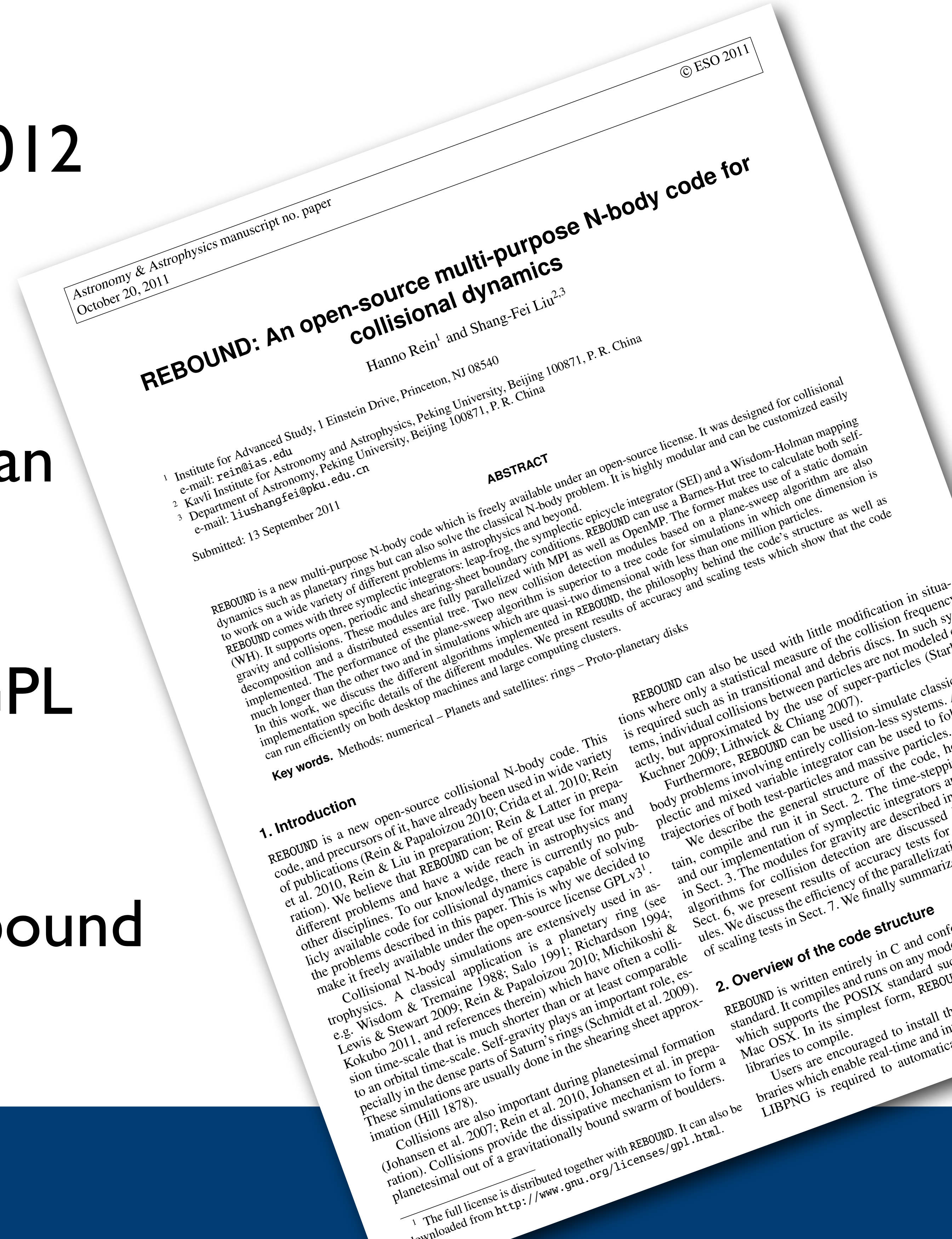
Institute for Advanced Study

The collisional N-body code REBOUND and three applications to Saturn's Rings

Hanno Rein @ Kobe, March 2012

REBOUND

- Code description paper published by A&A, Rein & Liu 2012
- Multi-purpose N-body code
- First public N-body code that can be used for granular dynamics
- Written in C99, open source, GPL
- Freely available at http://github.com/hannorein/rebound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REBOUND: An open-source multi-purpose N-body code for collisional dynamics

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ABSTRACT

REBOUND is a new multi-purpose N-body code which is freely available under an open-source license. It was designed for collisional dynamics such as planetary rings but can also solve the classical N-body problem. It is highly modular and can be customized easily to work on a wide variety of different problems in astrophysics and beyond.
REBOUND comes with three symplectic integrators: leap-frog, the symplectic epicycle integrator (SEI) and a Wisdom-Holman mapping (WH). It supports open, periodic and shearing-sheet boundary conditions. REBOUND can use a Barnes-Hut tree to calculate both self-gravity and collisions. These modules are fully parallelized with MPI as well as OpenMP. The former makes use of a static domain decomposition and a distributed essential tree. Two new collision detection modules based on a plane-sweep algorithm are also implemented. The performance of the plane-sweep algorithm is superior to a tree code for simulations in which one dimension is much longer than the other two and in simulations which are quasi-two dimensional with less than one million particles.
In this work, we discuss the different algorithms implemented in REBOUND, the philosophy behind the code's structure as well as implementation specific details of the different modules. We present results of accuracy and scaling tests which show that the code can run efficiently on both desktop machines and large computing clusters.

Key words. Methods: numerical – Planets and satellites: rings – Proto-planetary disks

1. Introduction

REBOUND is a new open-source collisional N-body code. This code, and precursors of it, have already been used in wide variety of publications (Rein & Papaloizou 2010; Crida et al. 2010; Rein et al. 2010, Rein & Liu in preparation; Rein & Latter in preparation). We believe that REBOUND can be of great use for many different problems and have a wide reach in astrophysics and other disciplines. To our knowledge, there is currently no publicly available code for collisional dynamics capable of solving the problems described in this paper. This is why we decided to make it freely available under the open-source license GPLv3[1].

Collisional N-body simulations are extensively used in astrophysics. A classical application is a planetary ring (see e.g. Wisdom & Tremaine 1988; Salo 1991; Richardson 1994; Lewis & Stewart 2009; Rein & Papaloizou 2010; Michikoshi & Kokubo 2011, and references therein) which have often a collision time-scale that is much shorter than or at least comparable to an orbital time-scale. Self-gravity plays an important role, especially in the dense parts of Saturn's rings (Schmidt et al. 2009). These simulations are usually done in the shearing sheet approximation (Hill 1878).

Collisions are also important during planetesimal formation (Johansen et al. 2007; Rein et al. 2010, Johansen et al. in preparation). Collisions provide the dissipative mechanism to form a planetesimal out of a gravitationally bound swarm of boulders.

1 The full license is distributed together with REBOUND. It can also be downloaded from http://www.gnu.org/licenses/gpl.html.

REBOUND modules

Geometry
- Open boundary conditions
- Periodic boundary conditions
- Shearing sheet / Hill's approximation

Integrators
- Leap frog
- Symplectic Epicycle integrator (SEI)
- Wisdom-Holman mapping (WH)

Gravity
- Direct summation, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- FFT method, $O(N \log(N))$
- GRAPE, hardware accelerated, $O(N^2)$

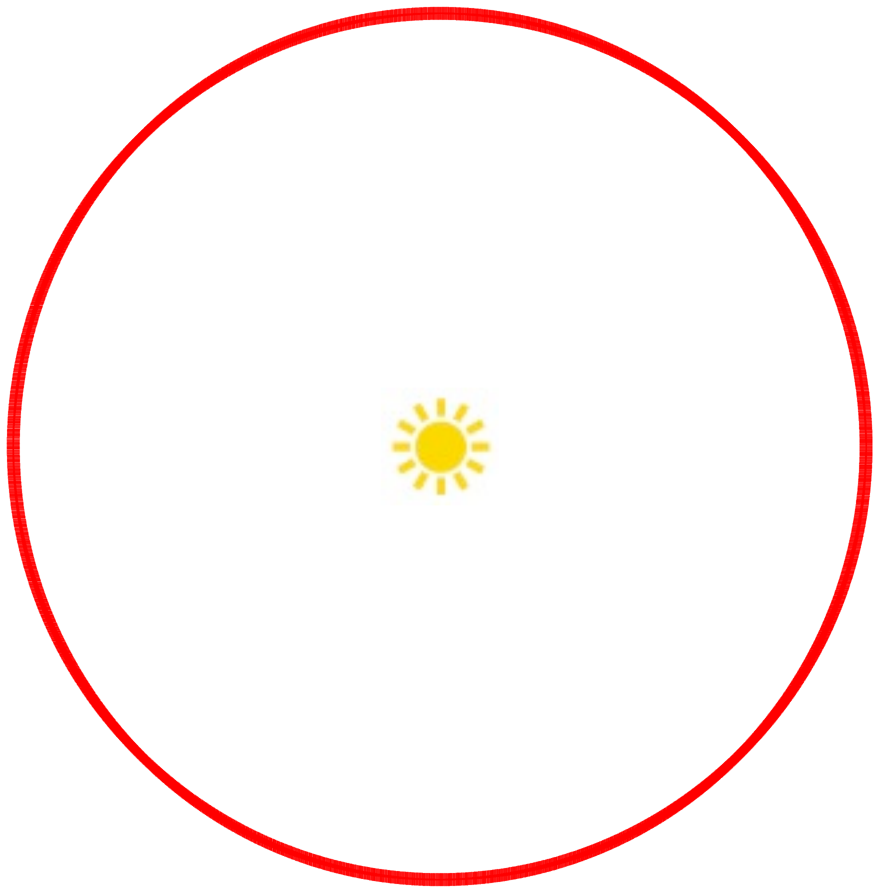
Collision detection
- Direct nearest neighbor search, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- Plane sweep algorithm, $O(N)$ or $O(N^2)$

Real-time visualization
- OpenGL

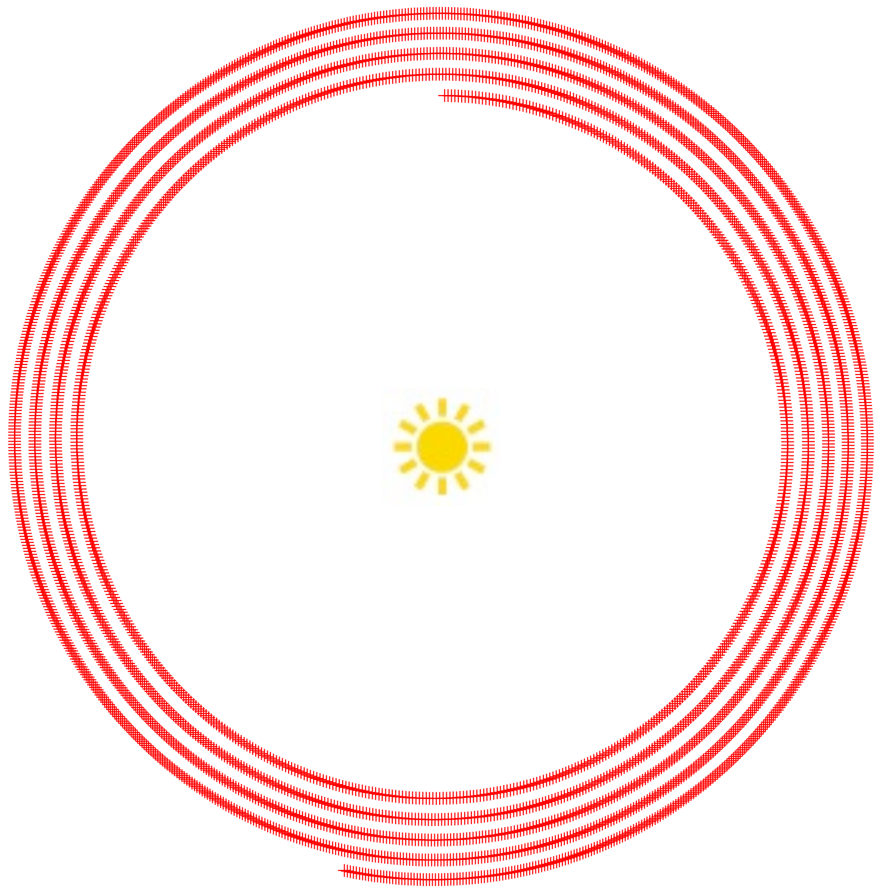
Symplectic integrators

Integrators

- REBOUND uses symplectic integrators
- Symplectic integrators mimic symmetries that are manifest in the Hamiltonian such as energy, momentum, angular momentum



Symplectic integrator



Non-symplectic integrator

Symplectic integrator: Leap-frog

$$H = \underbrace{\frac{1}{2}p^2}_{\text{Drift}} + \underbrace{\Phi(x)}_{\text{Kick}}$$

$$x \rightarrow x + v\,\frac{\Delta t}{2}$$

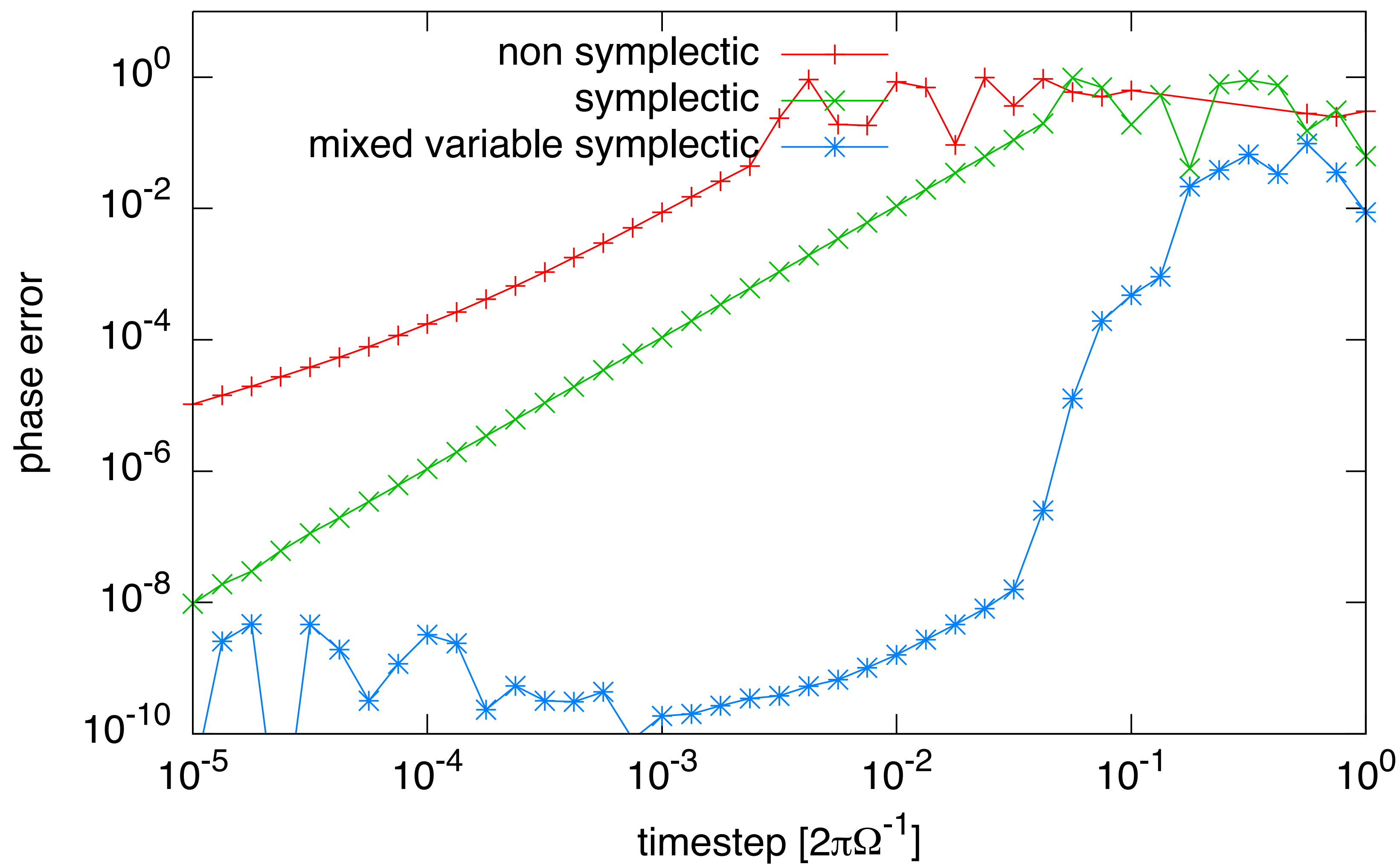
$$v \rightarrow v + \nabla\Phi\,\Delta t$$

$$x \rightarrow x + v\,\frac{\Delta t}{2}$$

Mixed variable symplectic integrator

- MVS give another huge enhancement in accuracy
- Can be used whenever motion is dominated by one process and slightly perturbed by another process

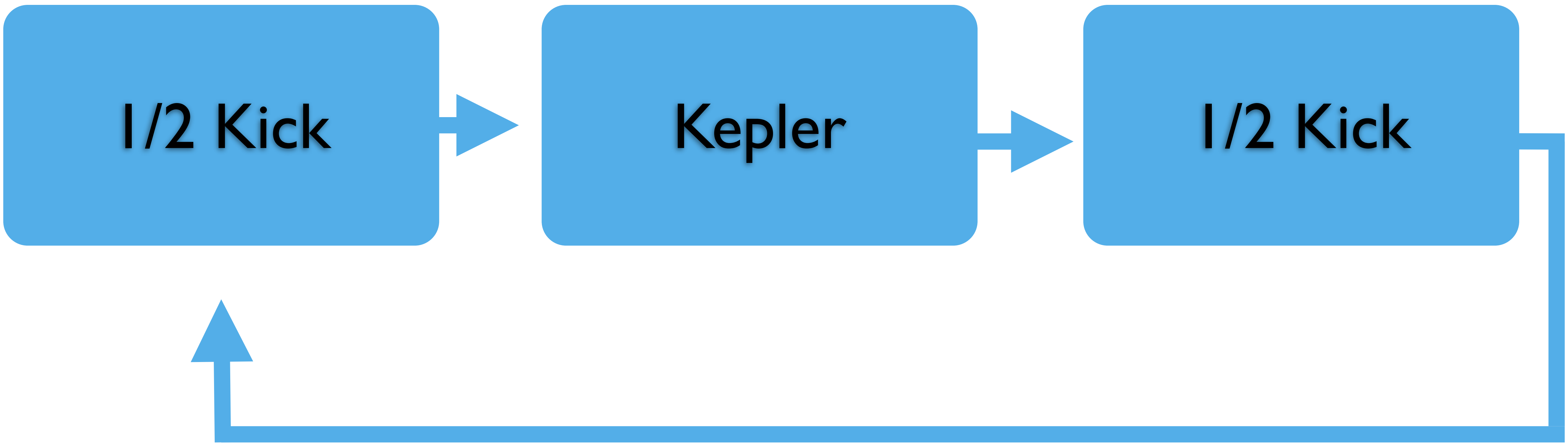
$$\mathrm{Error} = \epsilon\,(\Delta t)^{p+1}\ [H_0, H_{\mathrm{pert}}]$$



Rein & Tremaine 2011

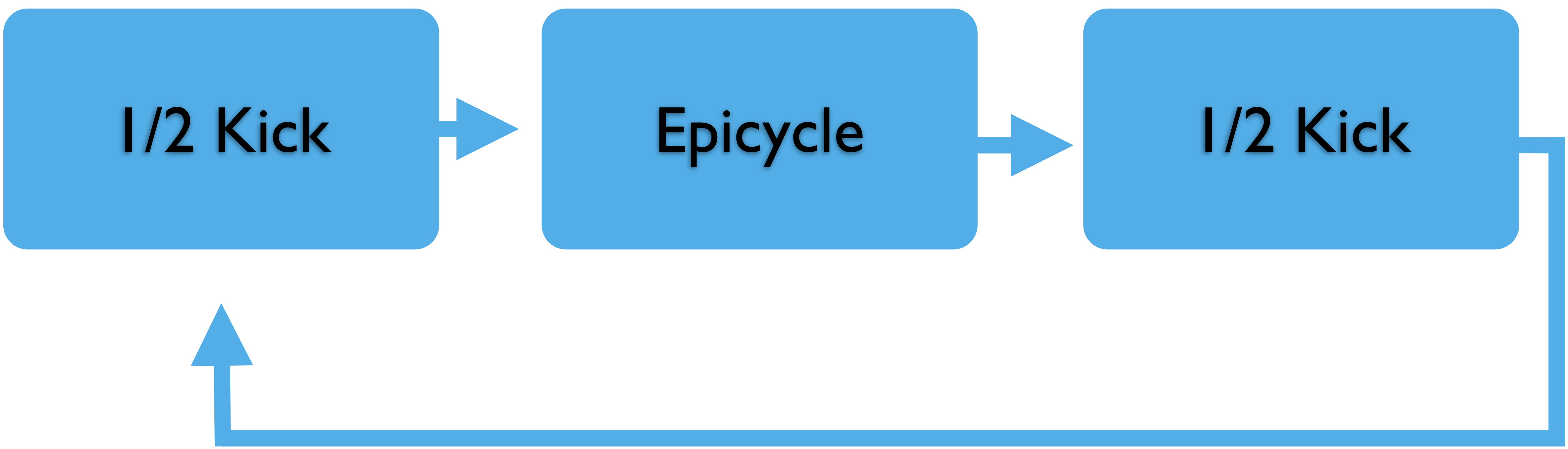
Mixed variable symplectic integrator

$$H = \underbrace{\frac{1}{2}p^2 + \Phi_{\text{Kepler}}(x)}_{\text{Kepler}} + \underbrace{\Phi_{\text{Other}}(x)}_{\text{Kick}}$$



Symplectic Epicycle Integrator

$$H = \underbrace{\frac{1}{2}p^2 + \Omega(p \times r)e_z + \frac{1}{2}\Omega^2\left[r^2 - 3(r \cdot e_x)^2\right]}_{\text{Epicycle}} + \underbrace{\Phi(r)}_{\text{Kick}}$$

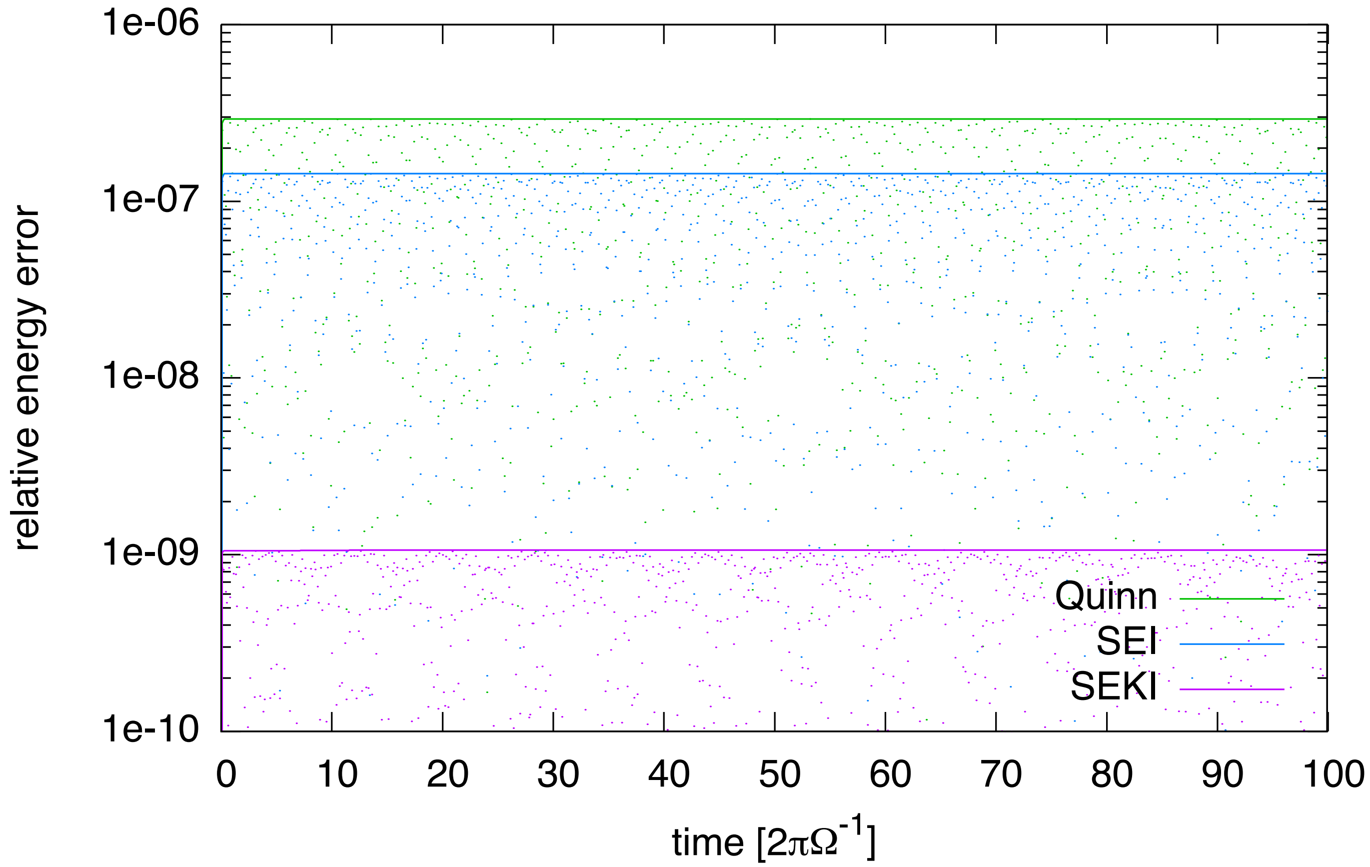


Symplectic Epicycle Integrator: Rotation

- Solving for the orbital motion involves a rotation.
- Formally $\det(D) = 1$, but due to floating point precision $\det(D) \sim 1$ only.
- Trick: Use three shear operators instead of one rotation.

$$\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\tan\frac{1}{2}\phi & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \sin\phi \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\tan\frac{1}{2}\phi & 1 \end{pmatrix}$$

- $\det(D) = 1$ exactly for each shear operator, even in floating point precision.
- No long term trend linear trend anymore!



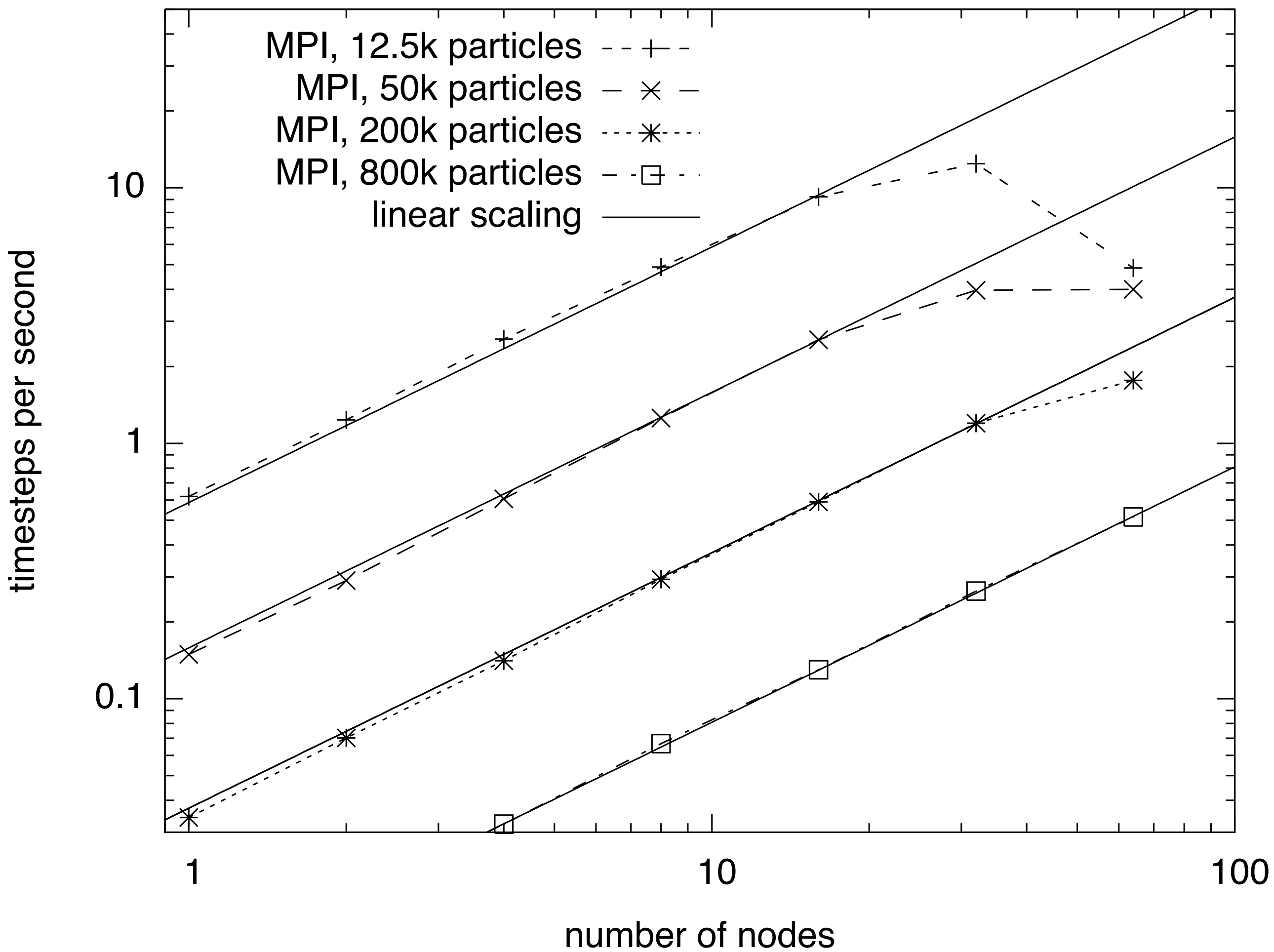
Take home message I

Symplectic integrators are awesome.

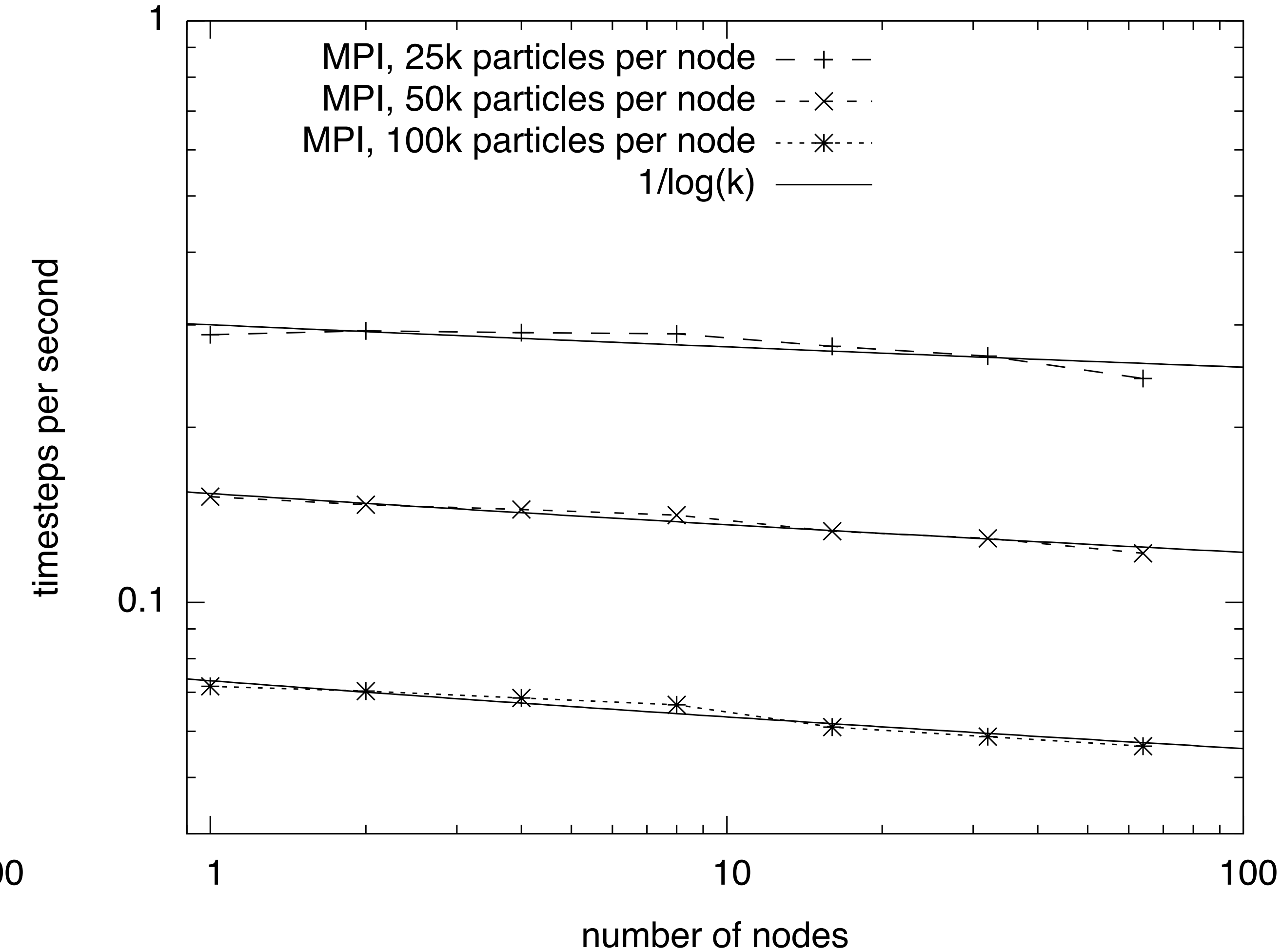
REBOUND Demo

REBOUND scalings using a tree

strong



weak

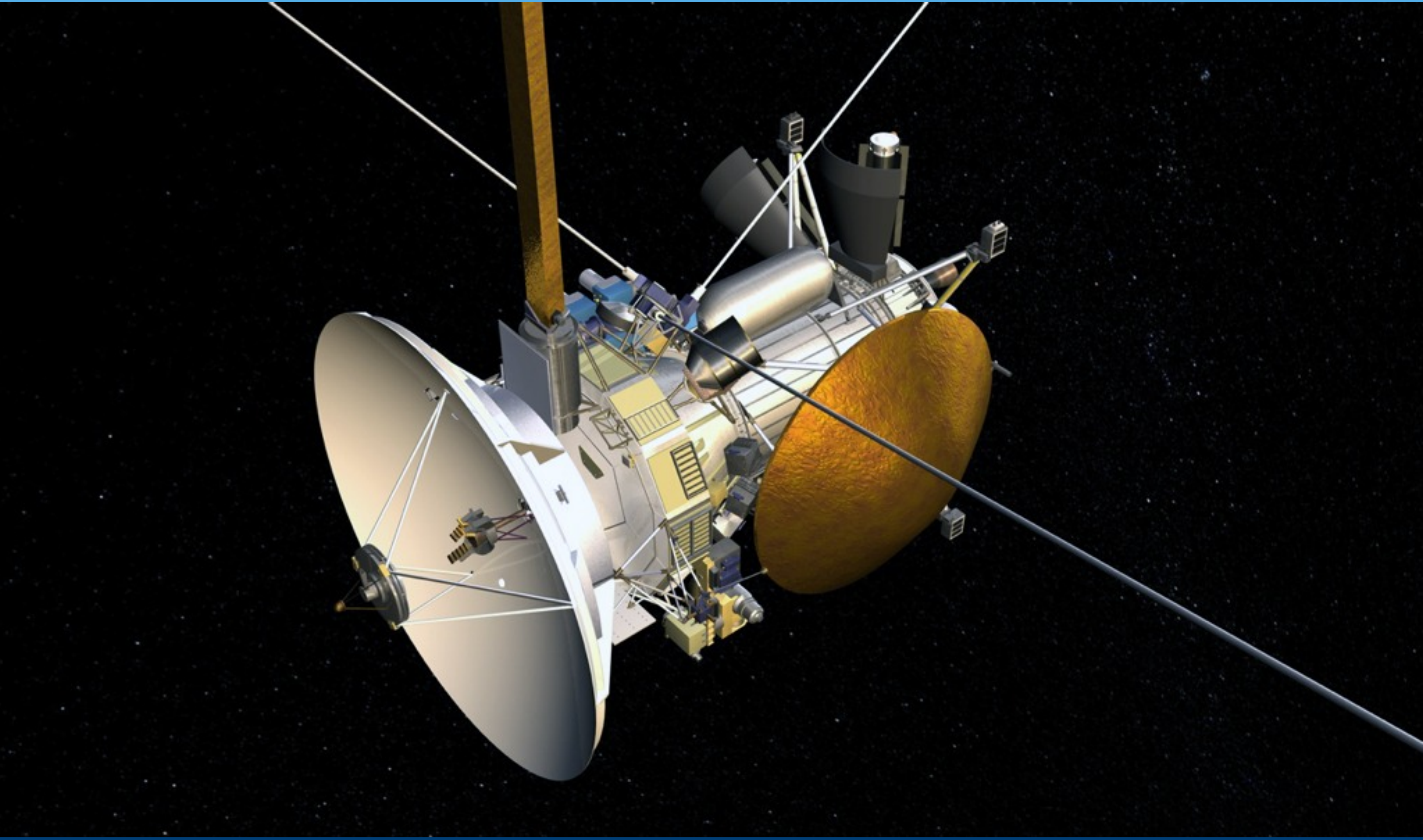


Take home message II

Download and play with REBOUND.

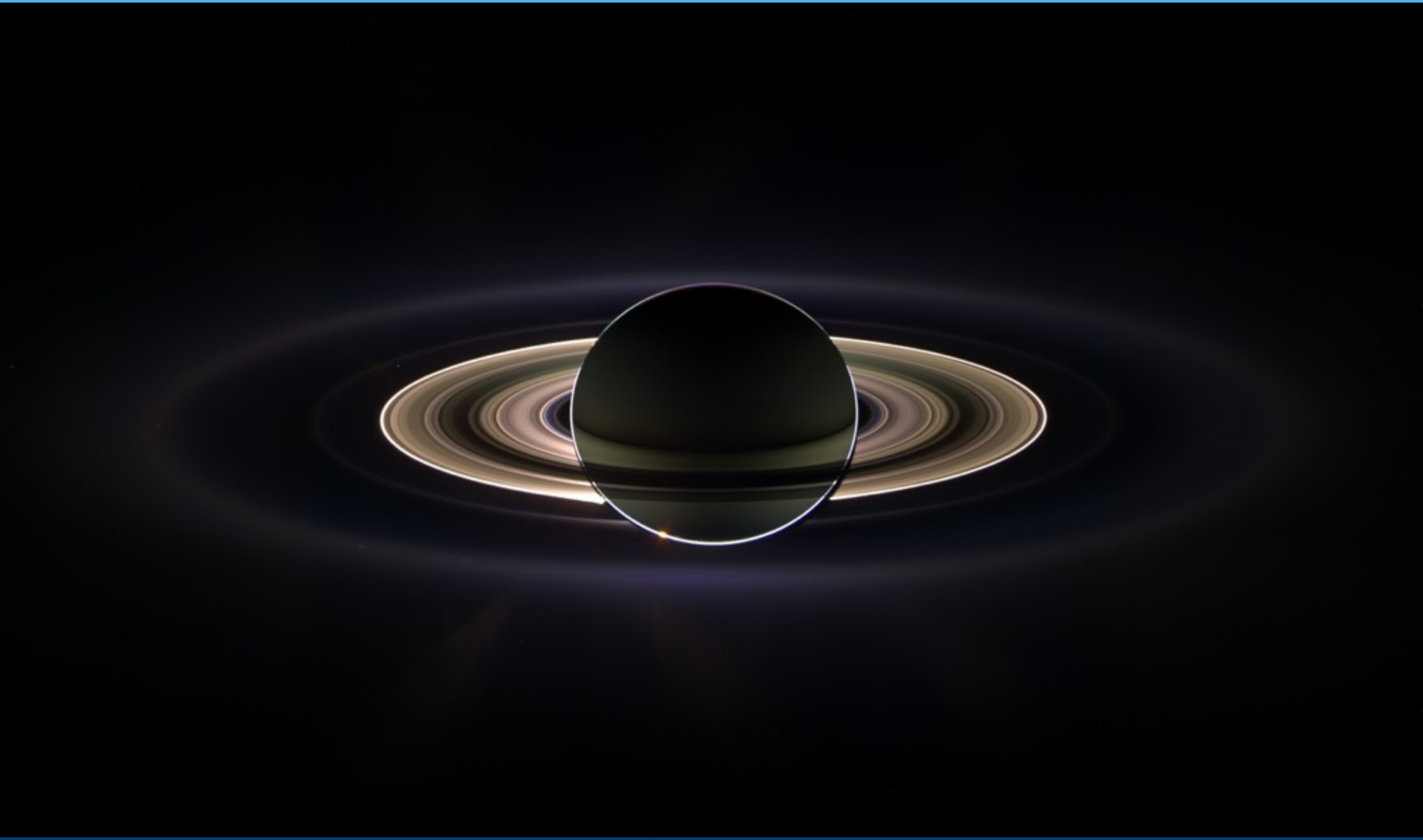
Saturn's Rings

Cassini spacecraft



Credit: JPL/Gordon Morrison

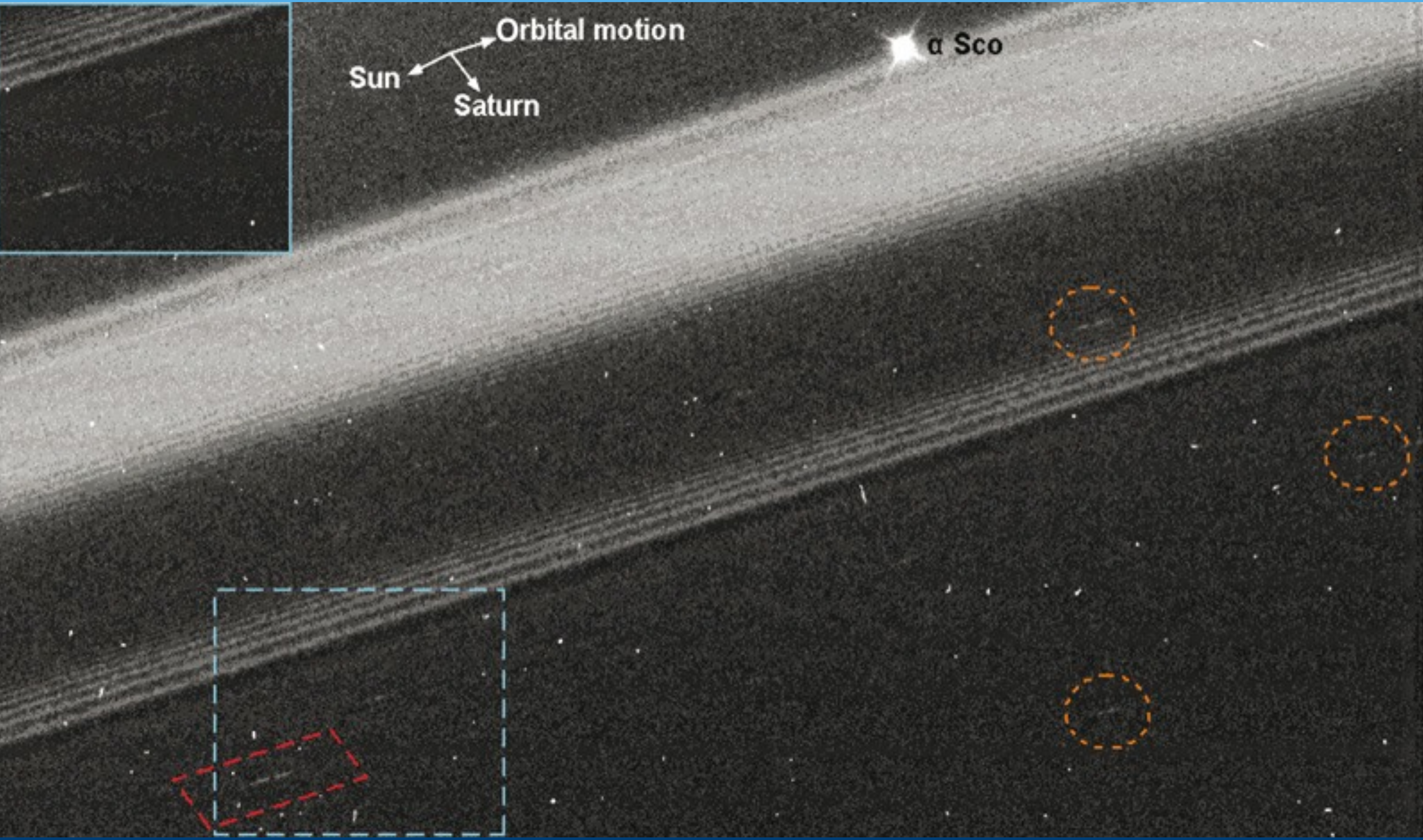
Cassini spacecraft



NASA/JPL/Space Science Institute

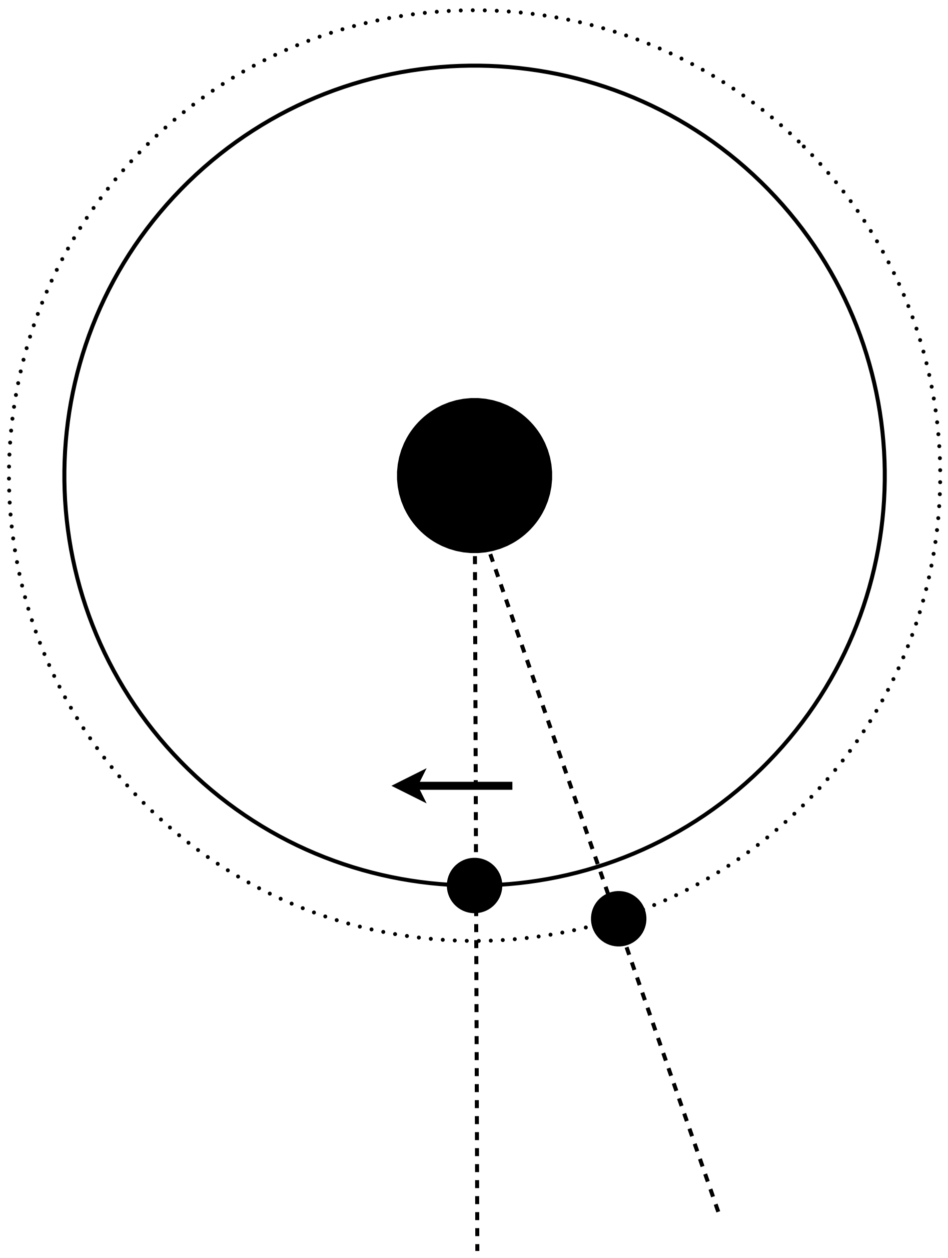
Moonlets in Saturn's Rings

Propeller structures in A-ring



Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006

Longitude residual



Mean motion [rad/s]

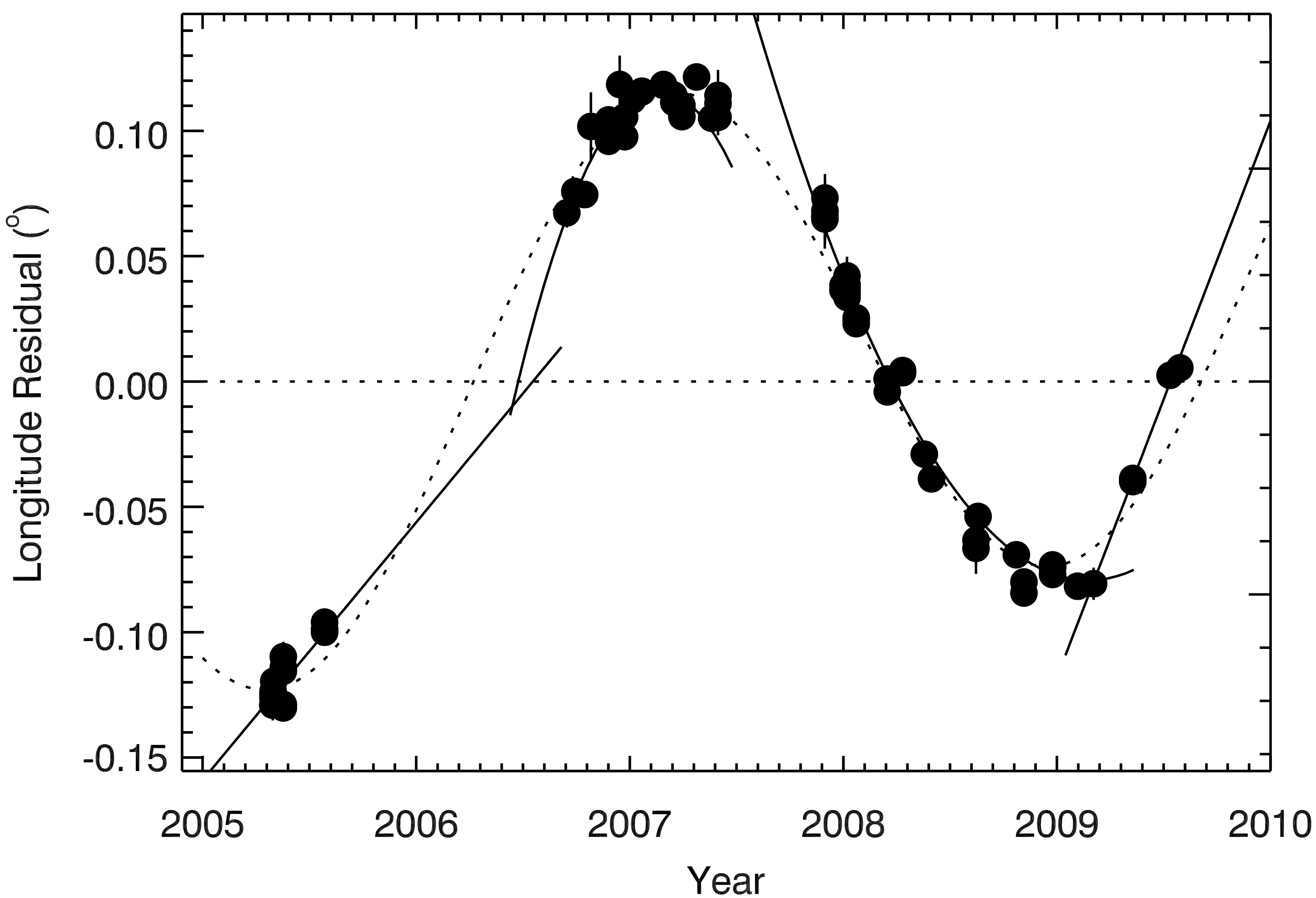
$$n = \sqrt{\frac{GM}{a^3}}$$

Mean longitude [rad]

$$\lambda = n\,t$$

$$\lambda(t) - \lambda_0(t) = \int_0^t (n_0 + n'(t'))\,dt' - \underbrace{\int_0^t n_0\,dt'}_{n_0 t}$$

Observational evidence of non-Keplerian motion



Random walk

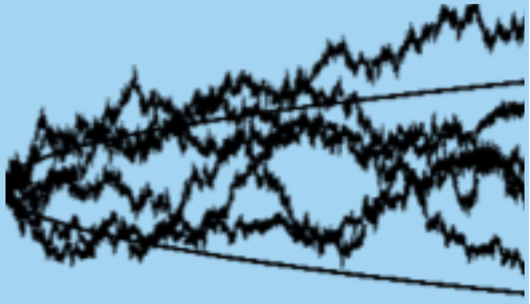
Analytic model

Describing evolution in a statistical manner
Partly based on Rein & Papaloizou 2009

$$\Delta a = \sqrt{4\frac{Dt}{n^2}}$$
$$\Delta e = \sqrt{2.5\frac{\gamma Dt}{n^2 a^2}}$$

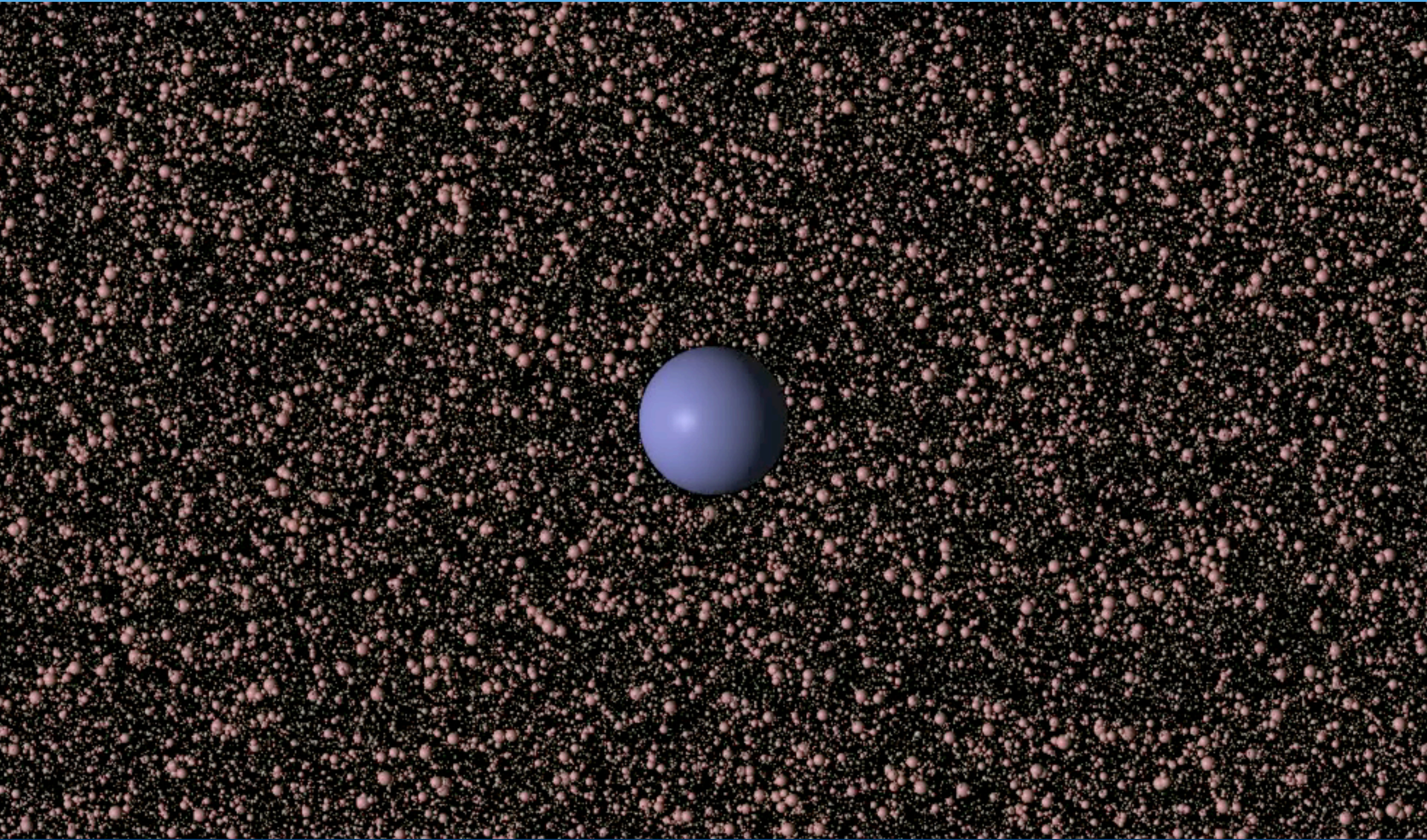
N-body simulations

Measuring random forces or integrating moonlet directly
Crida et al 2010, Rein & Papaloizou 2010



Rein & Papaloizou 2010, Crida et al 2010

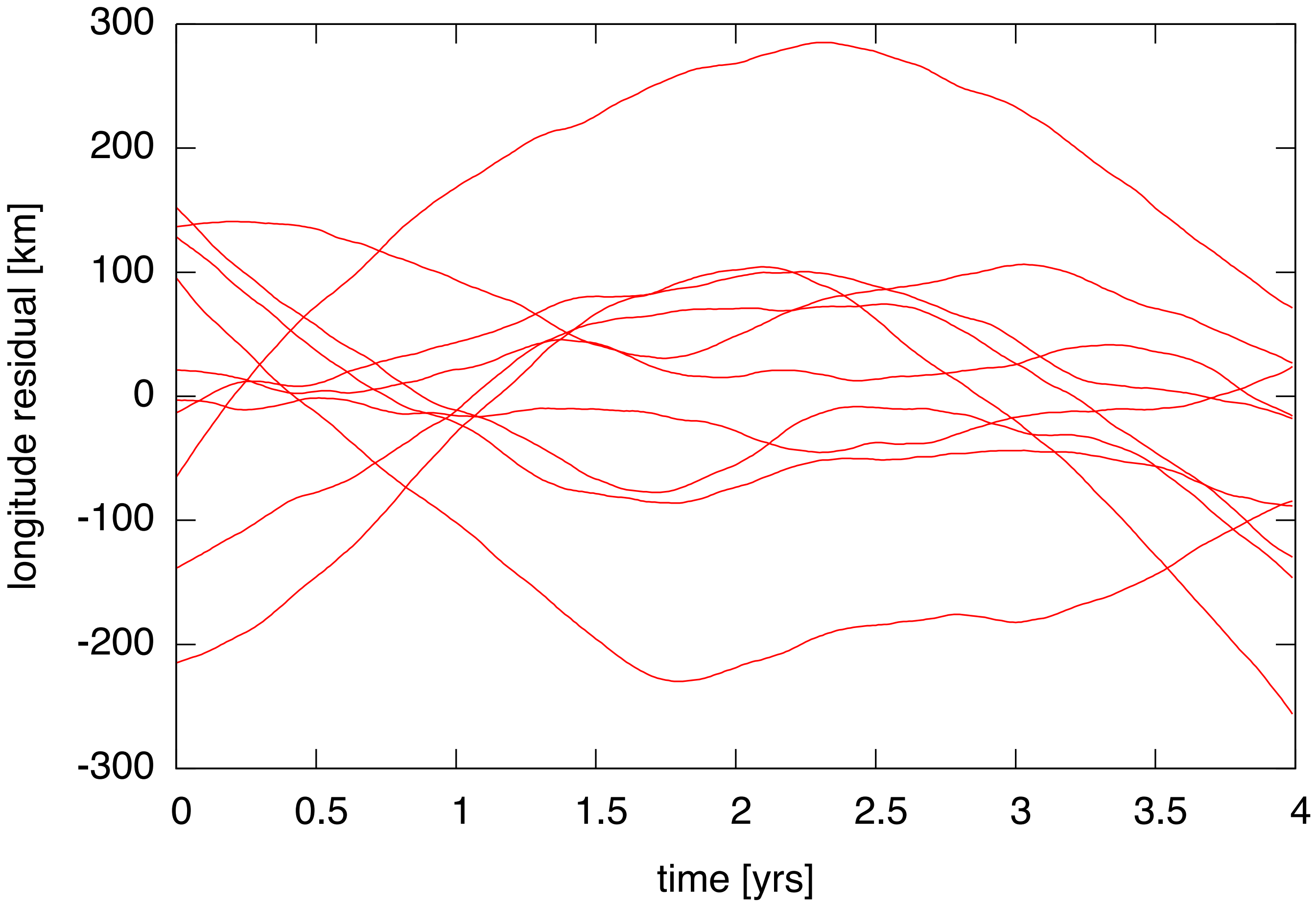
Random walk

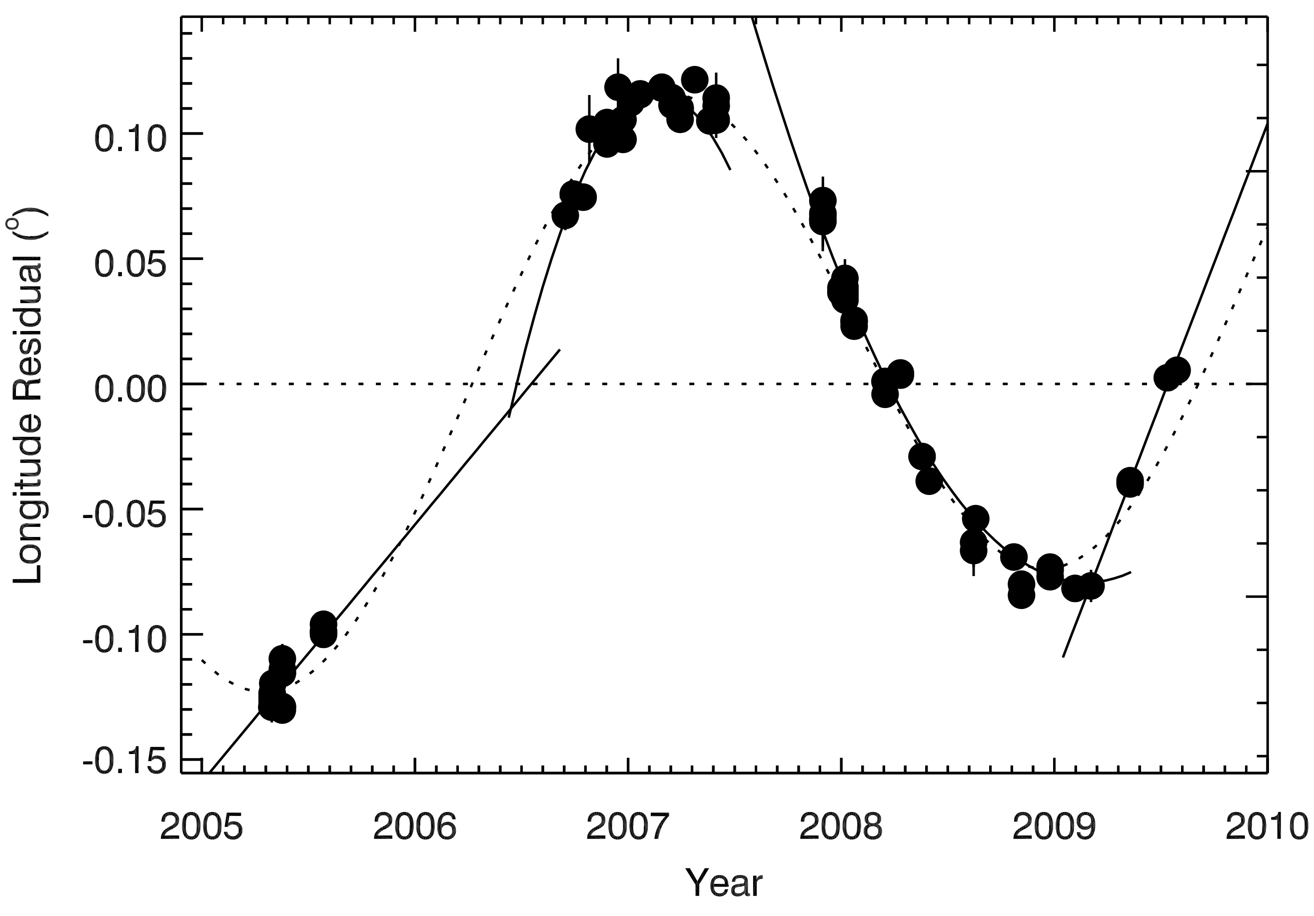


REBOUND code, Rein & Papaloizou 2010, Crida et al 2010

Results from simulations and observations

- Moonlet motion is undergoing a Levy flight.
- Over long time-scales this is just a random walk.
- But what we see within 5 years is a moonlet being kicked once or twice by other large bodies in the ring.
- Leads to constraints about size distribution.





Rein, Chiang & Pan (in prep)

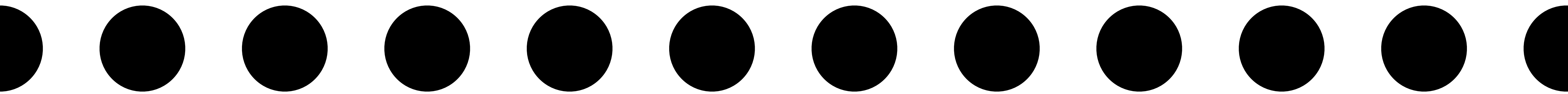
Take home message III

Moonlets in Saturn's Rings show
direct evidence of
disk satellite interaction.

Gravitational instability in a narrow ring

Gravitational instability in a narrow ring

- First studied by Maxwell 1859
- Idealized setup
- Equal mass, equally spaced particles
- Initially on circular orbits around central object



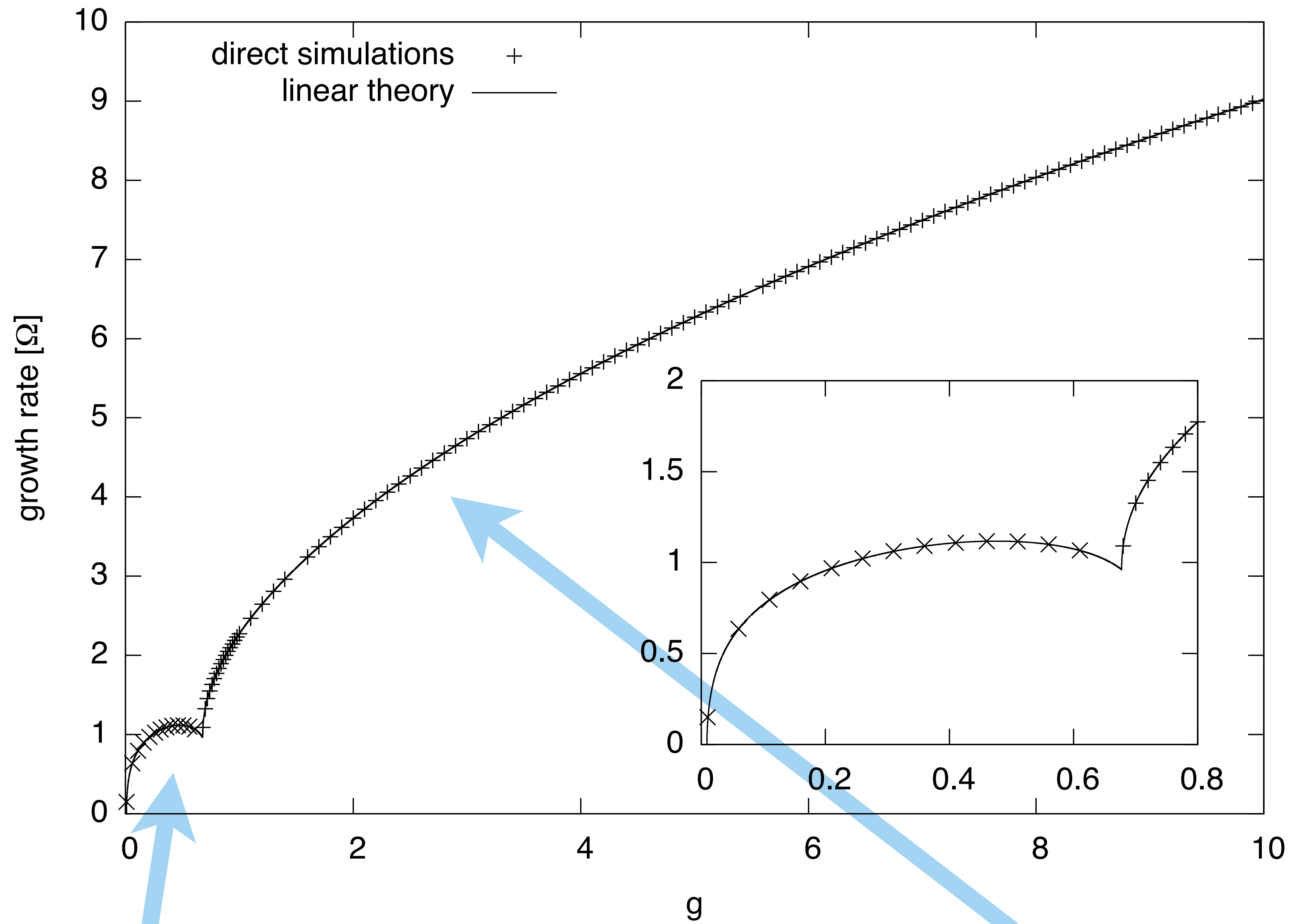
- Seed perturbations grow if the mass is above a critical value
- Two different modes, depending on particle mass and spacing

Growing epicycles	Longitudinal clumping

Latter, Rein & Ogilvie (submitted)

Latter, Rein & Ogilvie (submitted)

Analytic and numerical growth rates of the GI

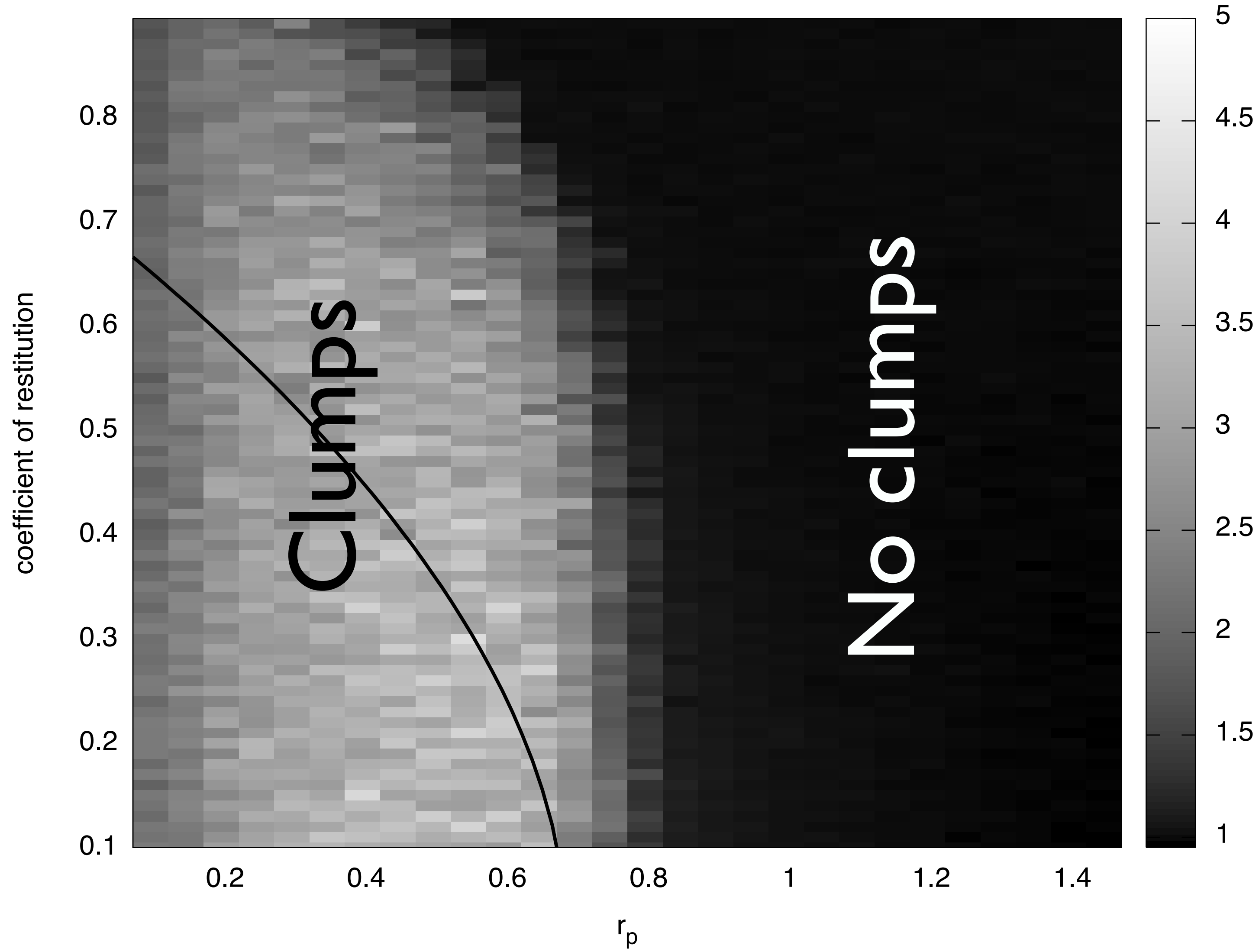


Growing epicycles

Longitudinal clumping

Long term evolution

- Hot ring or clumps
- Independent of initial mode of the instability
- Determined by coefficient of restitution and particle density

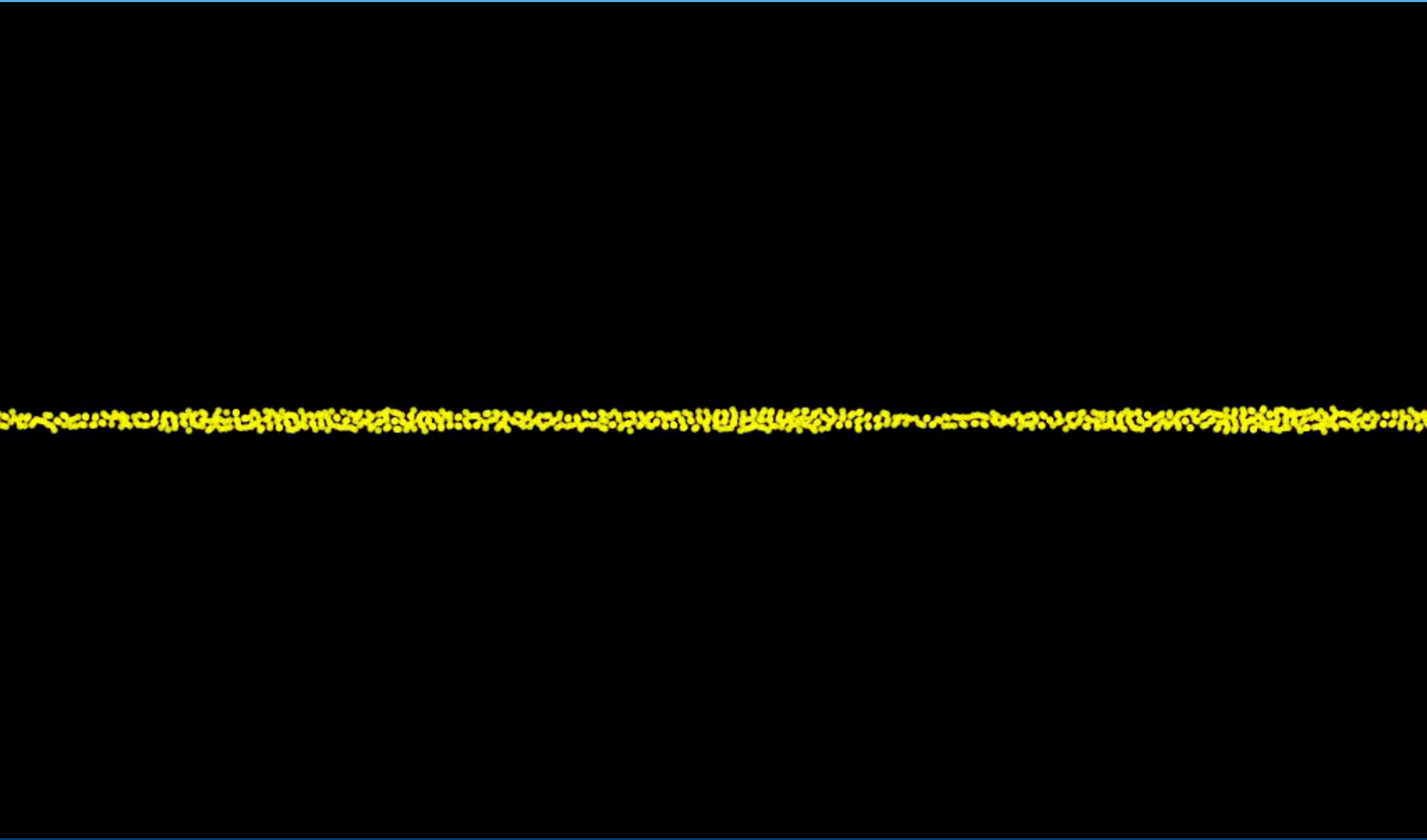


Take home message IV

Latter, Rein & Ogilvie 2012 is easier to read than Maxwell 1859.

Viscous over-stability in Saturn's rings

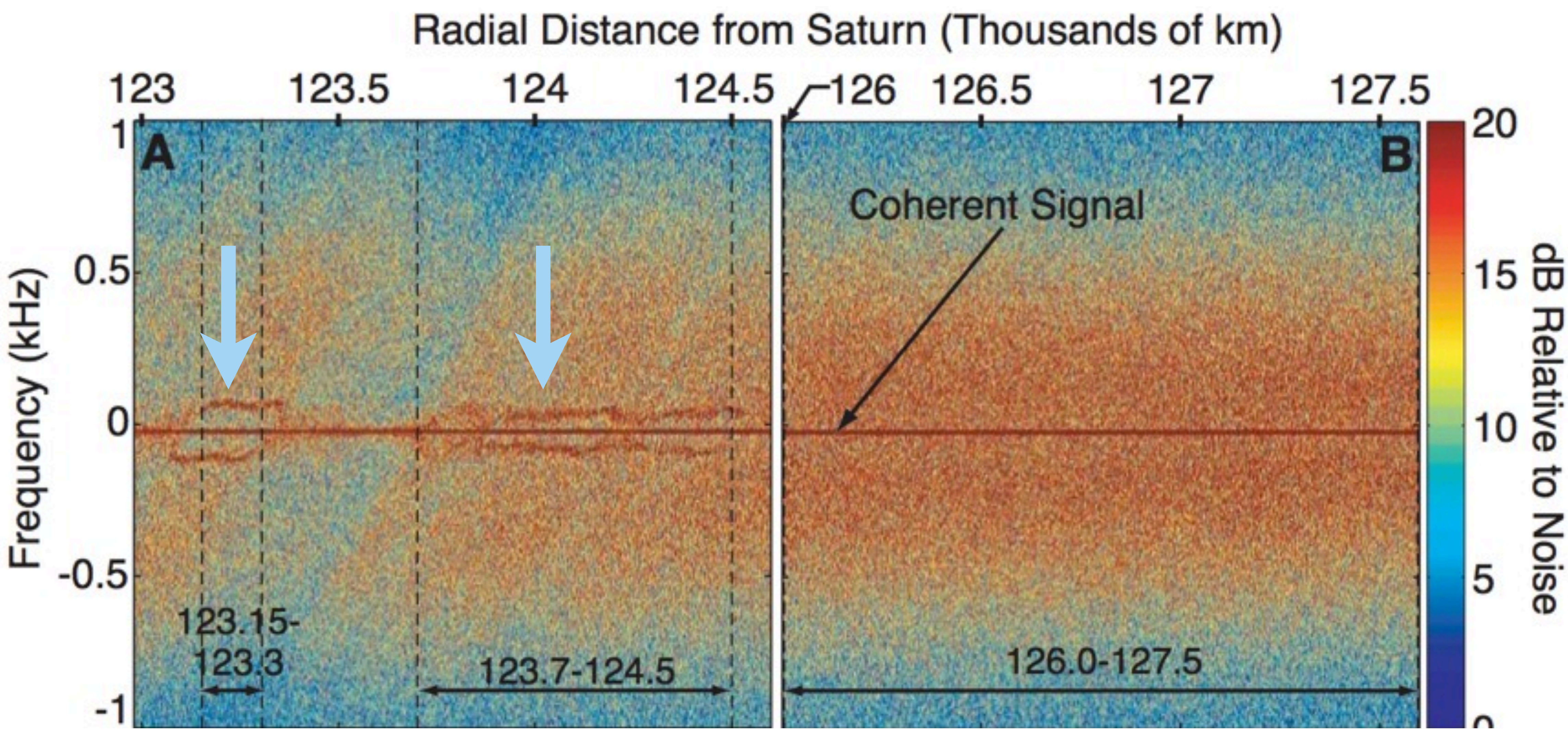
Close-up view of the viscous over-stability



Rein & Latter (in prep)

Observations

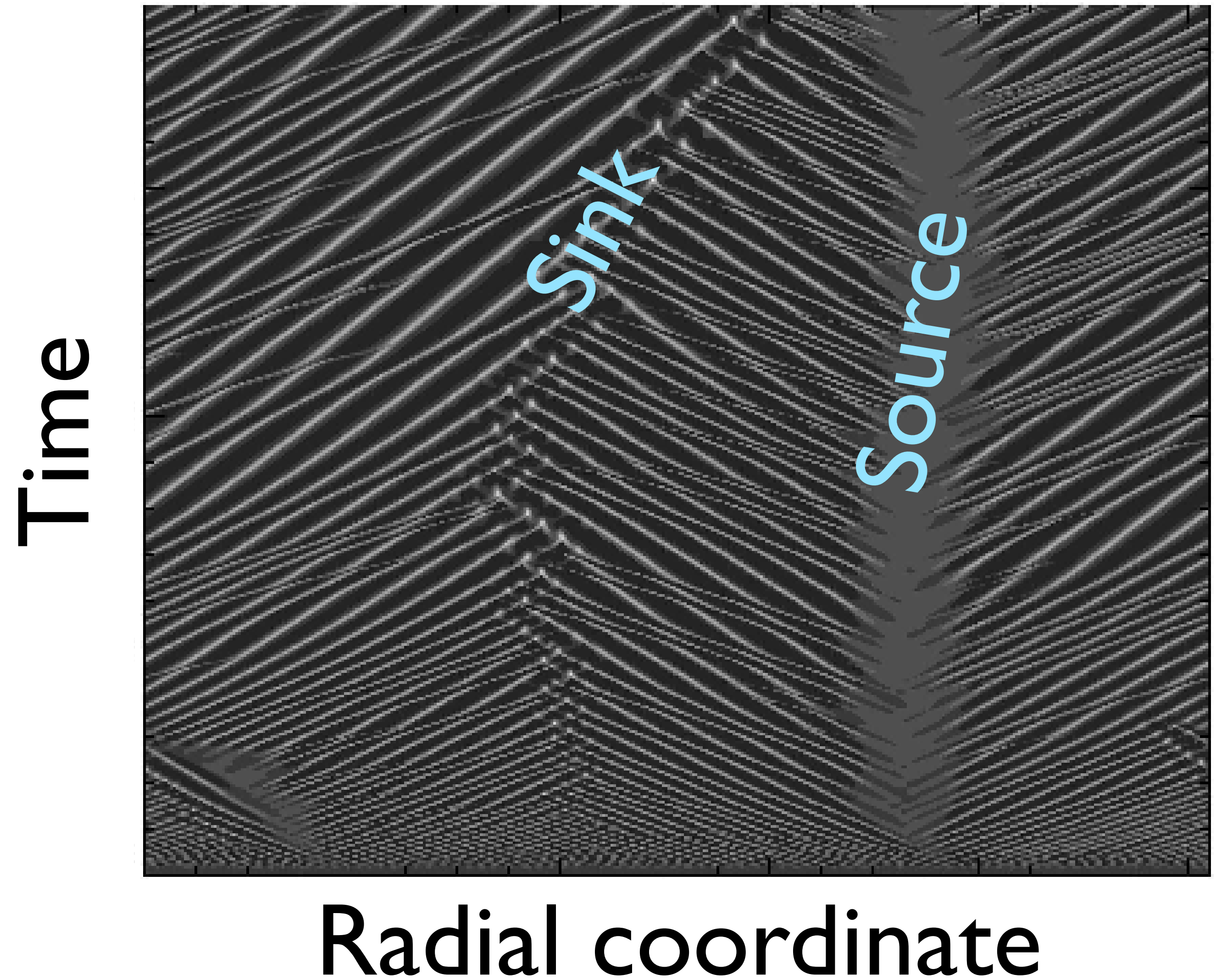
- Observational evidence for small scale structures
- Typical size ~100m



Rein & Latter (in prep), Thomson 2010

Previous work

- Both analytic calculations and hydrodynamic simulations show non-linear wave-train solutions.
- Rich dynamics with sources and sinks of wave-trains.



Latter & Ogilvie 2009, Salo (in press), etc

Numerical simulations with REBOUND

Symplectic Epicycle Integrator

- Fast
- High accuracy
- No long term drifts (important)

Plane-sweep algorithm

- Fast
- O(N) for elongated boxes

Direct particle simulations of Saturn's Rings

- Longest integration time ever done*
- Widest boxes ever done*

* to my knowledge, Rein & Latter (in prep)

Long term, wide box simulations

Work in progress...

Take home message V

Our simulations are big enough to directly study the non-linear evolution of the viscous over-stability.

Conclusions

Conclusions / Take home messages

I. Symplectic integrators are awesome.

II. Download and play with REBOUND.

III. Moonlets in Saturn's Rings show direct evidence of disk satellite interaction.

IV. Latter, Rein & Ogilvie 2012 is easier to read than Maxwell 1859.

V. Our simulations are big enough to directly study the non-linear evolution of the viscous over-stability.