorbital modulation because its period is much longer than the orbital period.
- This modulation is intrepreted by a twisted, or warped accretion disk. When disk precess, the warp region would obscure our line of view to the central X-ray source.

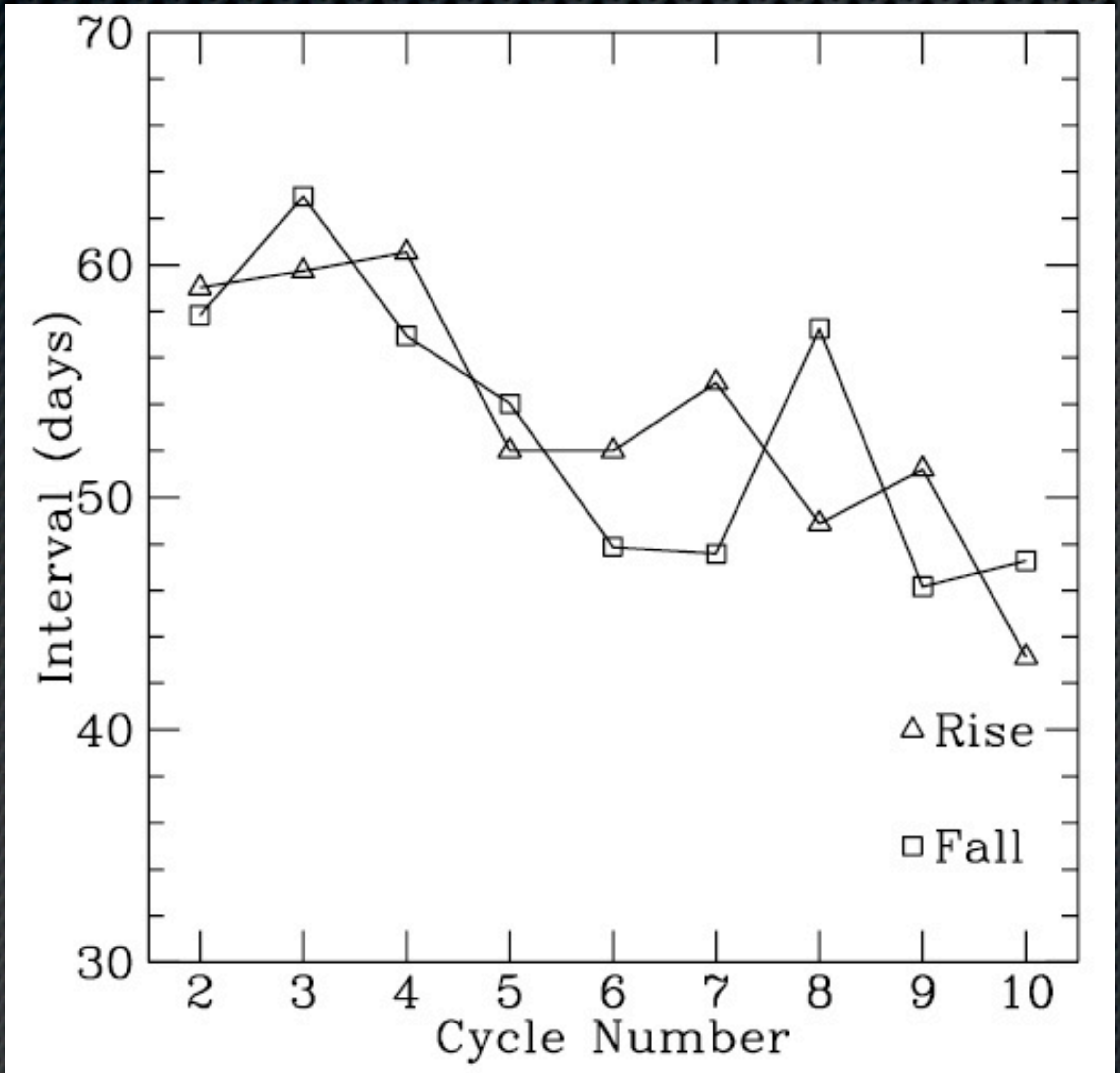
1984



Gruber et al. (1984)

- Gruber et al. (1984) shows that the cycle length of superorbital variation of SMC X-1 is highly variable.

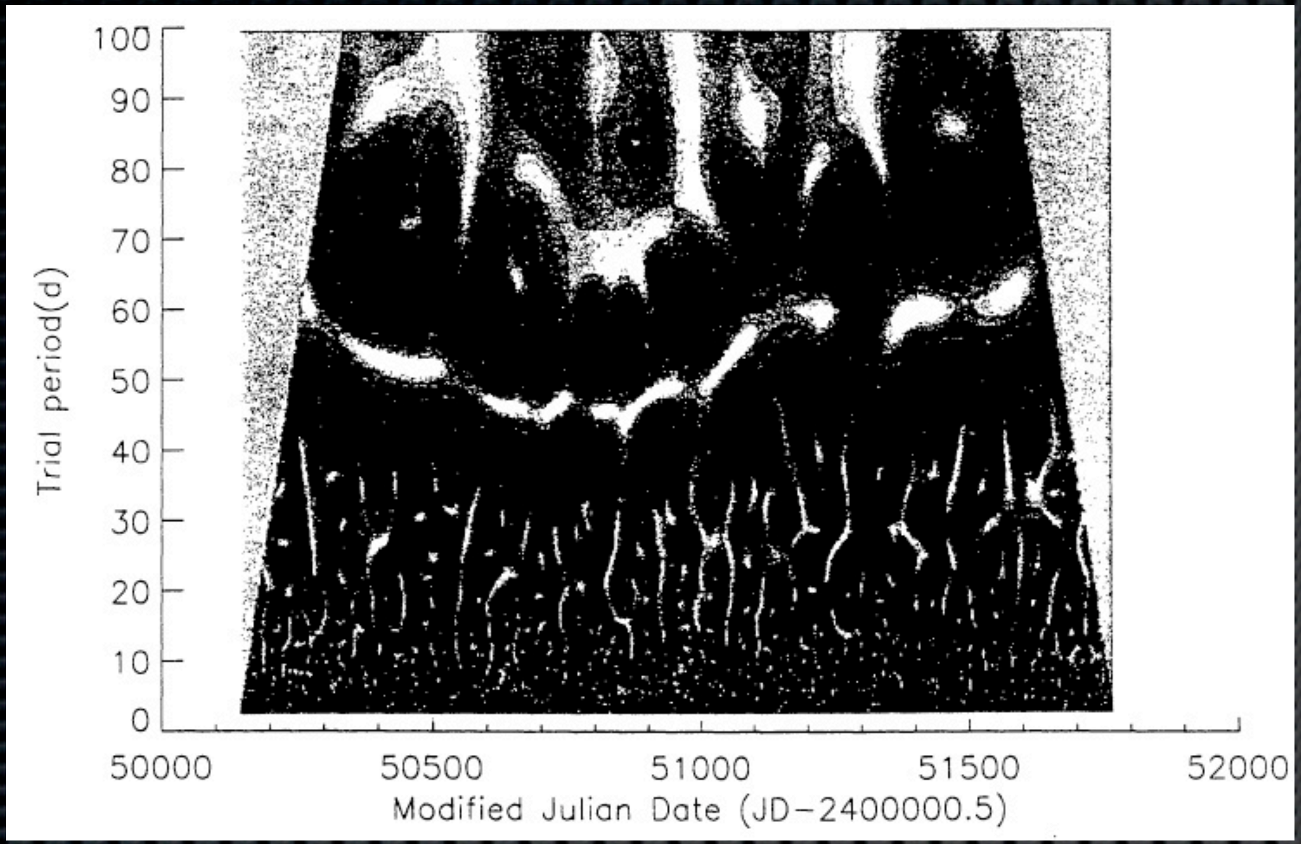
1998



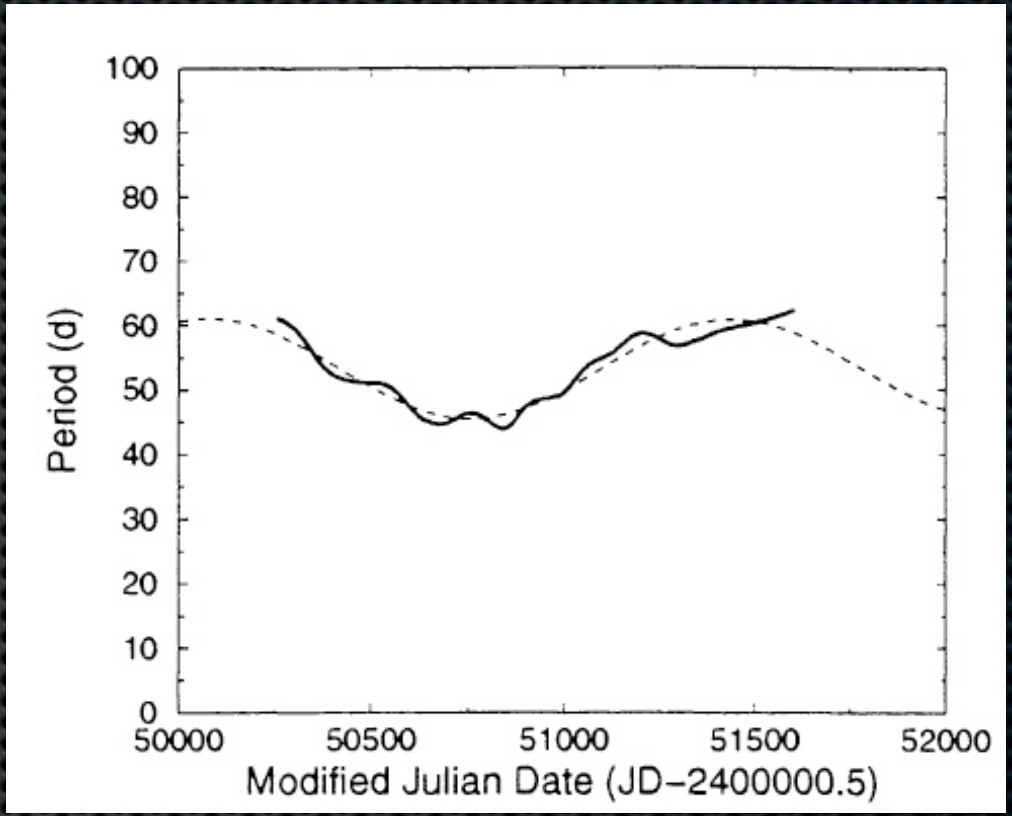
Wojdowski et al. (1998)

- Wojdowski et al. (1998) shows that time intervals between successive high-low transitions decreases with first 10 cycle RXTE observation.

2001



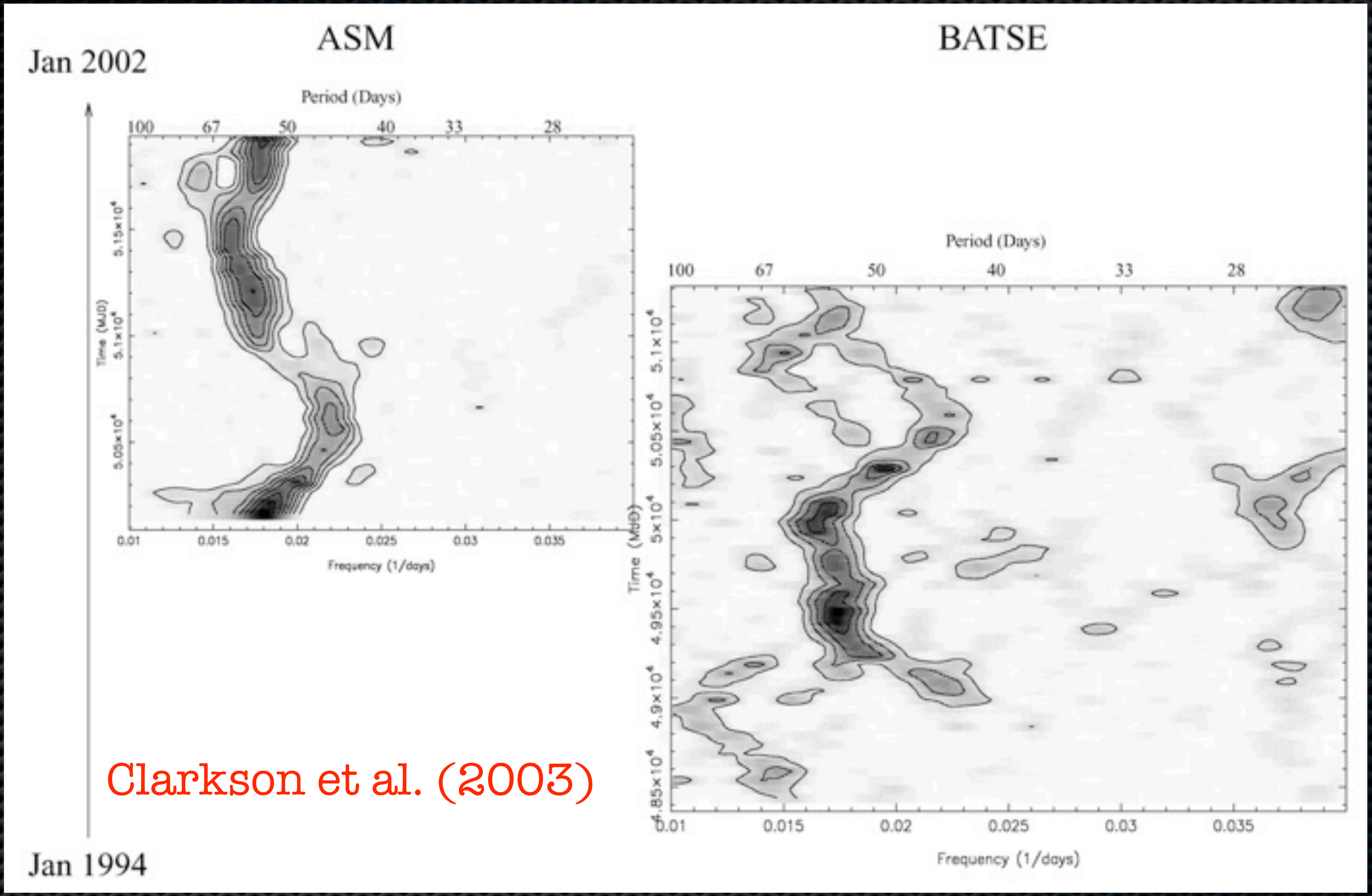
Ribo et al. (2001)



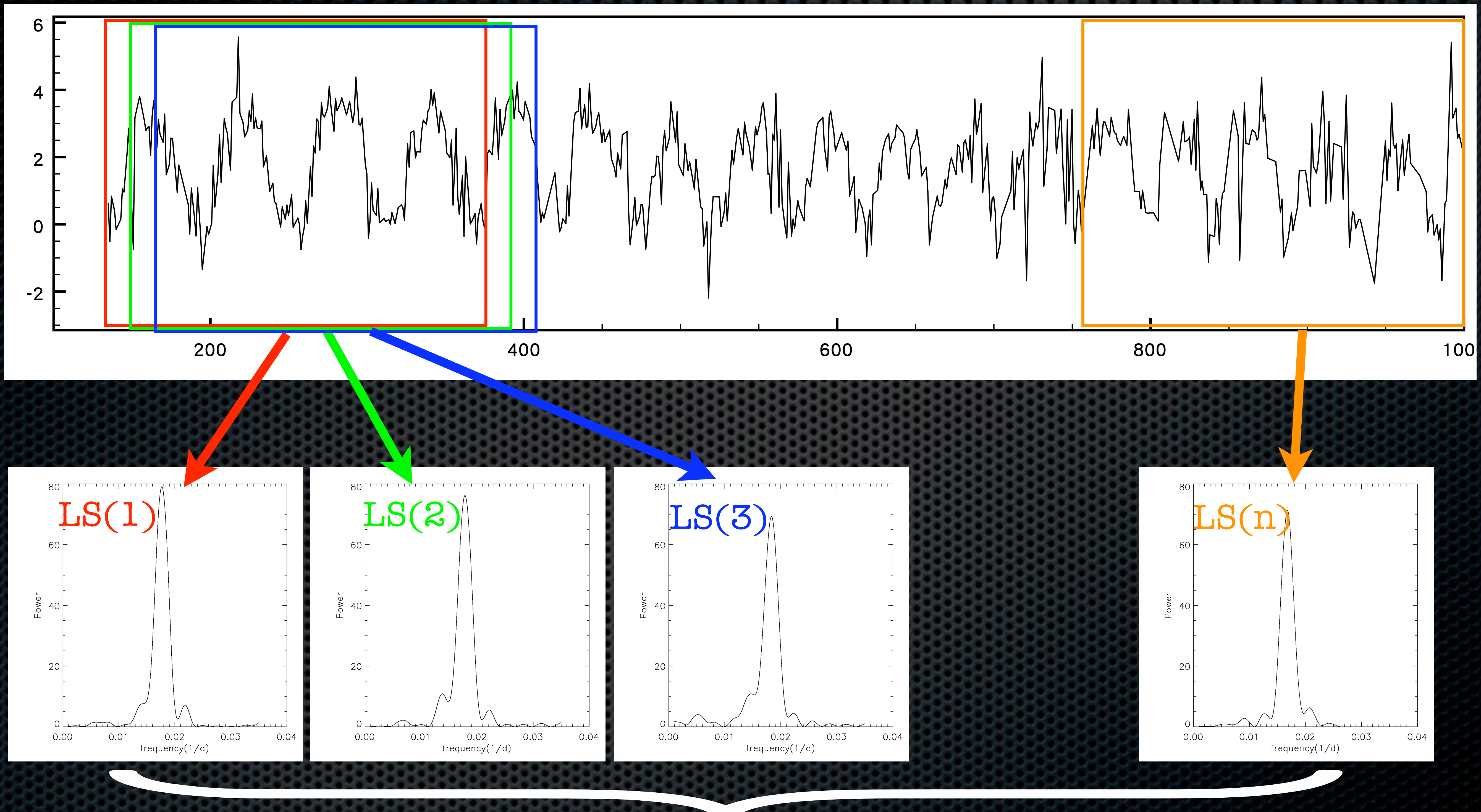
Ribo et al. (2001)

- Ribo et al. (2001) shows that the period varies between 60 to 45 days based on wavelet technique.
- The cycle length of period change is about 1421d.

2003

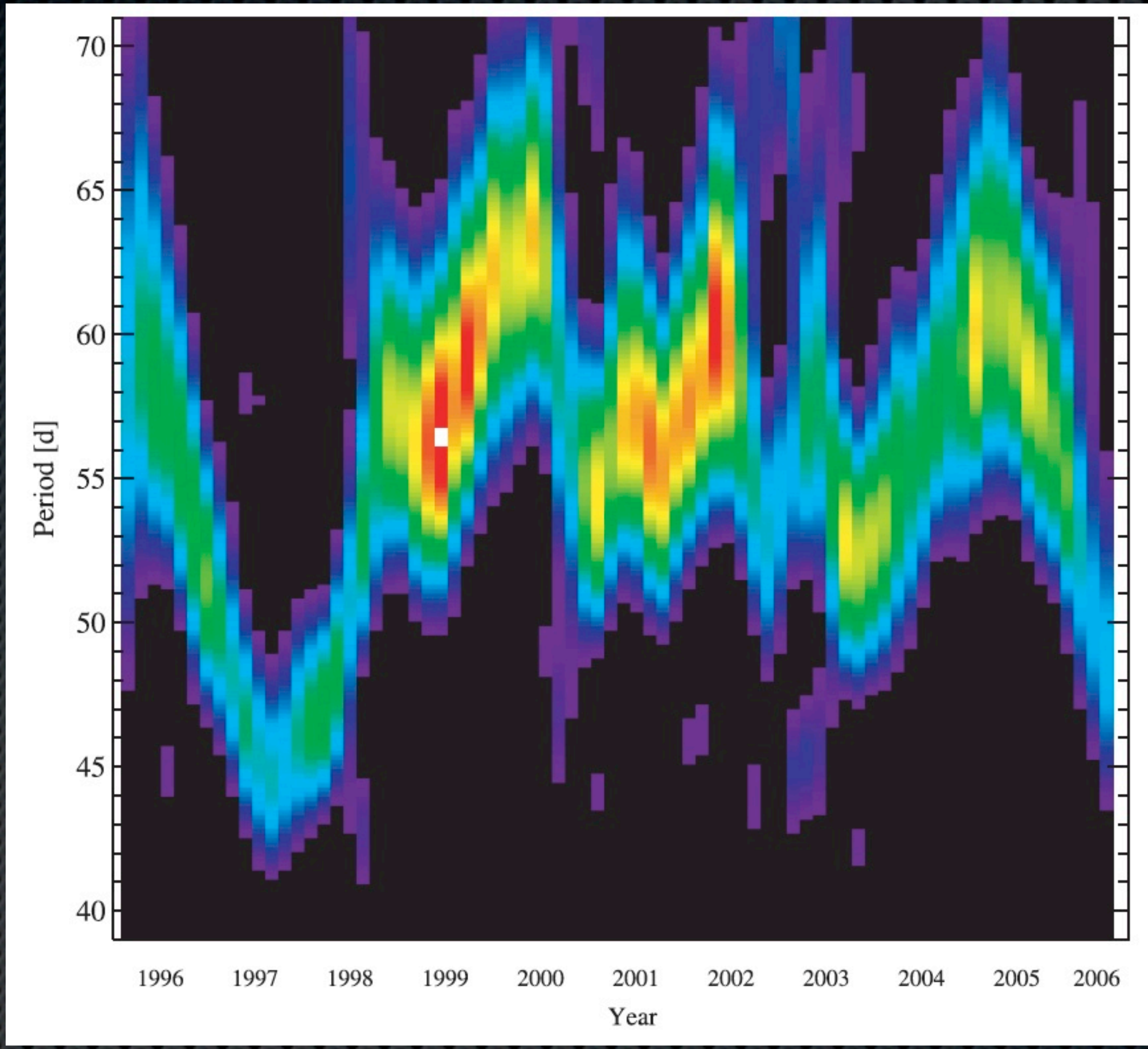


- Clarkson et al. (2003) analyzed the variation by using of dynamic power spectrum and shows that the period changed from 60d to 40d and then return to its former values, on a time scale of approximately 1600d.



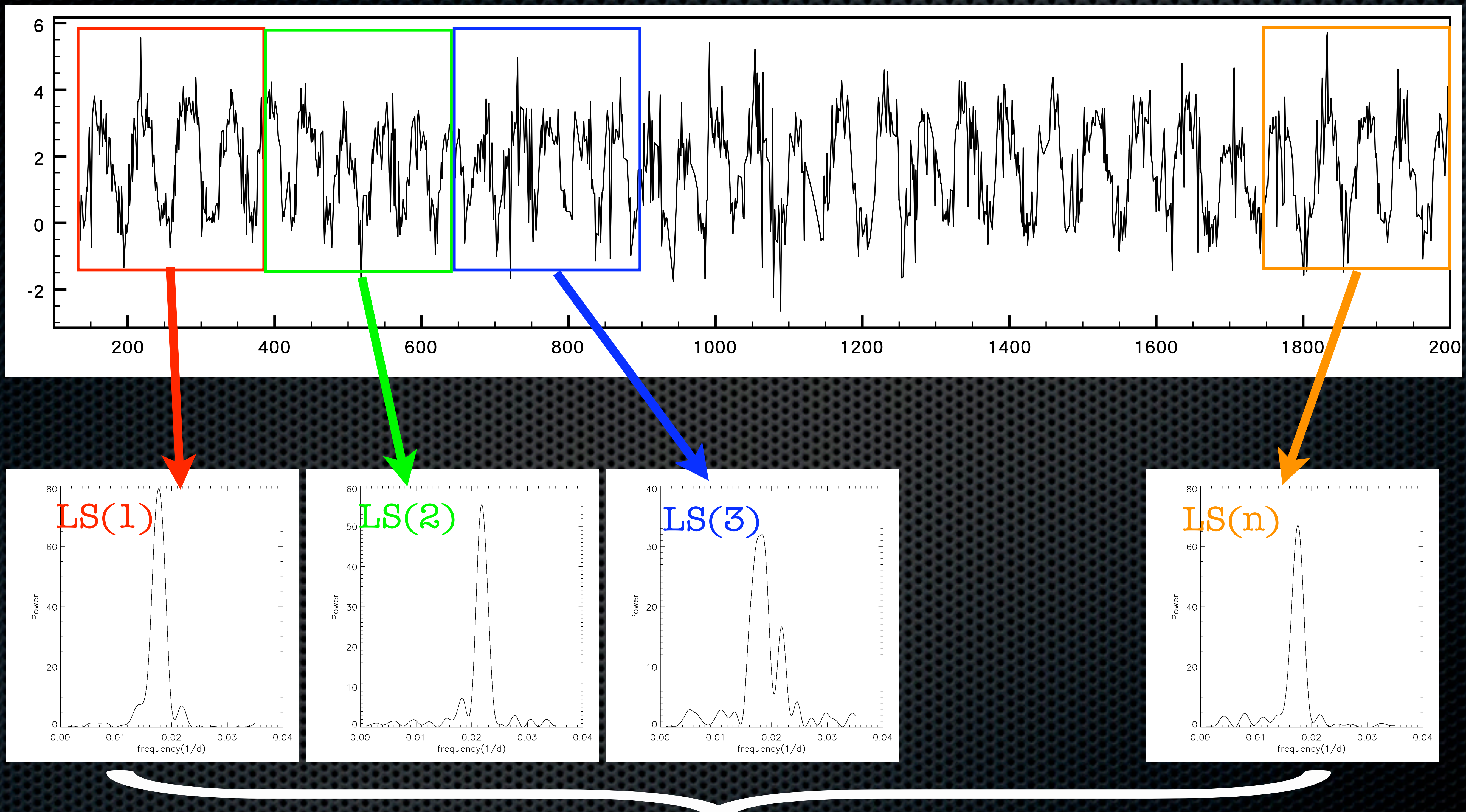
Generate a 3D time-frequency-power map

2007



Trowbridge et al. (2007)

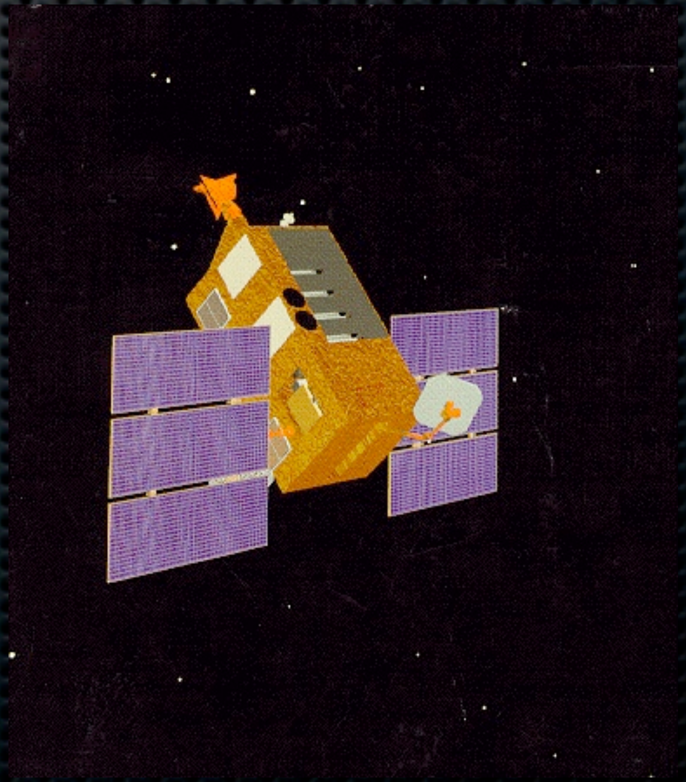
- Trowbridge et al. (2007) used slide Lomb-Scargle periodogram to analyze the superorbital variation of SMC X-1. They observe a sharp variation in period in contrast to Clarkson et al. (2003)



Generate a 3D time-frequency-power map

Observation: RXTE ASM

- The Rossi X-ray Timing Explorer (RXTE) was launched in 1995.
- RXTE consists of All-Sky Monitor (ASM), Proportional Counter Array (PCA), and High Energy X-ray Timing Experiment (HEXTE).
- The All Sky Monitor (ASM) continuously monitor the whole sky per 90 min.
- The energy range is 1.3 to 12.1 keV, which is divided into three bands of 1.3-3.0, 3.0-5.0, 5.0-12.0 keV.
- This mission will cease this year.



RXTE GOF

ASM Source Catalog

RXTE FAQ

For the latest ASM weather map results results,
visit the RXTE ASM X-ray Weather Map page.

For the latest calibration news on the FITS products,
visit the ASM Events page.

For usage notes on the ASM Products,
visit the ASM Data Products page.

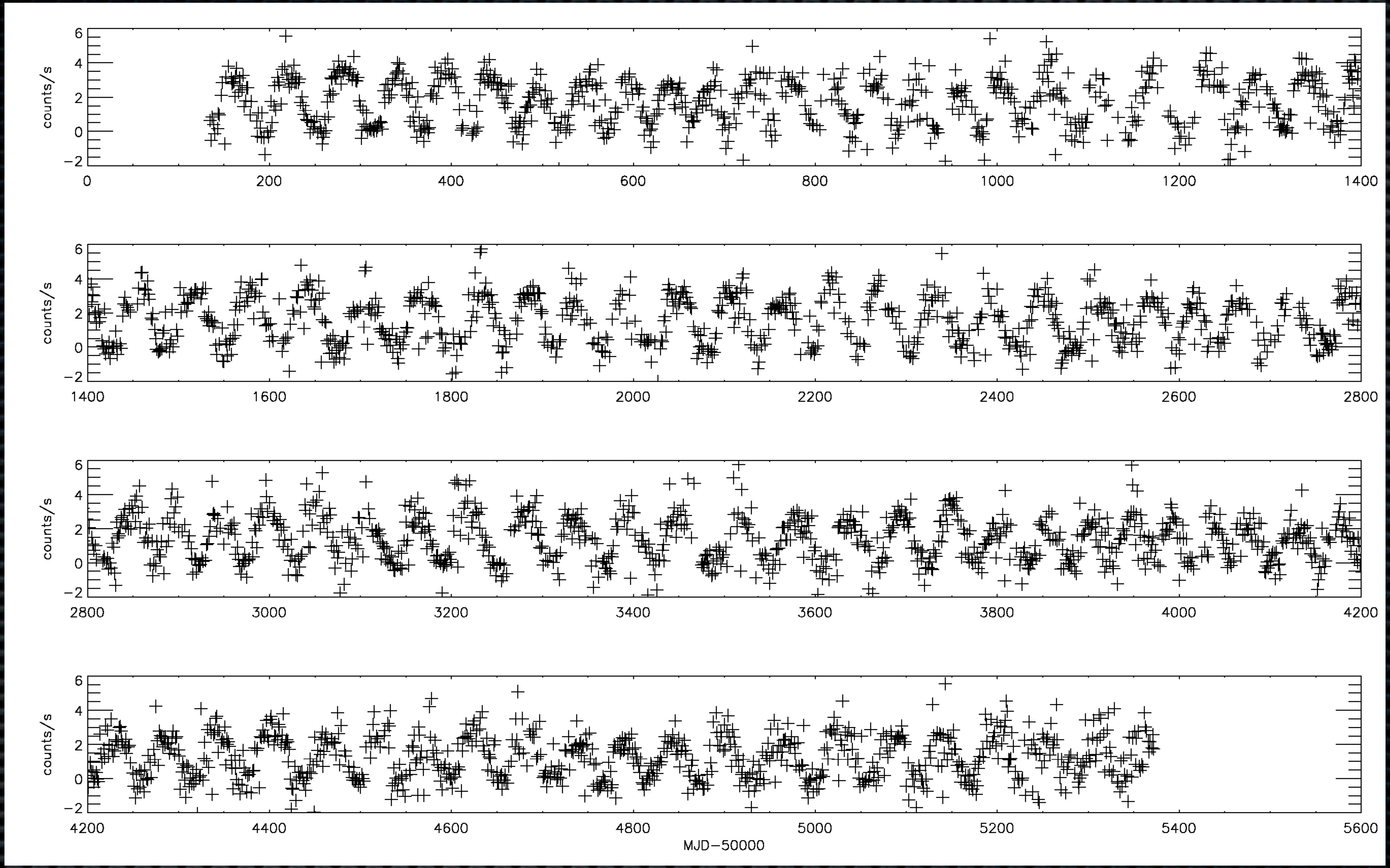
For a comprehensive 'how-to' on data analysis techniques,
visit to the ASM Recipe.

Below you will find a listing of all the sources that the XTE GOF and MIT ASM teams keep track of. The current total is 350 for each the color files and the lightcurves. This listing differs from the SOF's ASM Catalog in that this is a complete listing of all the sources the ASM tracks. The SOF catalog is only the ASM sources which are included on the ASM Weather Map.

Click on the links below to download the ASM definitive light curves and colors for the sources listed, or use Browse to search and retrieve the definitive ASM data products.

Source Name	RA (J2000)	Dec (J2000)	Lc	Col
1E1024.1-5733	156.4858	-57.8115	lc	col
1E1048.1-5937	162.5372	-59.8888	lc	col
1E1547.0-5408	237.7255	-54.3066	lc	col
1E1740.7-2942	265.9781	-29.7454	lc	col
1E1841-045	280.3306	-4.9364	lc	col
1E2259.0+5836	345.2865	58.8792	lc	col
1ES0033+595	8.9693	59.8346	lc	col
1ES0145+318	27.1238	14.0383	lc	col
1ES0229+200	38.2025	20.2881	lc	col
1ES0235+164	39.6621	16.6167	lc	col
1ES0347-121	57.3467	-11.9908	lc	col
1ES0806+524	122.4550	52.3162	lc	col
1ES0821-426	125.7893	-42.8548	lc	col

ASM Light Curve (1996-2010)



- In this research, we use eclipse removed, one-day binned light curve.
- The observation gaps are interpolated by piecewise cubic hermite interpolation method.

The Definition of Frequency

- Given the period of a wave as T, the frequency is defined as:

$$f = \frac{1}{T}$$

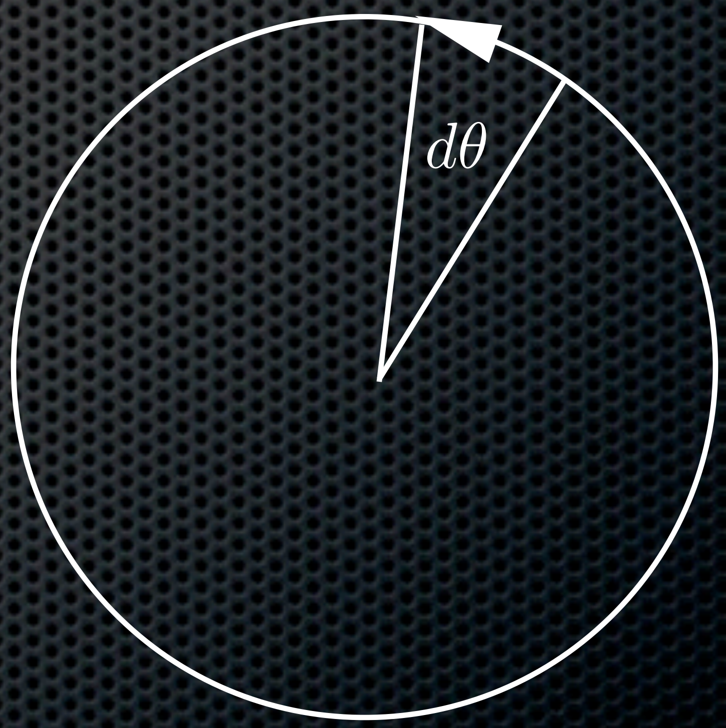
- Too crude
- Only work for simple sinusoidal waves
- Does not work for non-linear processes
- Does not apply to non-stationary processes

Instantaneous Frequency

$$Velocity = \frac{Distance}{Time}$$

Only valid for constant speed, or mean velocity

$$v = \frac{dx}{dt}$$



$$Frequency = \frac{1}{Period}$$

Mean Frequency

$$\omega = \frac{d\theta}{dt}$$

θ is the (angular) phase function

→ How to define θ?

Hilbert Transform

For any time series $x(t)$, we can always have its Hilbert Transform as

$$y(t) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{x(t')}{t-t'}dt'$$

where P indicates the Cauchy principal value. With this definition, x(t) and y(t) form the complex conjugate pair, so we can have an analytic signal z(t) as

$$z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)}$$

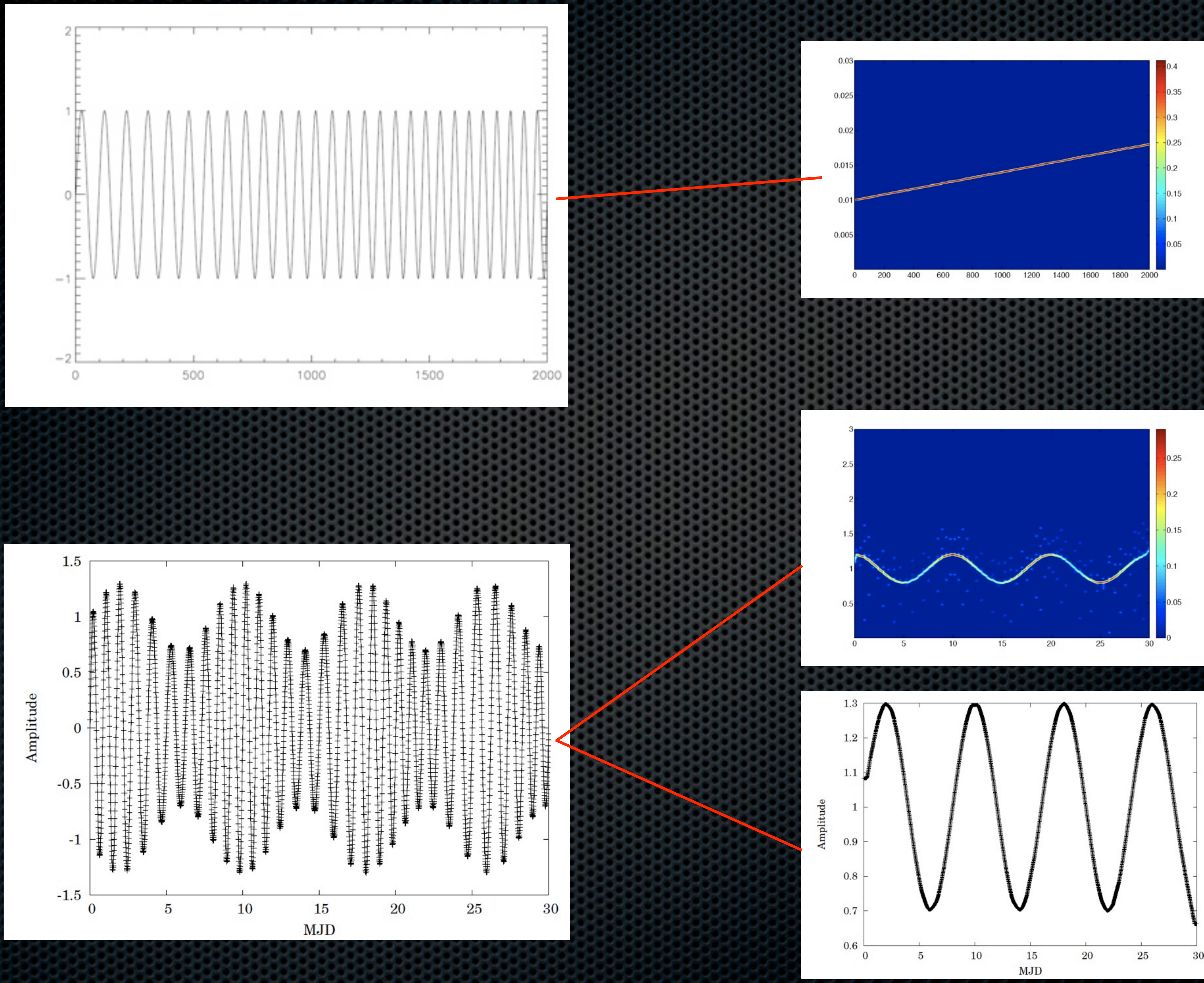
where

$$a(t) = \sqrt{x^2(t) + y^2(t)}, \theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right).$$

In this definition, a(t) is the amplitude of this modulation.

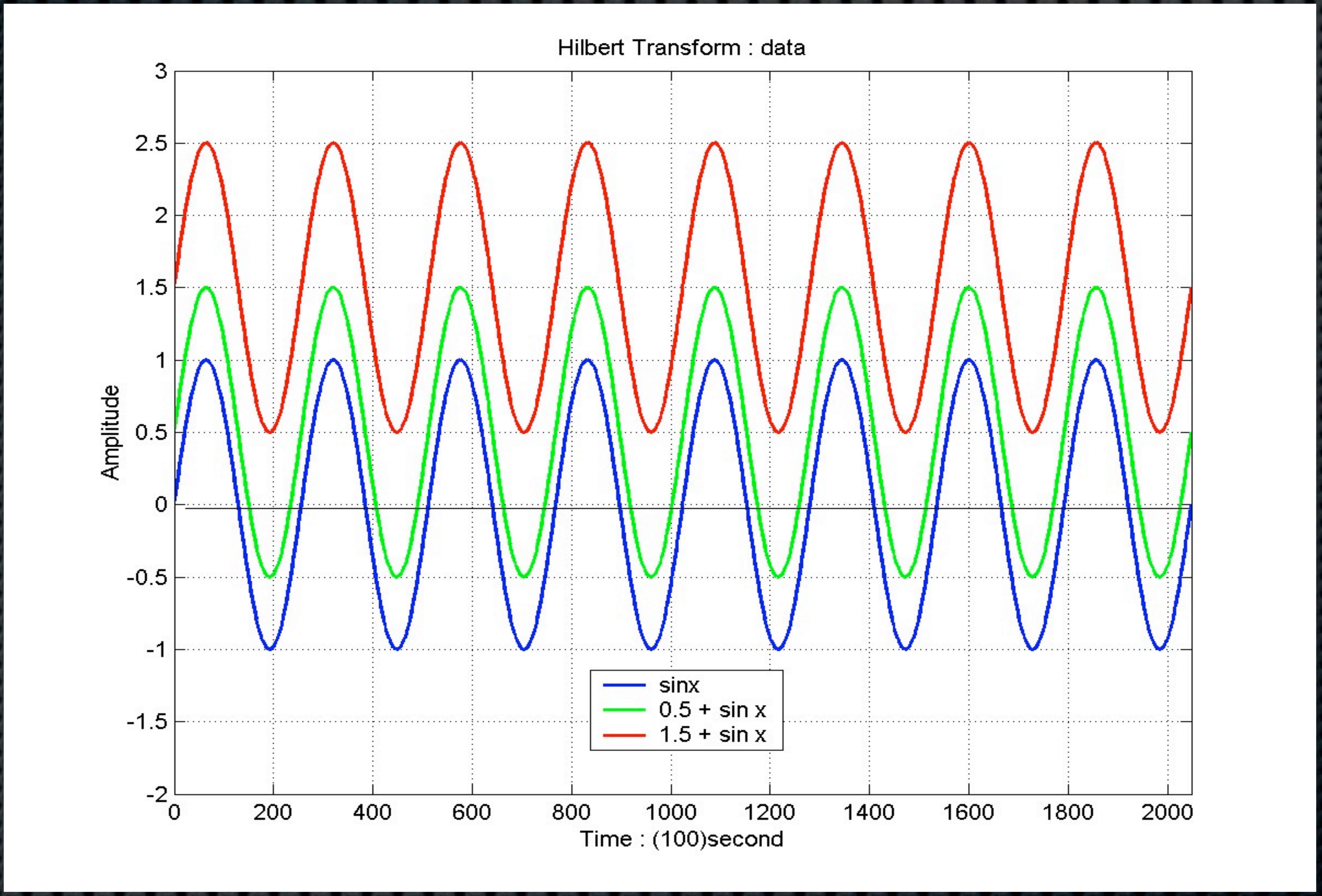
With the well defined phase angle, we can define the instantaneous frequency as

$$\omega = \frac{d\theta}{dt}$$

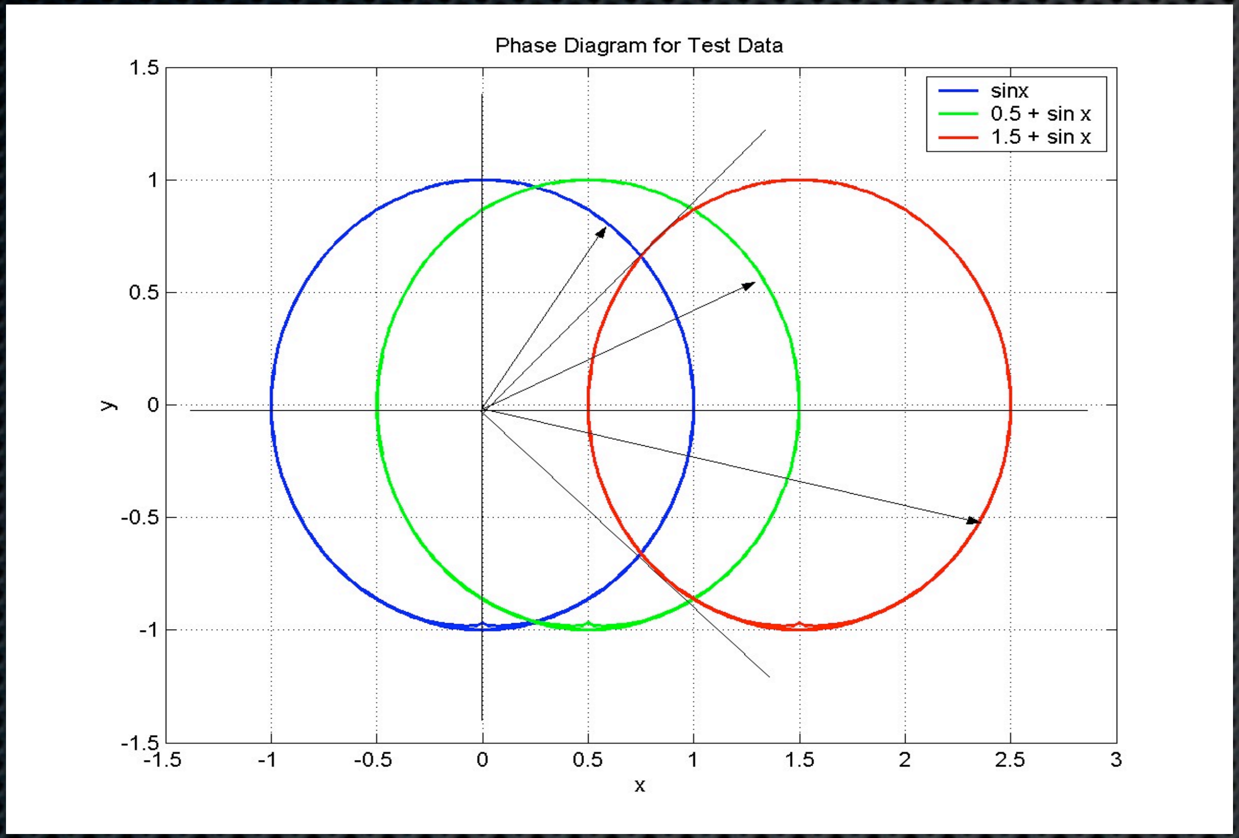


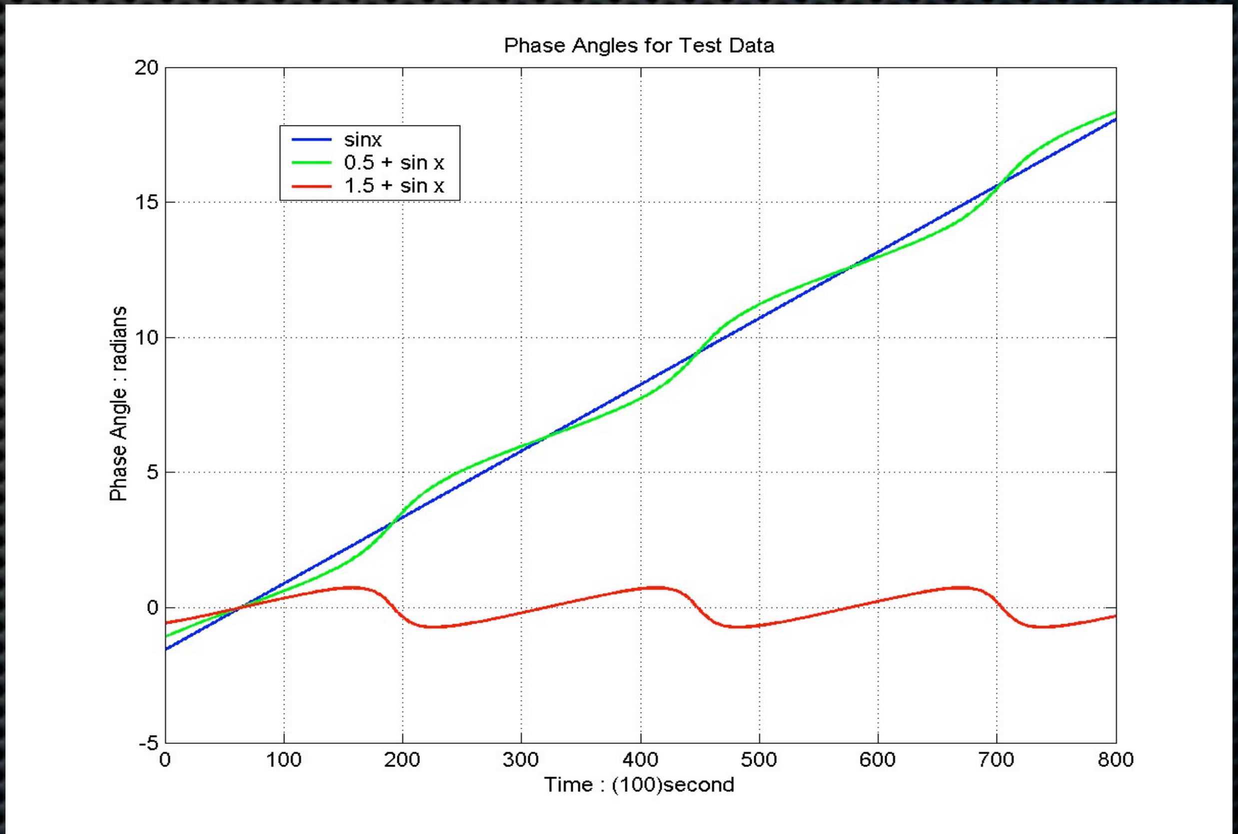
Amplitude
MJD
Amplitude
MJD

However...

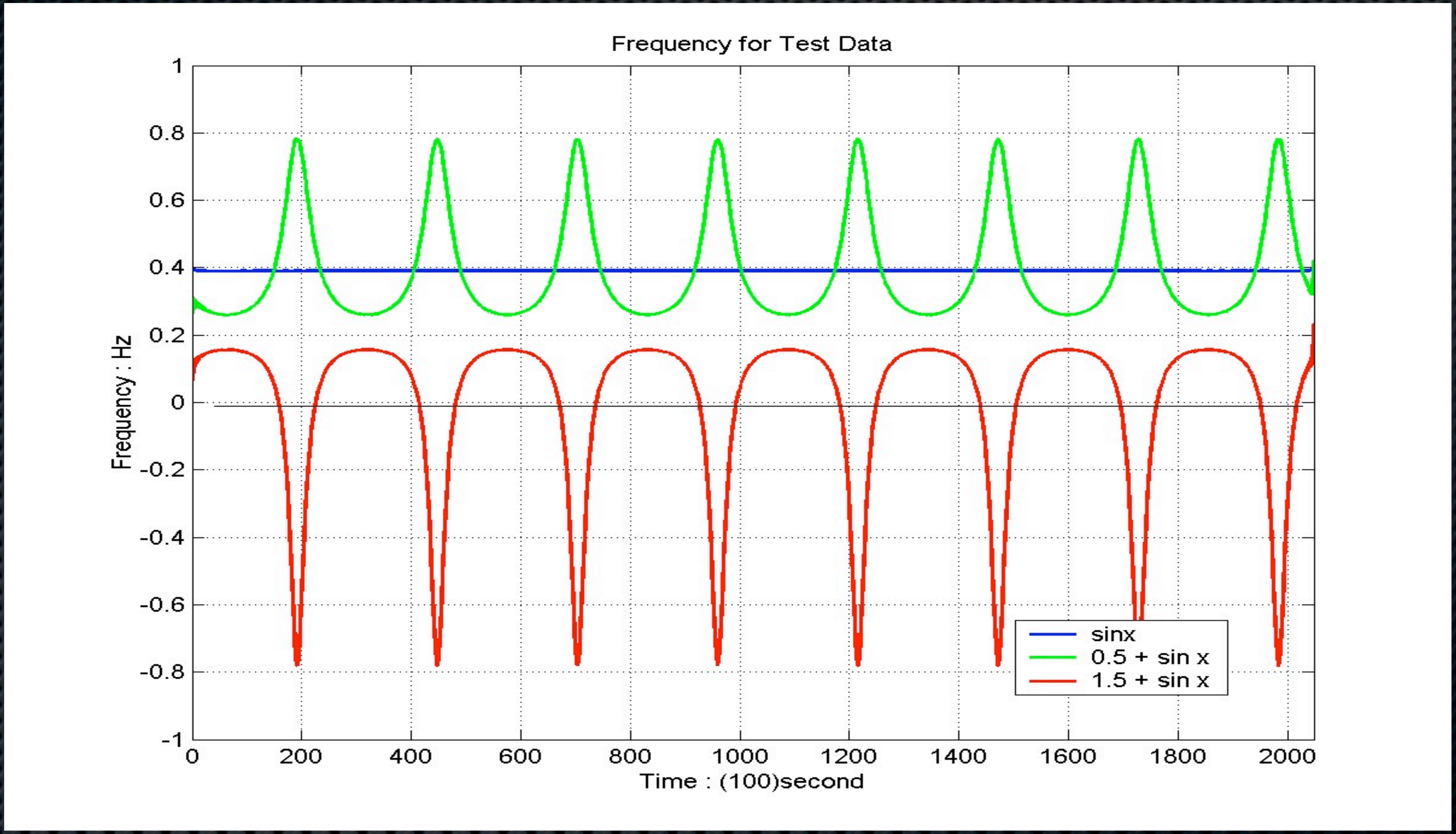

Hilbert Transform : data
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Amplitude
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Time : (100)second
sinx
0.5 + sin x
1.5 + sin x

Hilbert Transform for sin(ωt)+b





The Instantaneous Frequency

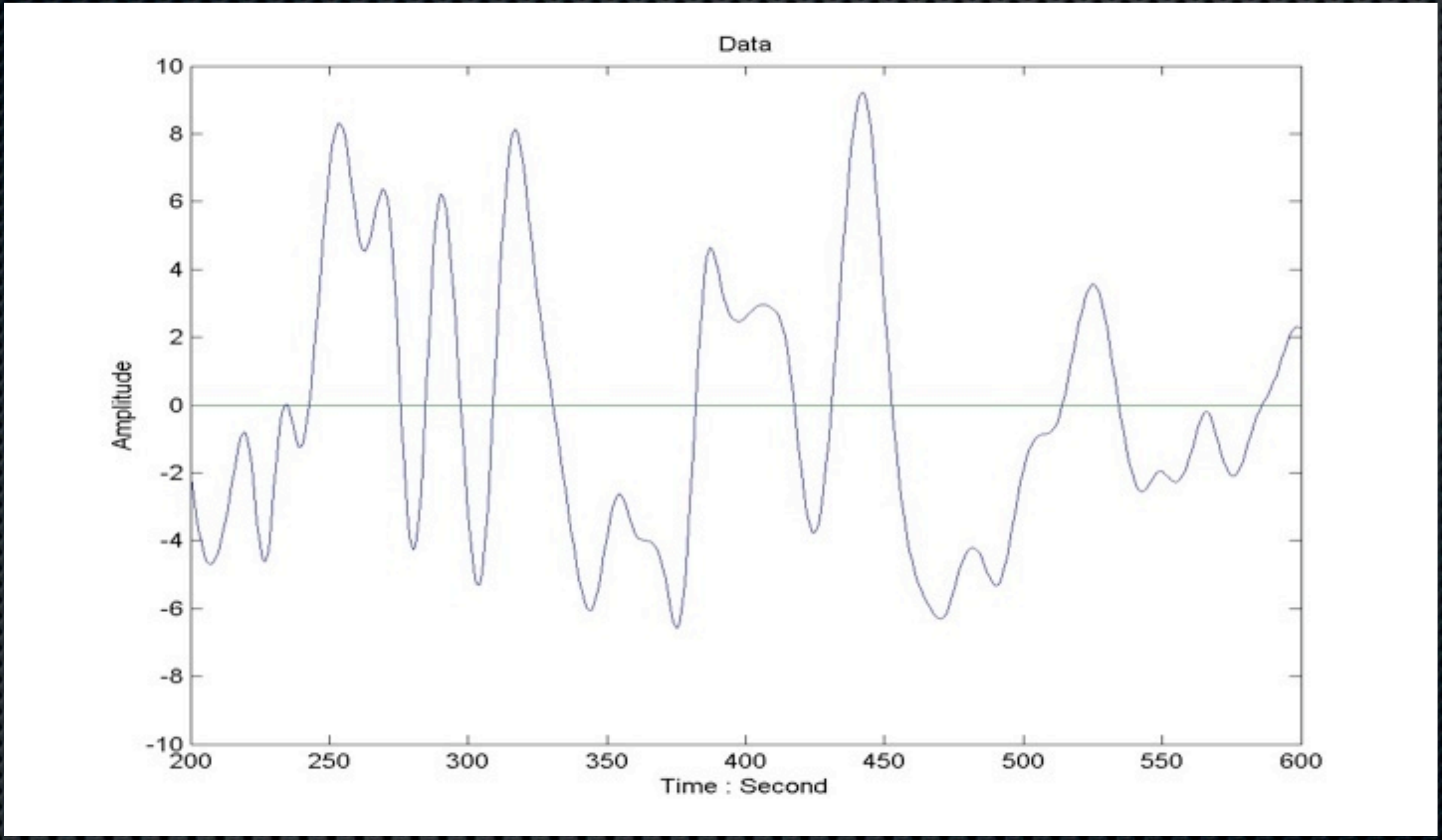


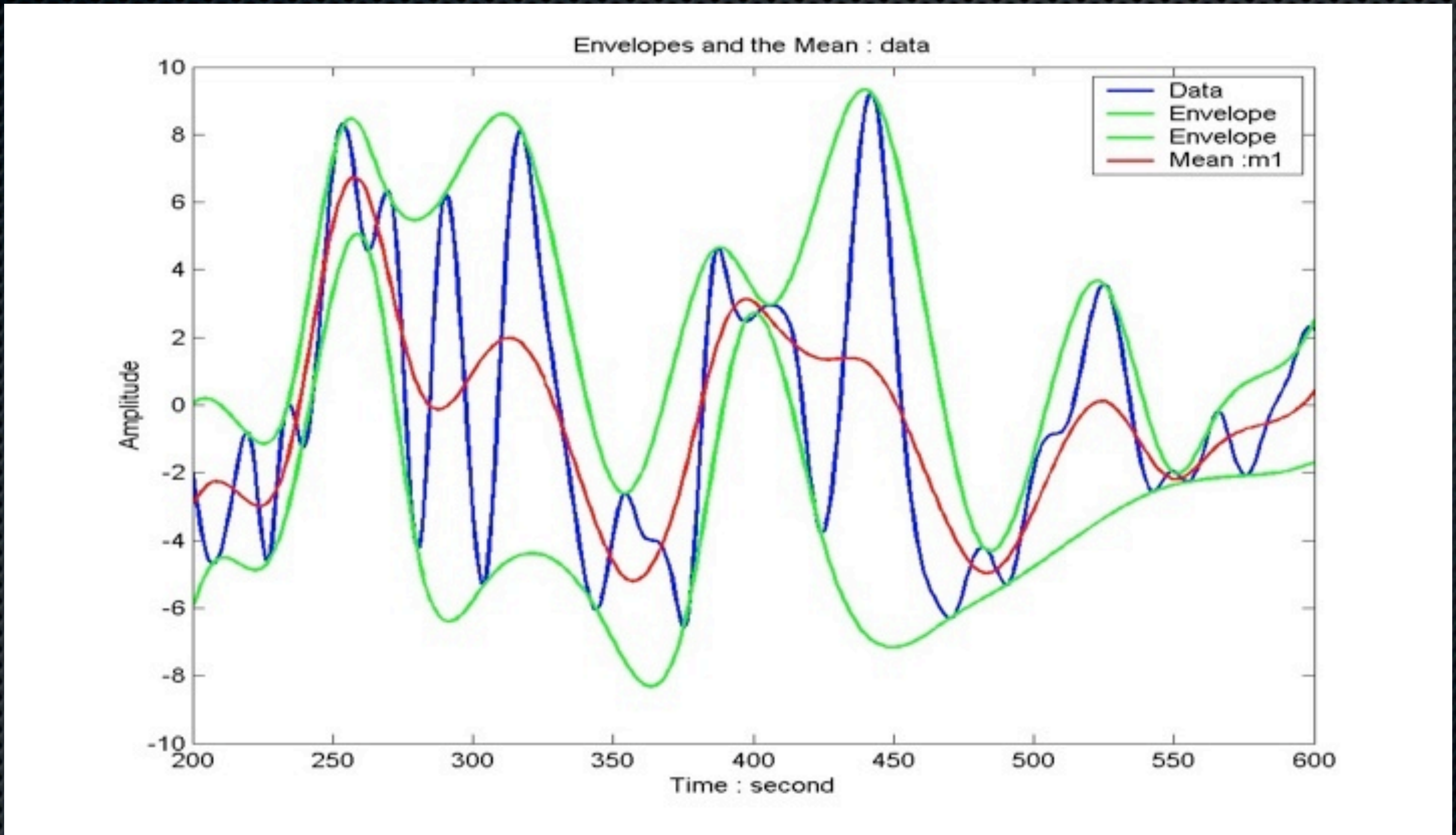
The Intrinsic Mode Function

- Physically, the necessary conditions to define a meaningful instantaneous frequency are that the functions are symmetric with respect to the *local zero mean*, and have the same number of *zero crossings* and *extrema*.
- Based on the observations, Huang et al. (1998) propose a class of functions designated as intrinsic mode functions following the definition:
  - In the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one
  - At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

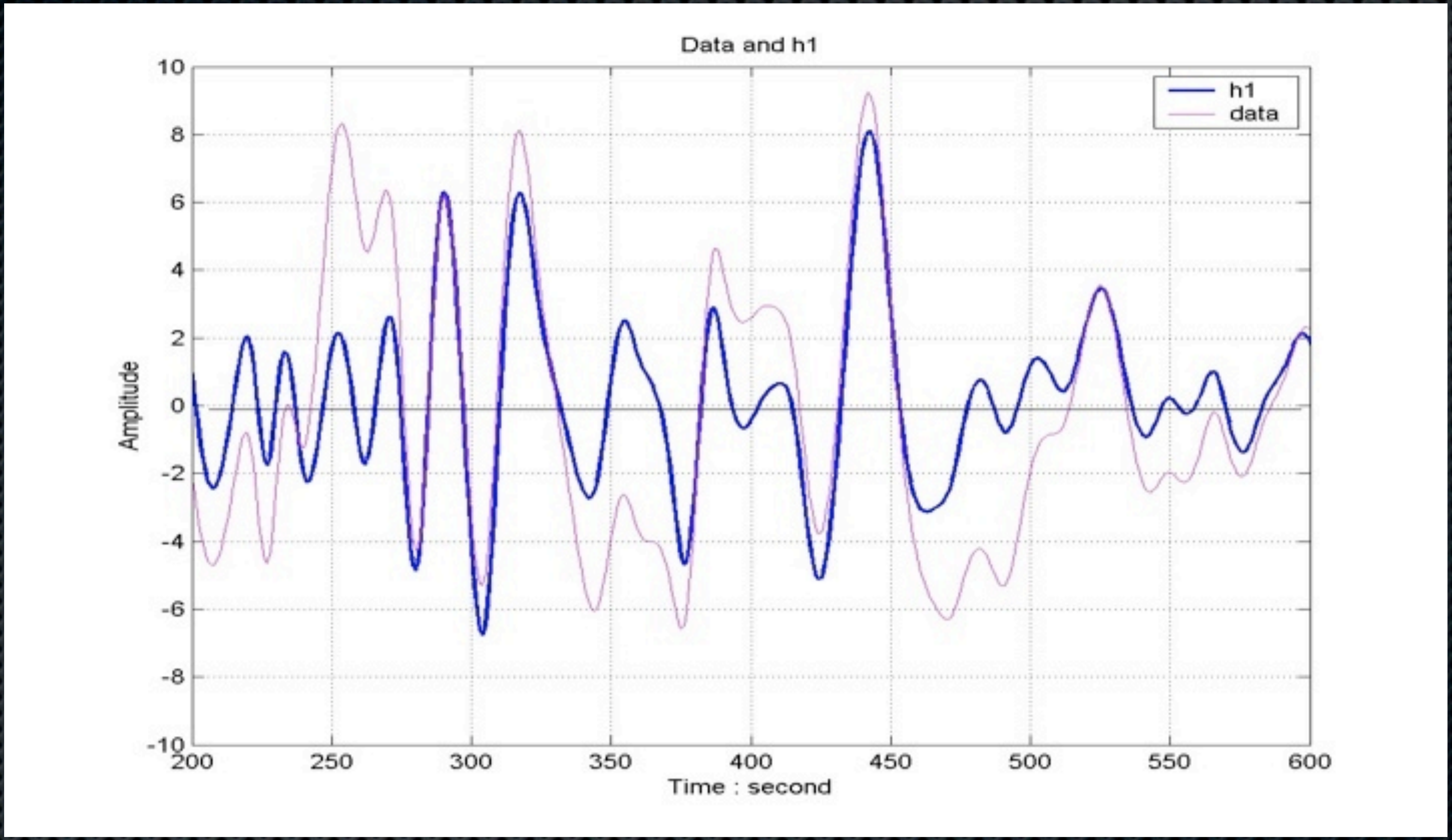
Empirical Mode Decomposition (EMD)

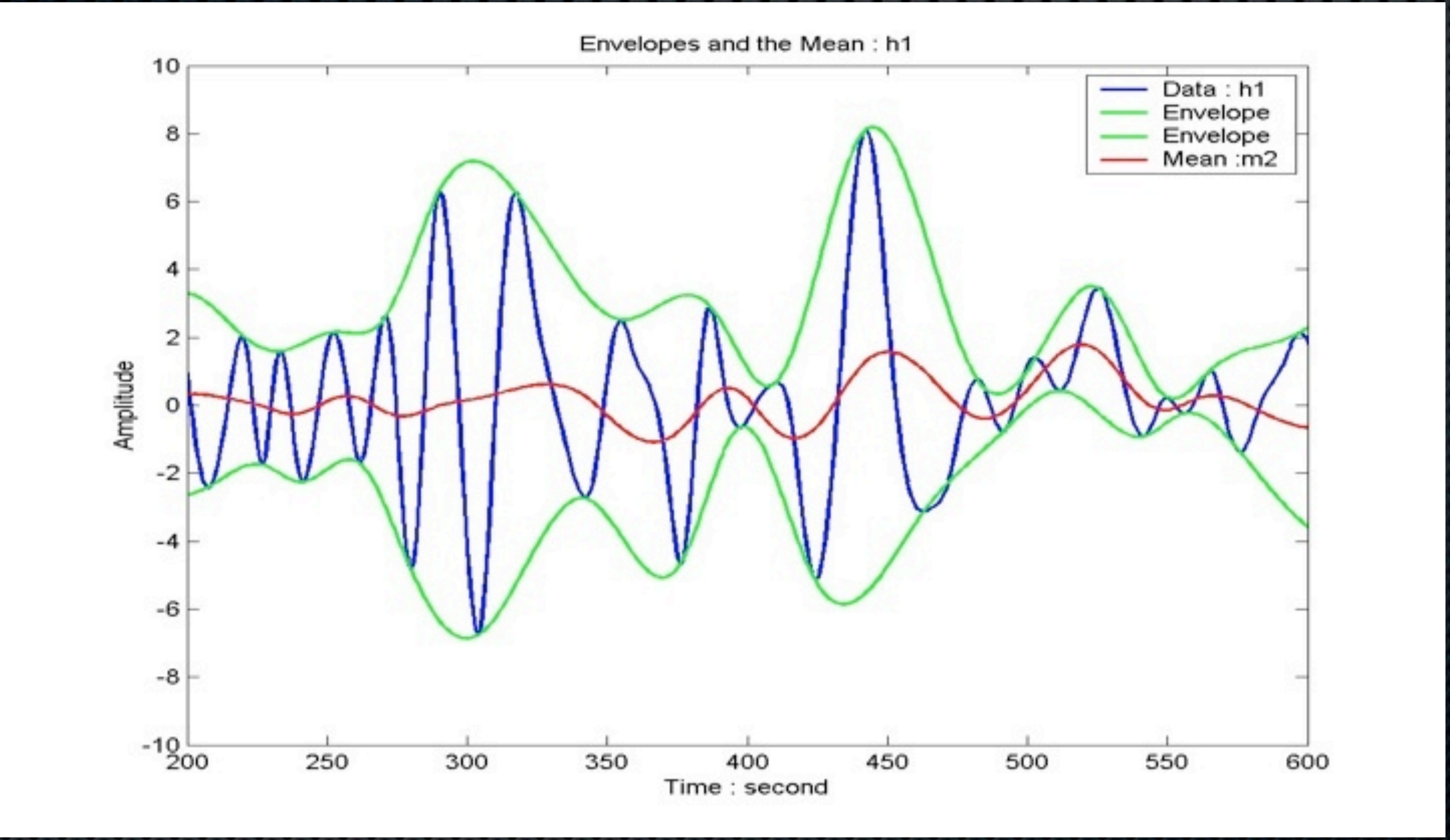
- A sifting process proposed by Norden E. Huang et al. in 1998.
- Because Hilbert Transform can only be applied on IMFs, we have to decompose the original data into IMFs.
- To get an IMF, the procedure is described as follows:
  - Connect the local maxima with cubic spline function to get the upper envelope.
  - Repeat step 1 for local minima to produce the lower envelope.
  - Take the mean value of upper and lower envelopes.
  - Substract the data with the mean value.
  - Treat the difference as data, repeat step 1 to 4 until the result satisfied the condition of IMF.

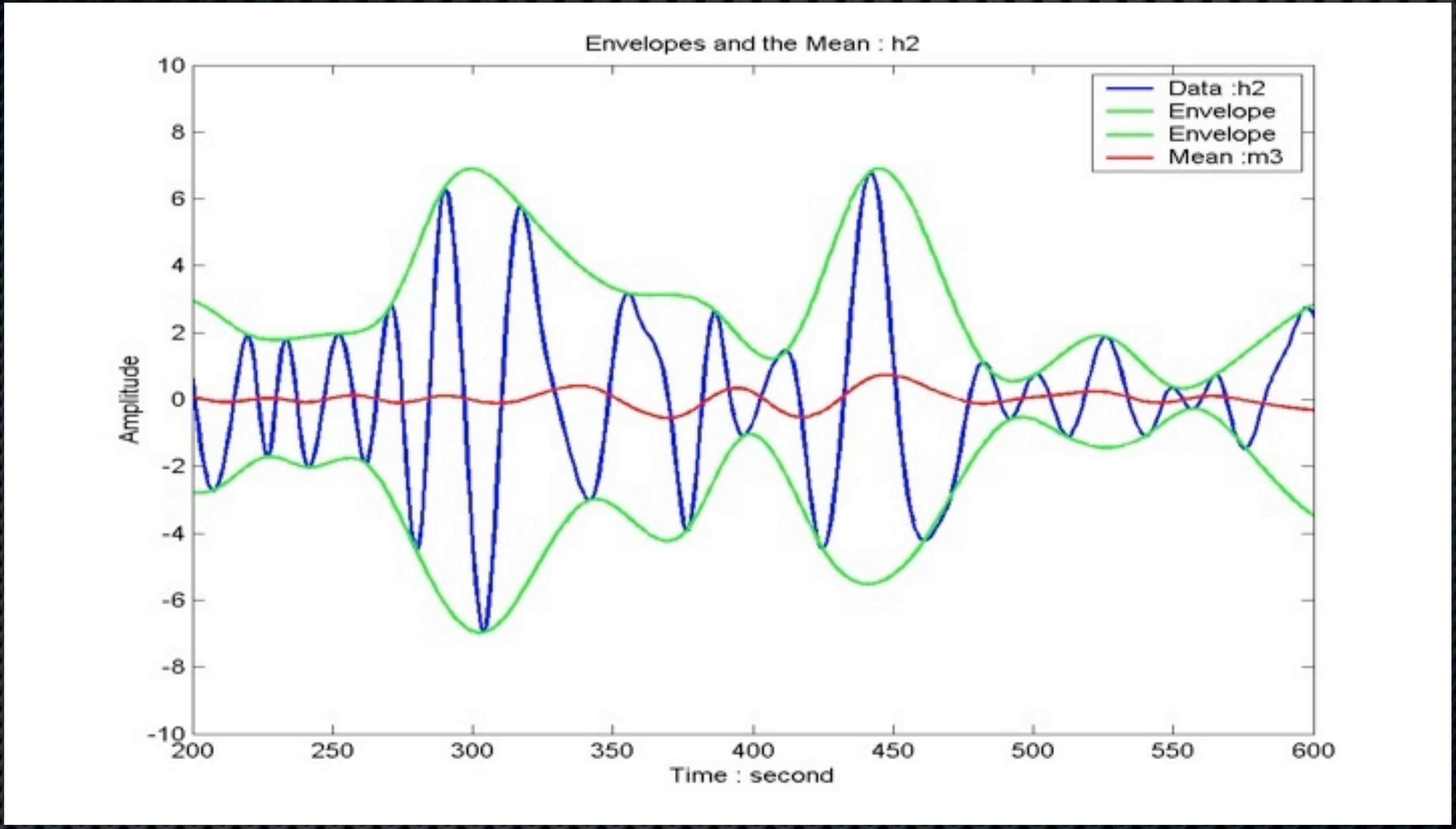

Data
Amplitude
Time : Second
10
8
6
4
2
0
-2
-4
-6
-8
-10
200
250
300
350
400
450
500
550
600



Envelopes and the Mean : data
Data
Envelope
Envelope
Mean :m1
Amplitude
Time : second
10
8
6
4
2
0
-2
-4
-6
-8
-10
200
250
300
350
400
450
500
550
600


Data and h1
h1
data
Amplitude
Time : second
10
8
6
4
2
0
-2
-4
-6
-8
-10
200
250
300
350
400
450
500
550
600


Envelopes and the Mean : h1
Data : h1
Envelope
Envelope
Mean :m2
Amplitude
Time : second
10
8
6
4
2
0
-2
-4
-6
-8
-10
200
250
300
350
400
450
500
550
600


Envelopes and the Mean : h2
Data :h2
Envelope
Envelope
Mean :m3
10
8
6
4
2
0
-2
-4
-6
-8
-10
Amplitude
200
250
300
350
400
450
500
550
600
Time : second

To get one IMF

$$x(t) - m_1 = h_1$$

$$h_1 - m_2 = h_2$$

$$\vdots$$

$$h_{k-1} - m_k = h_k \quad \Rightarrow h_k = c_1$$

$$x(t) - c_1 = r_1$$

Treat r_1 as data:

$$\vdots$$

$$r_1 - c_2 = r_2$$

$$\vdots$$

$$r_{n-1} - c_n = r_n$$

where r_n has no enough extrema to making envelope

$$x(t) = c_1 + r_1 = c_1 + c_2 + r_2 = \ldots = \sum_{i=1}^{n} c_i + r_n$$

Compare with Fourier Transform

- Having obtained the intrinsic mode function components, we will have no difficulties in applying the Hilbert transform to each component, and computing the instantaneous frequency.
- After performing the Hilbert transform on each IMF component, we can express the data in the following form:

$$x(t) = \sum_{j=1}^{n} a_j(t) \exp\left[i \int \omega_j(t) dt\right].$$

- Compare with the Fourier transform:

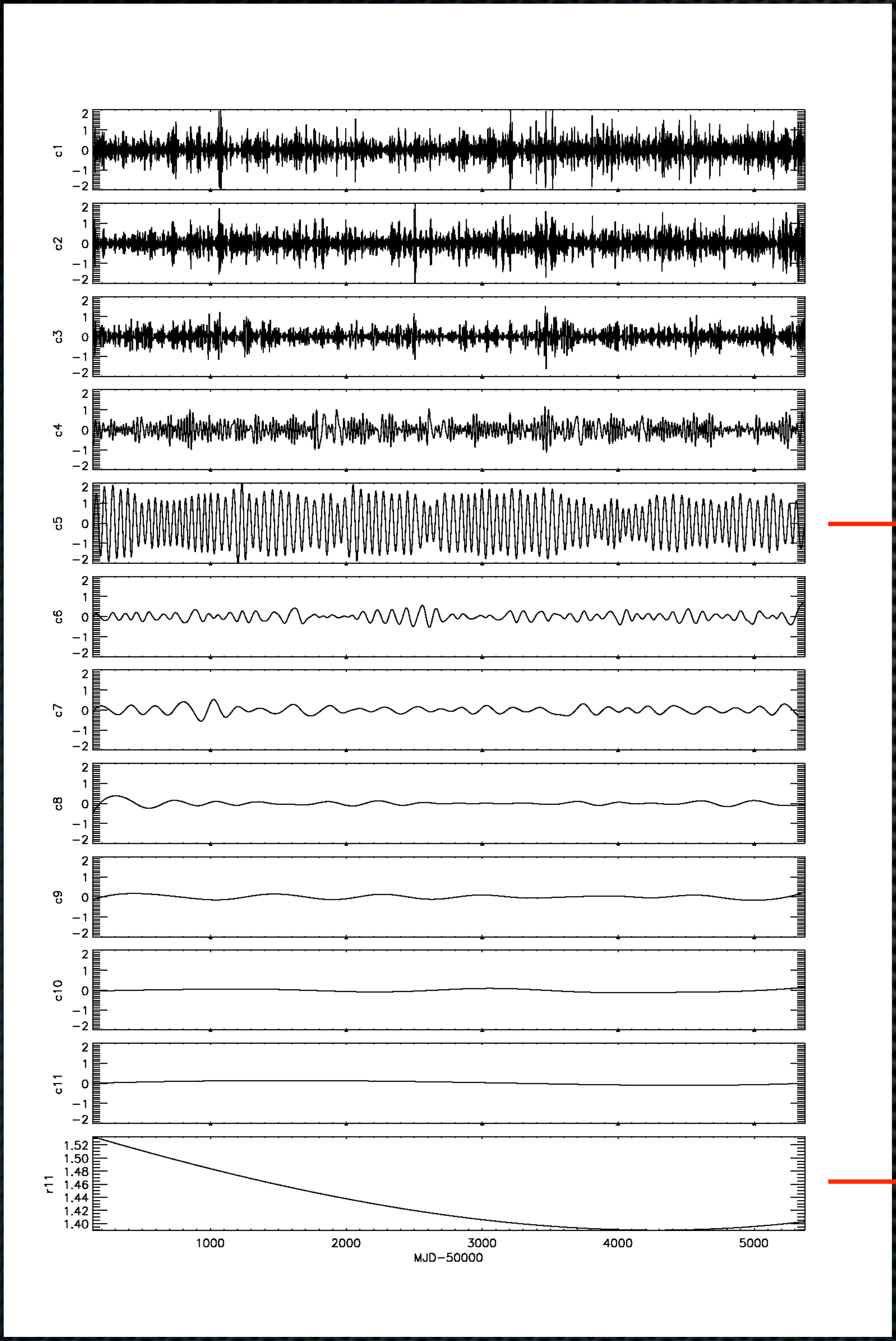
$$x(t) = \sum_{j=1}^{\infty} a_j e^{i\omega_j t}$$

- The major difference is that the amplitudes and frequencies are functions of time.

Ensemble EMD

- If the original data is noisy or contains high-frequency intermittent signals, the decomposition may suffer the mode mixing problem, i.e. the modulation signal with the same time scale across different IMFs
- This would cause many spurious, confused signals in the transition region.
- Based on study of white noise, Wu and Huang proposed a noise assisted data analysis method called Ensemble Empirical Mode Decomposition (EEMD)
- The procedure is described as follows:
  - Add a white noise series to the data.
  - Decomposed the data into IMFs by EMD.
  - repeat step 1 and 2 again and again.
  - Obtain the ensemble means of corresponding IMFs as the final result.

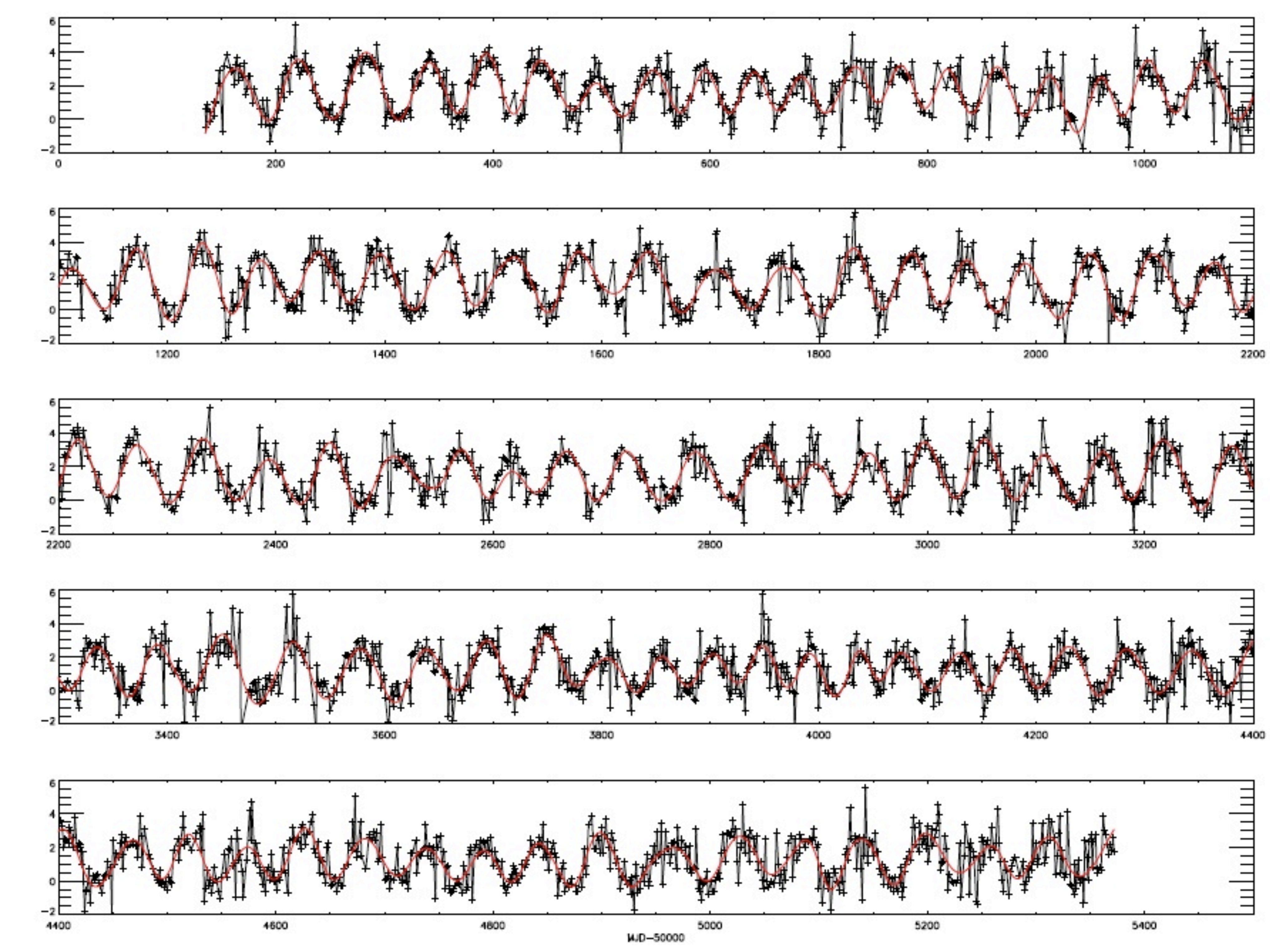
EEMD decomposition of SMC X-1



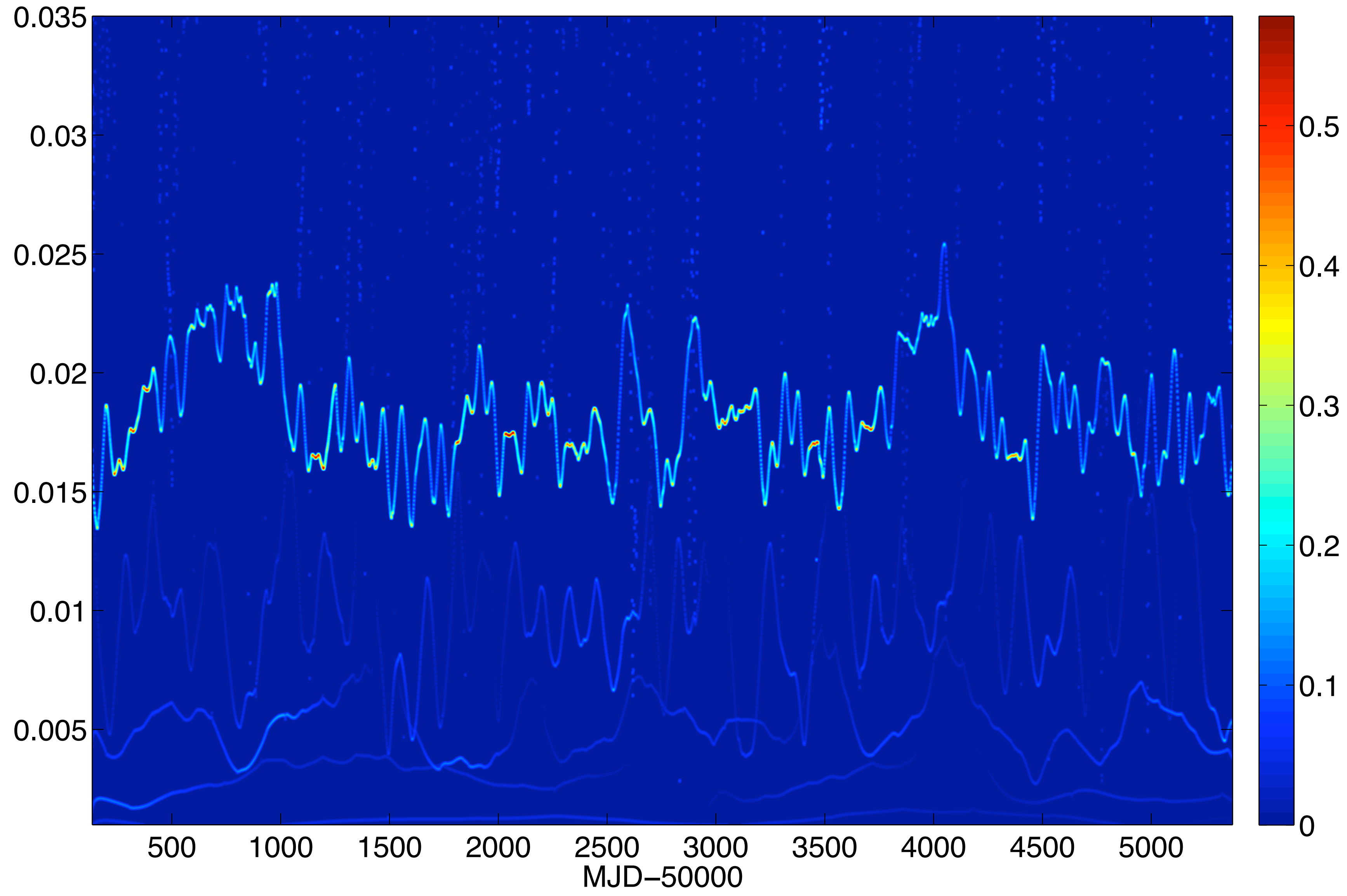
responsible for 40-65d superorbital modulation (by significant test)

decreasing trend

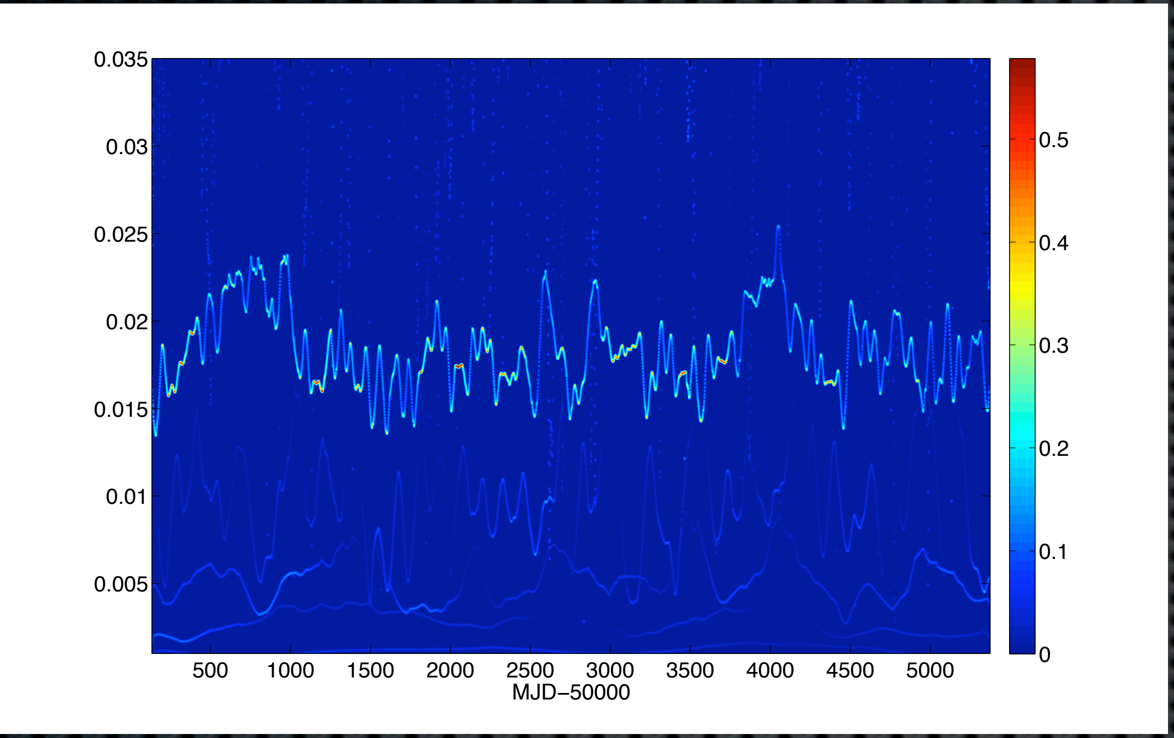
EMD Low-Pass Filter

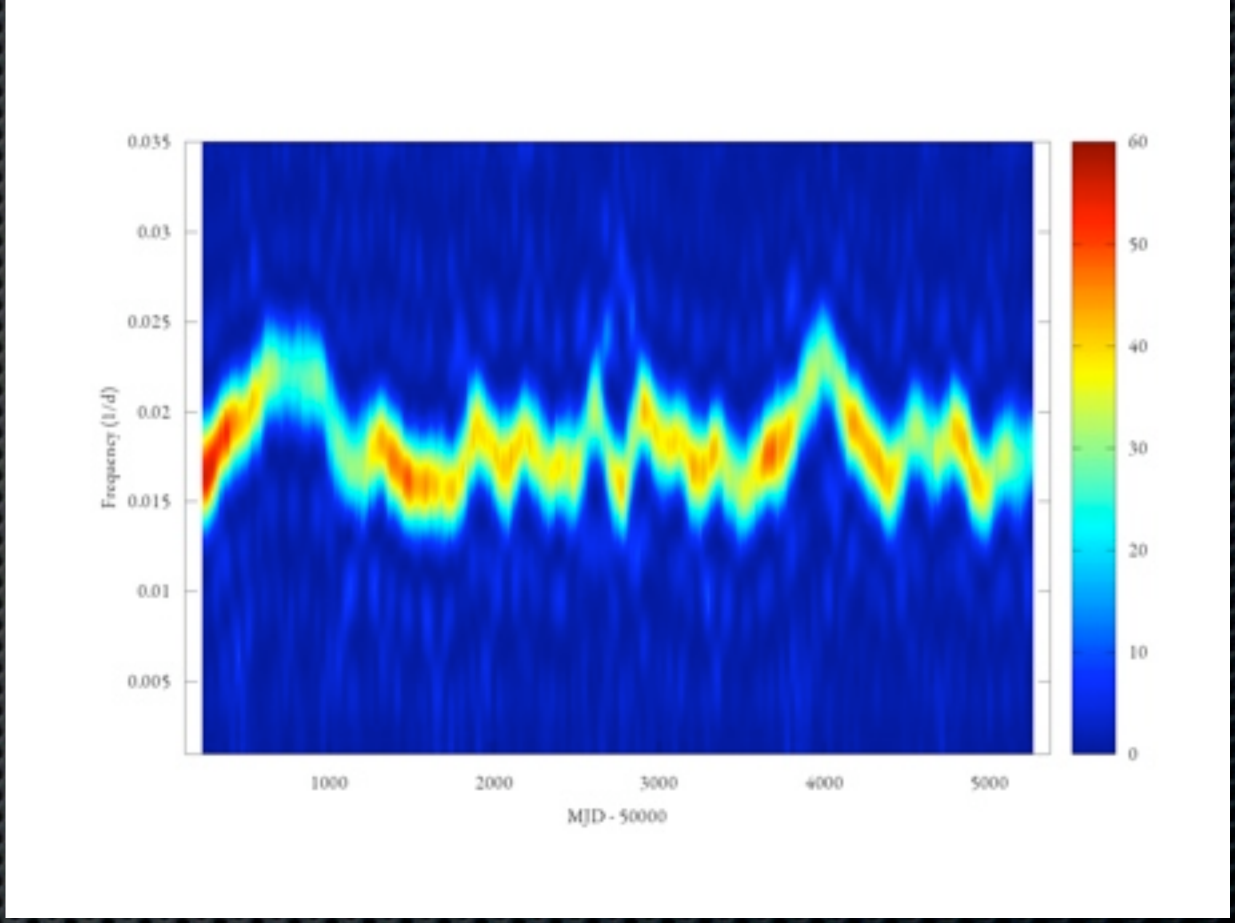


Hilbert Spectrum

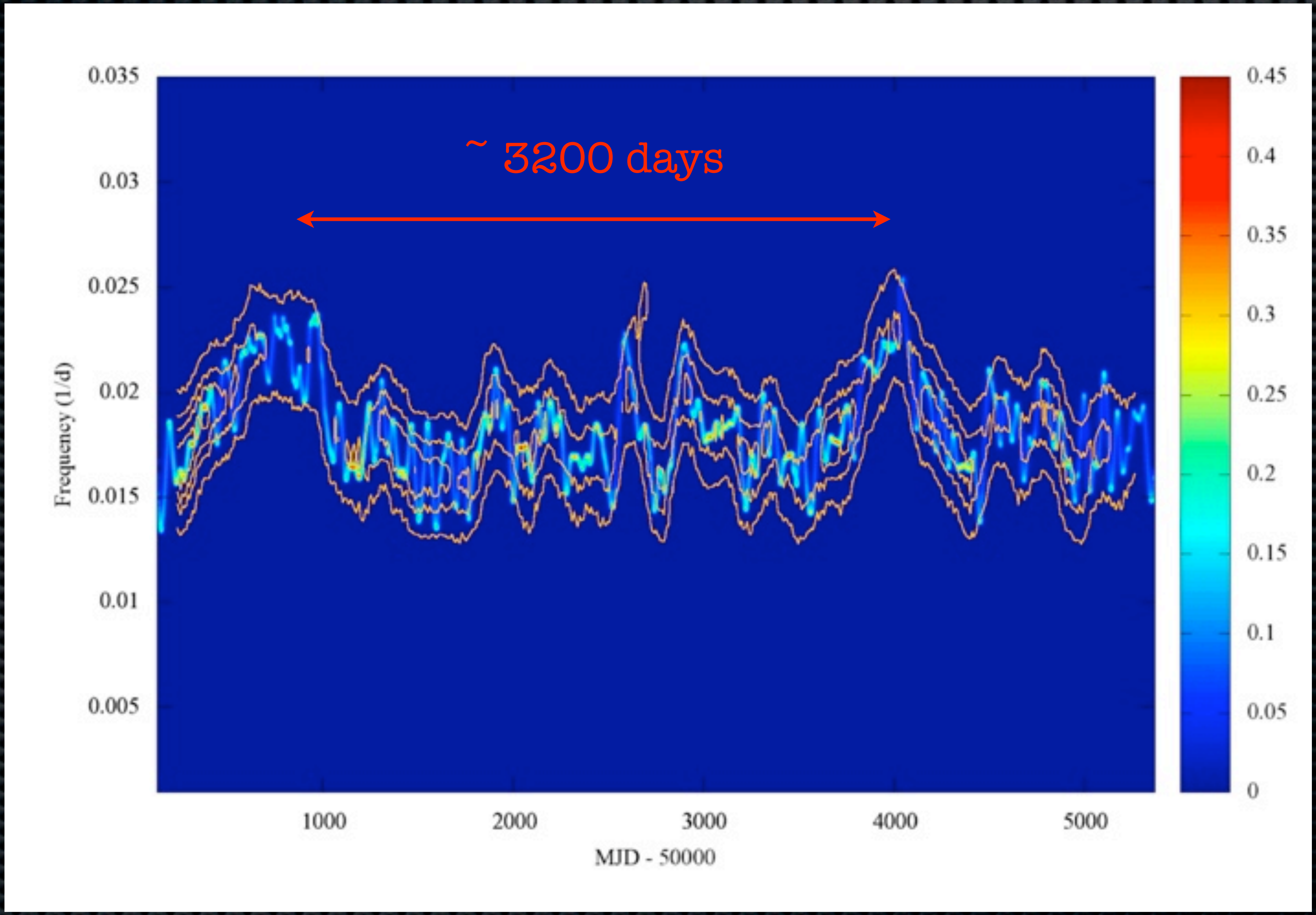


Hilbert Spectrum & Dynamic Power Spectrum

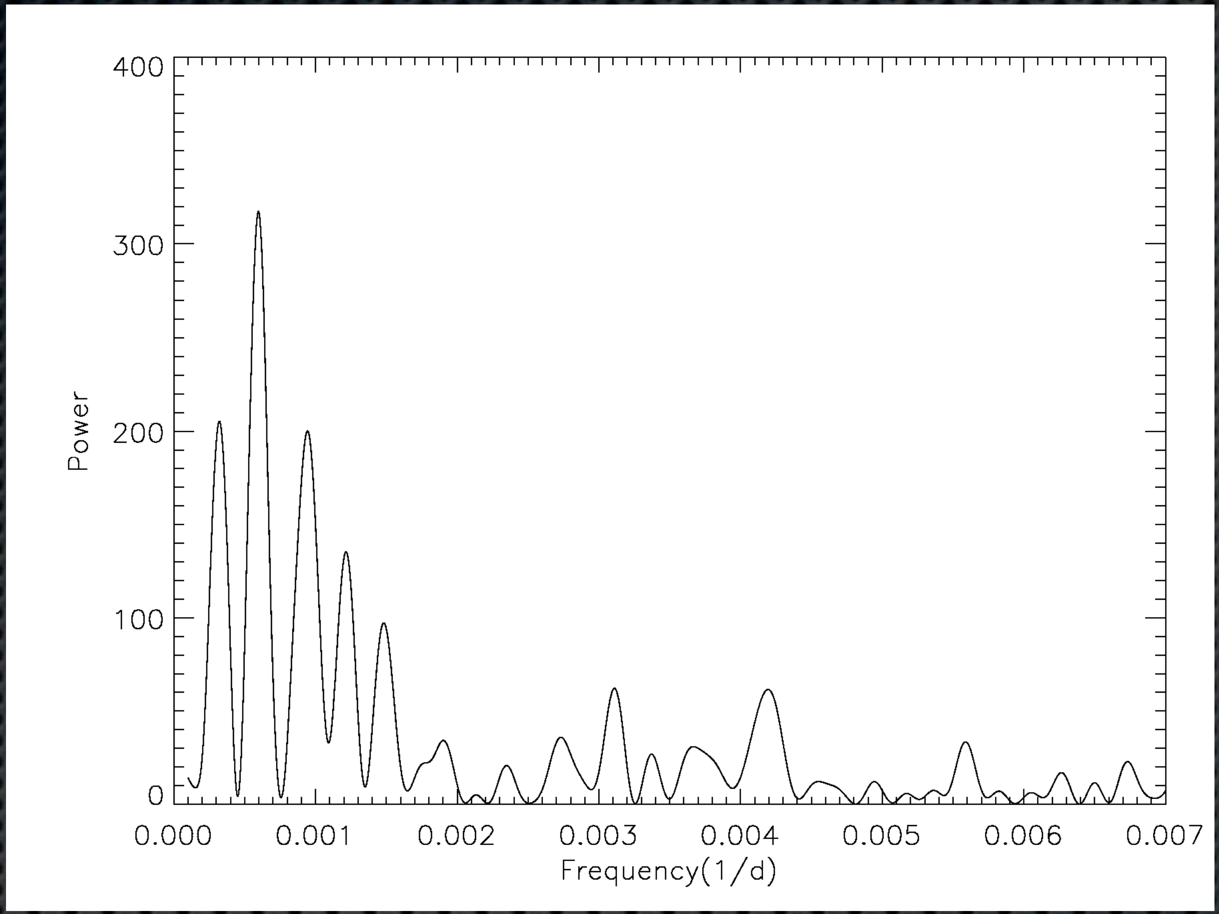




Hilbert Spectrum & Dynamic Power Spectrum

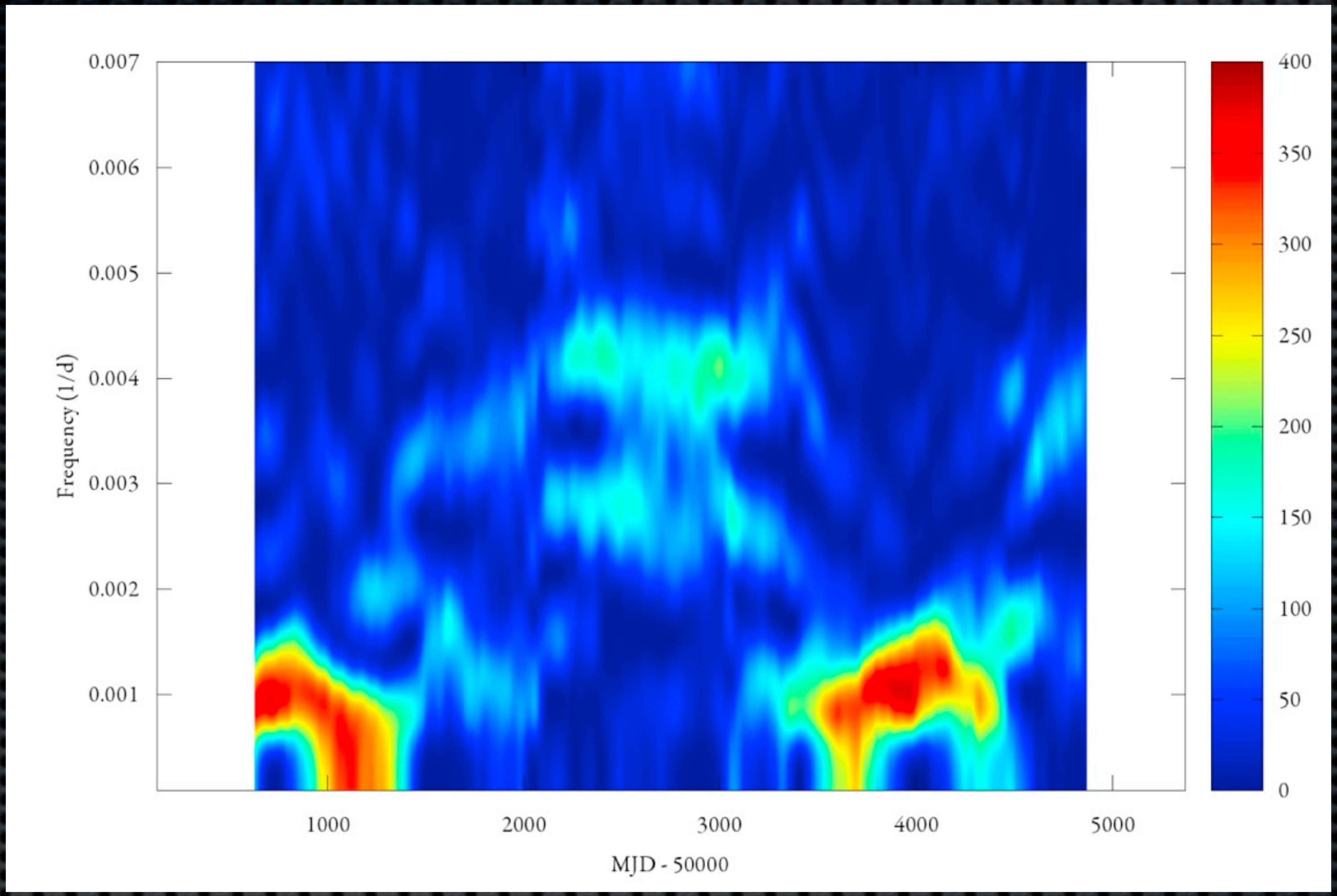


Periodicity of the Instantaneous Frequency



- The HHT provide a rigorous definition of instantaneous frequency. Thus, we can apply further timing analysis technique on the instantaneous frequencyu to obtain the periodicitied hidden in the frequency.
- The Lomb-Scargle periodogram of instantaneous frequency shows a lot of peaks that are harmonics of f=0.0003 (P ~ 3200d)
- The highest peak is P~1600d, which has been indicated by many previous studies.

Dynamic Power Spectrum of Instantaneous Frequency



- The most prominent peaks are located at around MJD 50,800 and MJD 54,000, which are correspond to the short-period state of the superorbital modulation in the time domain and the highest peaks in the frequency domain.
- During ~MJD 52,000 and ~MJD 53,500, the most prominent periodicities disappear and smaller periodicities from P ~ 240 d to P ~ 320 d are observed.

Superorbital Modulation Profile

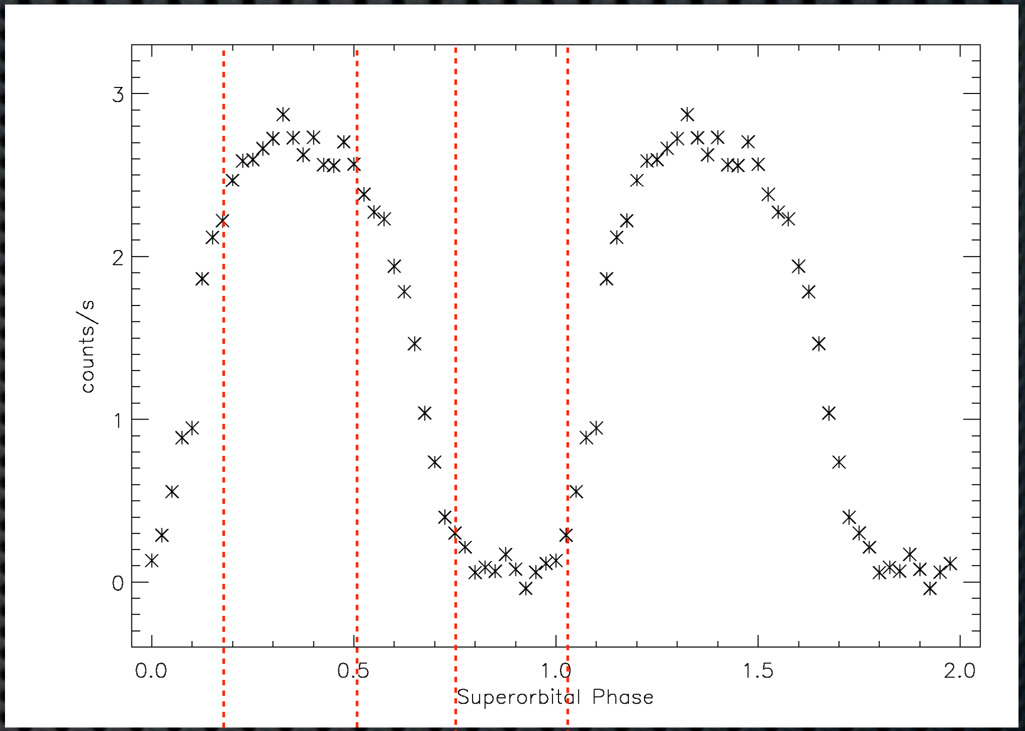
- It is difficult to derive a proper profile to describe the characteristics of superorbital modulation for highly variable cycle lengths from a light curve folded with a fixed period.

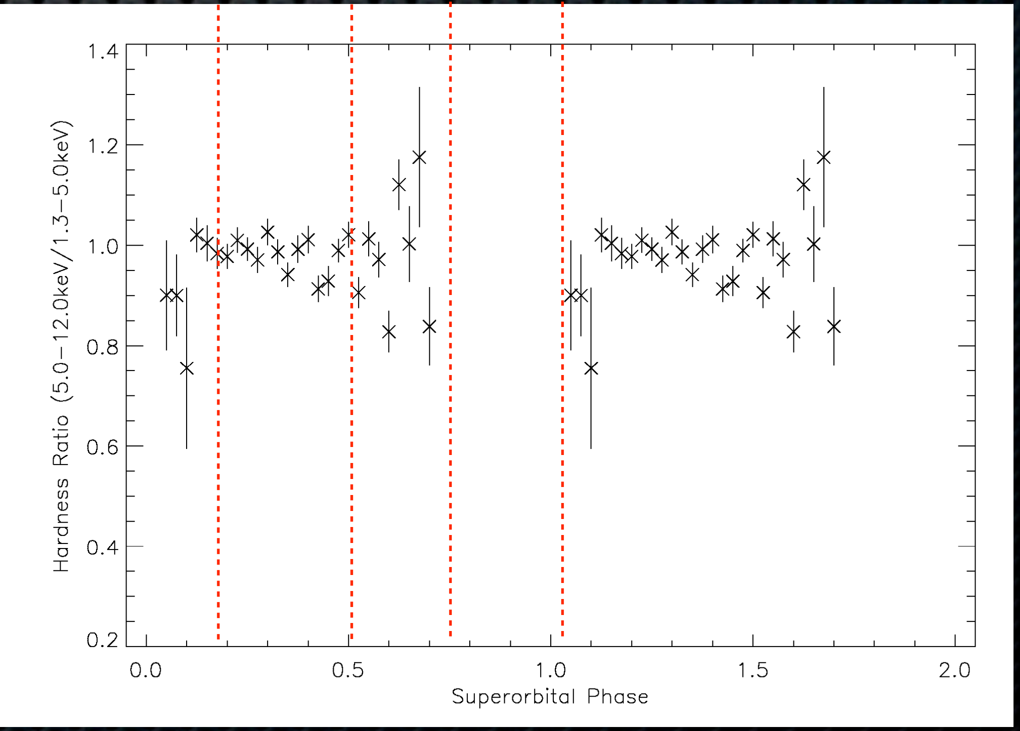
- The HHT gives us well-defined phase function. Thus, we can fold the light curve according to the phase to obtain the superorbital profile.

- The first data point is defined as phase zero.

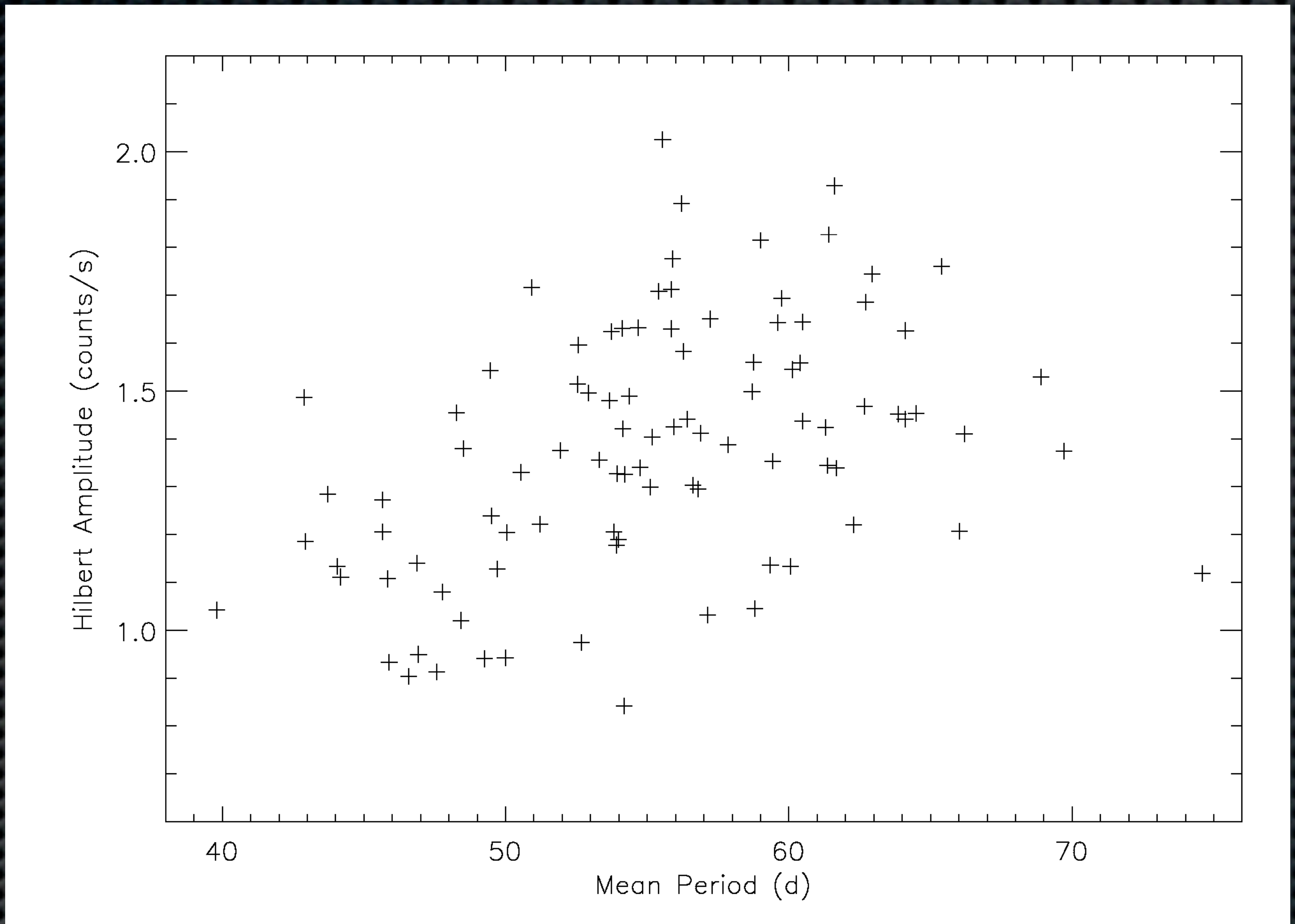
- The hardness ratio can also be folded with the same procedure.

- The hardness ratio in transition state do not appear to significantly deviate from the mean value as compared with that in the high state.





Correlation between Period and Amplitude



- The rank correlation coefficient is r = 0.46 with a null hypothesis probability value of 3.2×10^{-6}, indicating a significant correlation between the period and the amplitude.
- Similar correlation is observed in the superorbital modulation of Her X-1

Correlation between Period and Amplitude

$$P_p \sim \frac{\Sigma_d R_d^{3/2}}{L_d}$$

Wijers & Pringle (1999)

where P_p is the precession period of accretion disk, Σ_d is the surface density, R_d is the characteristic accretion disk radius, and L_d is the strength of the radiation field from the disk

If the radiation field from the disk is dominated by irradiation from a central source, then the amplitude of modulation will be correlate with L_d.

However, our result shows that precession period correlate with the amplitude, which is inconsistent with the prediction.

Summary & Future Work

- We successfully apply a recent developed time-frequency analysis method, the Hilbert-Huang Transform, on the ASM data of SMC X-1.
- The Hilbert spectrum is basically consistent with the dynamic power spectrum, but shows more detailed information in both time and frequency domains.
- By further timing analysis, we can find the periodicities hidden in the instantaneous frequency.
- With a well-defined phase derived by the HHT, we can get folded light curve and folded color curve even if the period changes dramatically.
- A correlation between superorbital period and modulation amplitude is obtained.

- Superorbital phase resolved spectral analysis
- Applying HHT on other sources & other fields.