第28回 Grain Formation Workshop
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宇宙空間における過冷却融液凝固過程：
数値計算によるアプローチ

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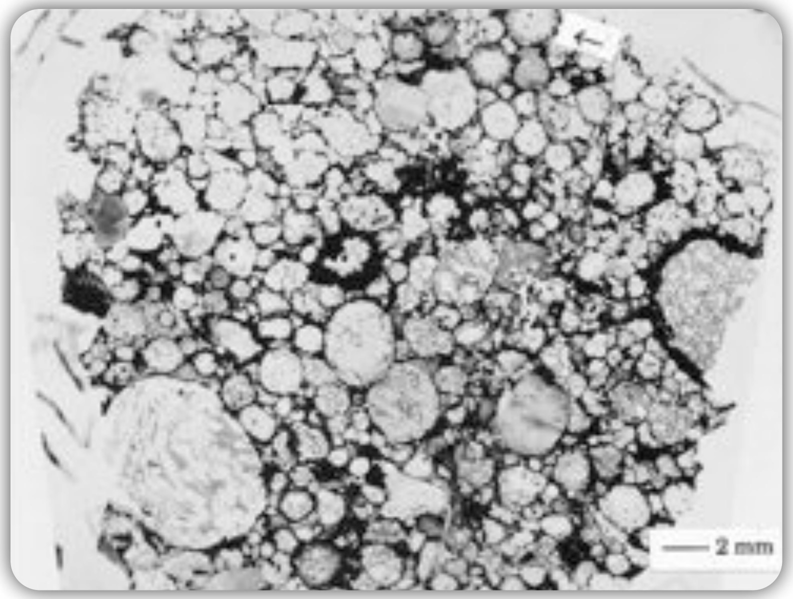
Planetary Dynamics

Introduction:

Chondrules

“olivine”
$(Mg_xFe_{1-x})_2SiO_4$

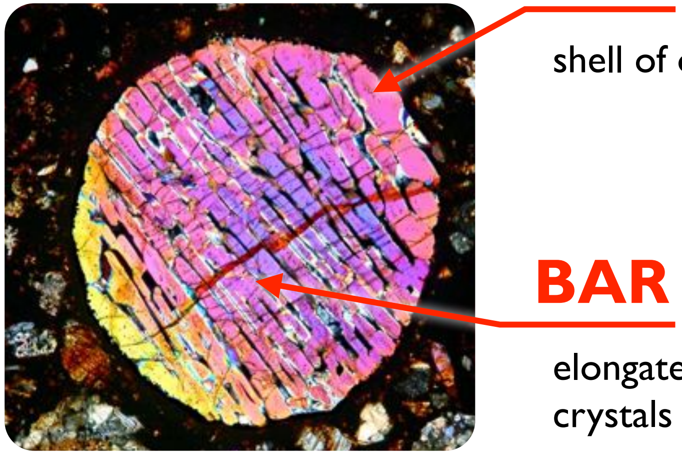


transmitted light image of thin section of Semarkona, LL3.0
(Connolly & Love 1998, Science 280, 62)

key stone of early solar nebula

unique solidification texture:

barred-olivine



shell of olivine crystal

elongated parallel crystals of olivine

(images from “http://jm-derochette.be/”)

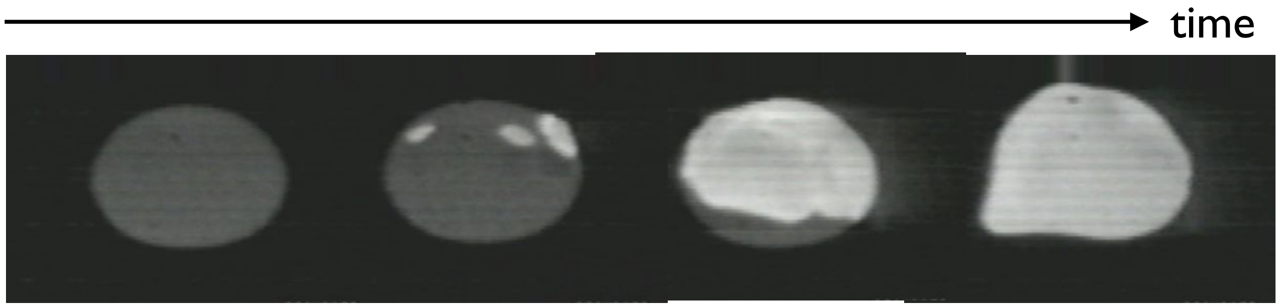
rim & bars

the same crystallographic orientation!

A mm-sized “magma” droplet cools to solidify in a short period of time. The solidification texture reflects the crystal growth process.

Introduction:

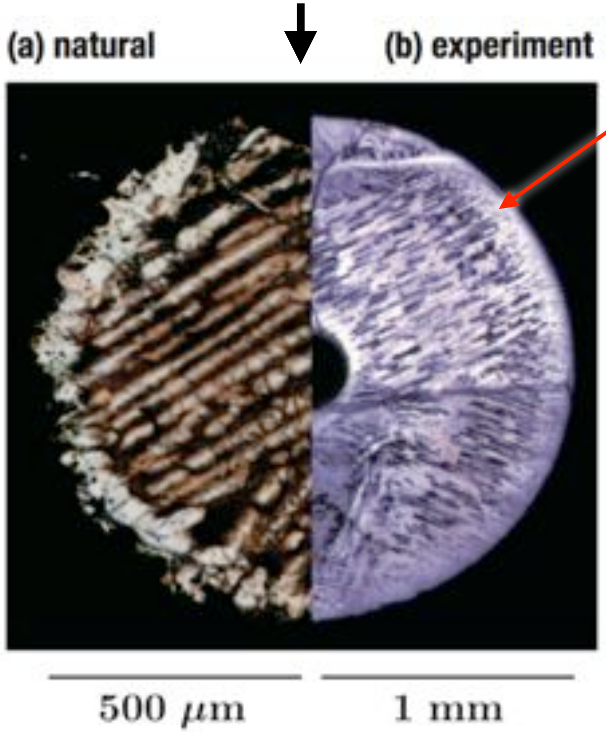
Condition for rim formation?



Ultra-high speed TV images of a rotating crystallizing forsterite (Mg_2SiO_4) melt. This crystallization process is completed within 0.1 s (Tsukamoto+1999, Antarct. Meteorites 24, 179).

- *levitation*
- *rapid cooling:*

  $R_{\mathrm{cool}} \sim 1000$ K/s
- *supercooling ~ 1000 K*



First reproduction of **RIM!**

Only bars (dendrites) inside chondrule were reproduced in 1990. However, it had no rim (Lofgren & Lanier 1990, Geochim. Cosmochim. Acta 54, 3537).

Question:

How large R_{cool} is required?

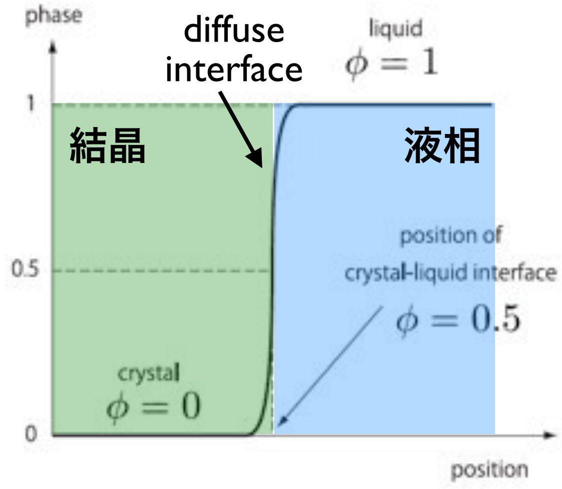
Strategy:

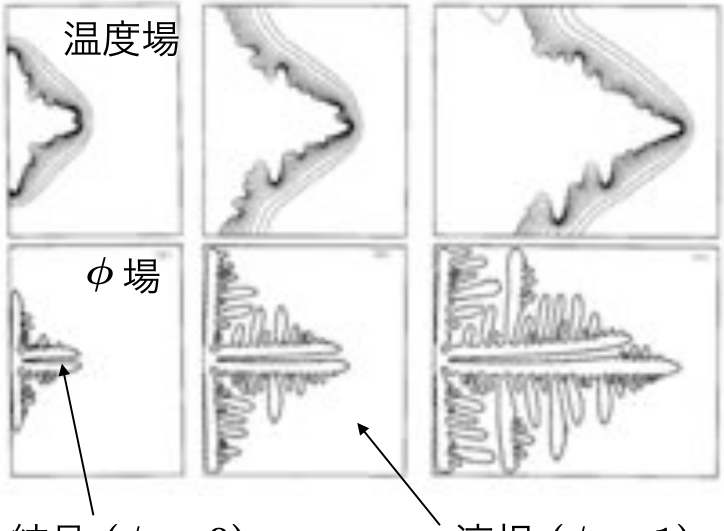
Phase-field simulations of crystallization of **“rapidly-cooling”** melt droplet

Method:

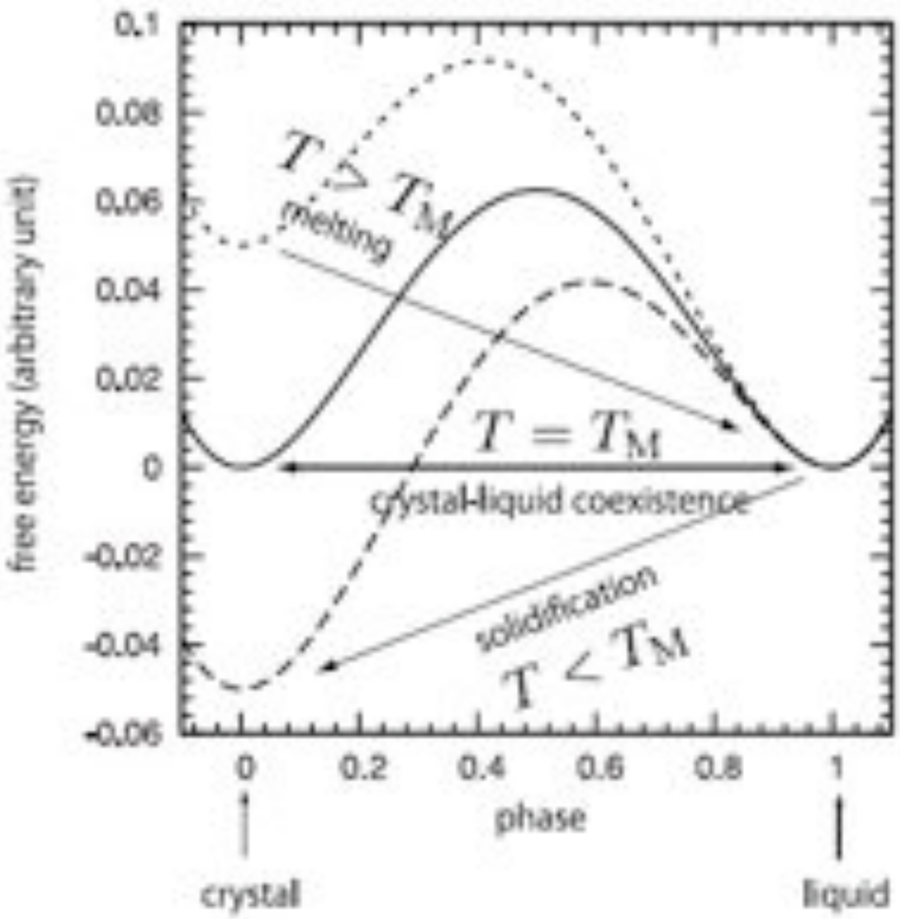
Phase-field method

フェーズ Φ による相の区別





double-well potential



複雑な界面形状の時間発展を
比較的容易に取り扱える (Sekerka 2007)

Method:

Phase-field equations

“release of latent heat”

“surface cooling”

- ***temperature, T:***

$$\int_{\mathrm{V}} \left[c_0 \frac{\partial T}{\partial t} + \boxed{L_0 p'(\phi) \frac{\partial \phi}{\partial t}} \right] dv = \oint_{\mathrm{S}} \kappa \boldsymbol{\nabla} T \cdot \boldsymbol{n}\, ds - \boxed{\oint_{\mathrm{sur}} \boldsymbol{q}_{\mathrm{s}} \cdot \boldsymbol{n}\, ds}$$

volumetric
heat capacity

latent heat of
crystallization
per unit volume

thermal
diffusivity

thickness of
interface

melting point

capillary length
(~ nm)

- ***phase, Φ:***

$$\frac{\delta}{\mu} \frac{\partial \phi}{\partial t} = \frac{\sqrt{2}}{12} \frac{T_{\mathrm{M}}}{T} p'(\phi)(T - T_{\mathrm{M}}) - \frac{T_{\mathrm{M}} \Gamma}{4\delta} g'(\phi) + T_{\mathrm{M}} \Gamma \delta \triangle \phi$$

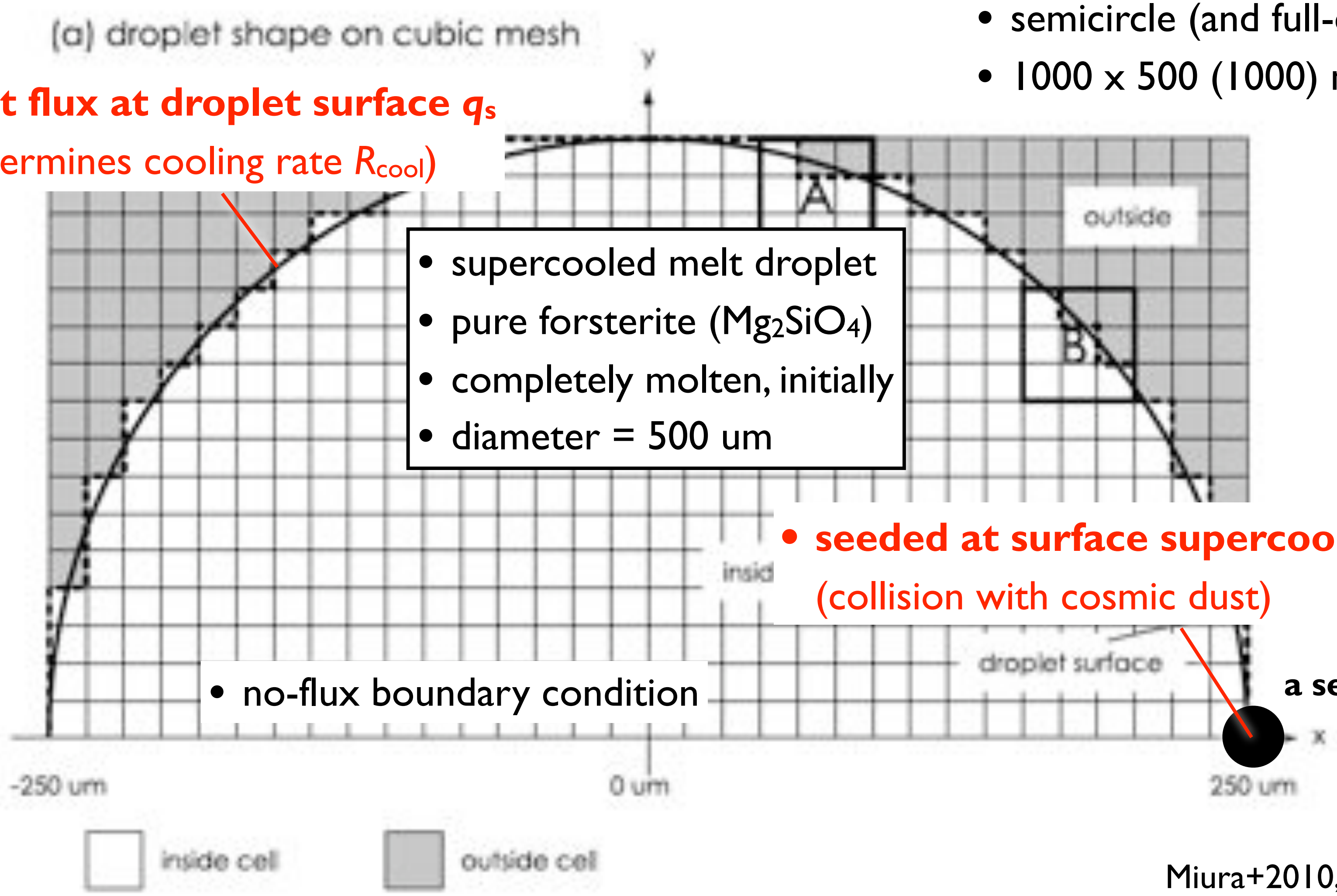
(model I of Wang+1993, Physica D 69, 189, modified)

***“simulating dynamics of crystal growth in a pure material
by solving two differential equations simultaneously”***

Method:

Computational settings

- 2D cubic cell
- semicircle (and full-circle)
- 1000 x 500 (1000) mesh



- **heat flux at droplet surface q_s**
  (determines cooling rate R_{cool})

- supercooled melt droplet
- pure forsterite (Mg_2SiO_4)
- completely molten, initially
- diameter = 500 um

- **seeded at surface supercooling ΔT_s**
  (collision with cosmic dust)

- no-flux boundary condition

a seed crystal

Miura+2010, in prep.

Result:

Cooling rate R_{cool} required rim formation

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temperature profile of droplet:

volumetric heat capacity

$$T(r) = T(0) - \frac{c_0 R_{\rm cool}}{4\kappa} r^2$$

temp. at radius *r*

temp. at center

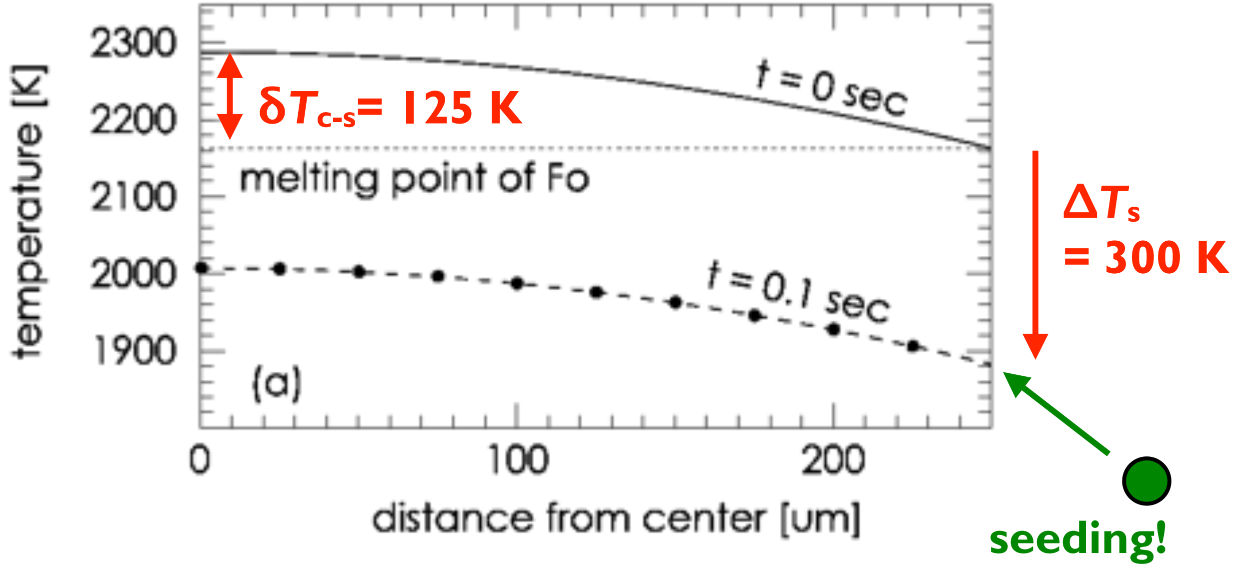
thermal conductivity

We assume that q_s (R_{cool}) is constant.

Temperature at the droplet surface is lower than that at the center (temperature difference $\delta T_{c\text{-}s}$).

When R_{cool} = 2800 K/s,

$\delta T_{c\text{-}s}$ = 125 K.



(Miura+2010, submitted)

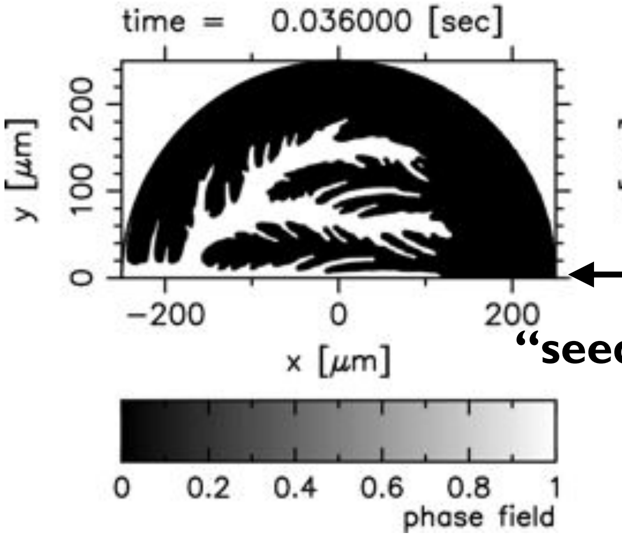
When the surface undercools by **300 K** below the melting point, we put a seed crystal at the surface to initiate crystallization.

Supercooling at the center:
= 175 K

Result:

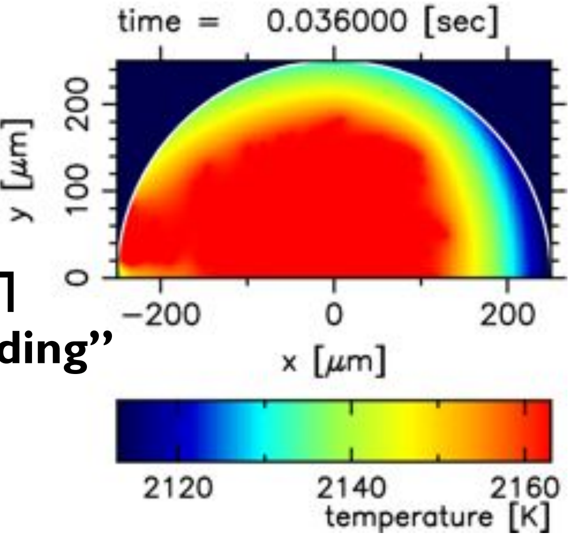
Crystallization pattern (R_{cool} = 2800 K s^{-1})

phase

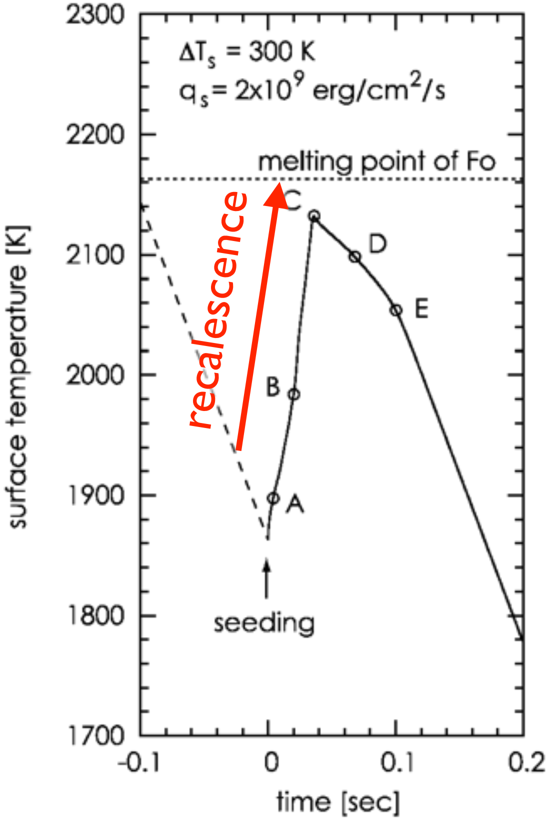


“seeding”

temperature



cooling curve



Miura+2010, submitted

- recalescence (rapid temperature increase)
- rapid crystallization (within ~ 0.1 sec)
- rapid growth along the surface ***(rim formation)***
- ***dendrite formation*** inside the droplet

double structure: rim (along the surface) + dendrite (inside the droplet)

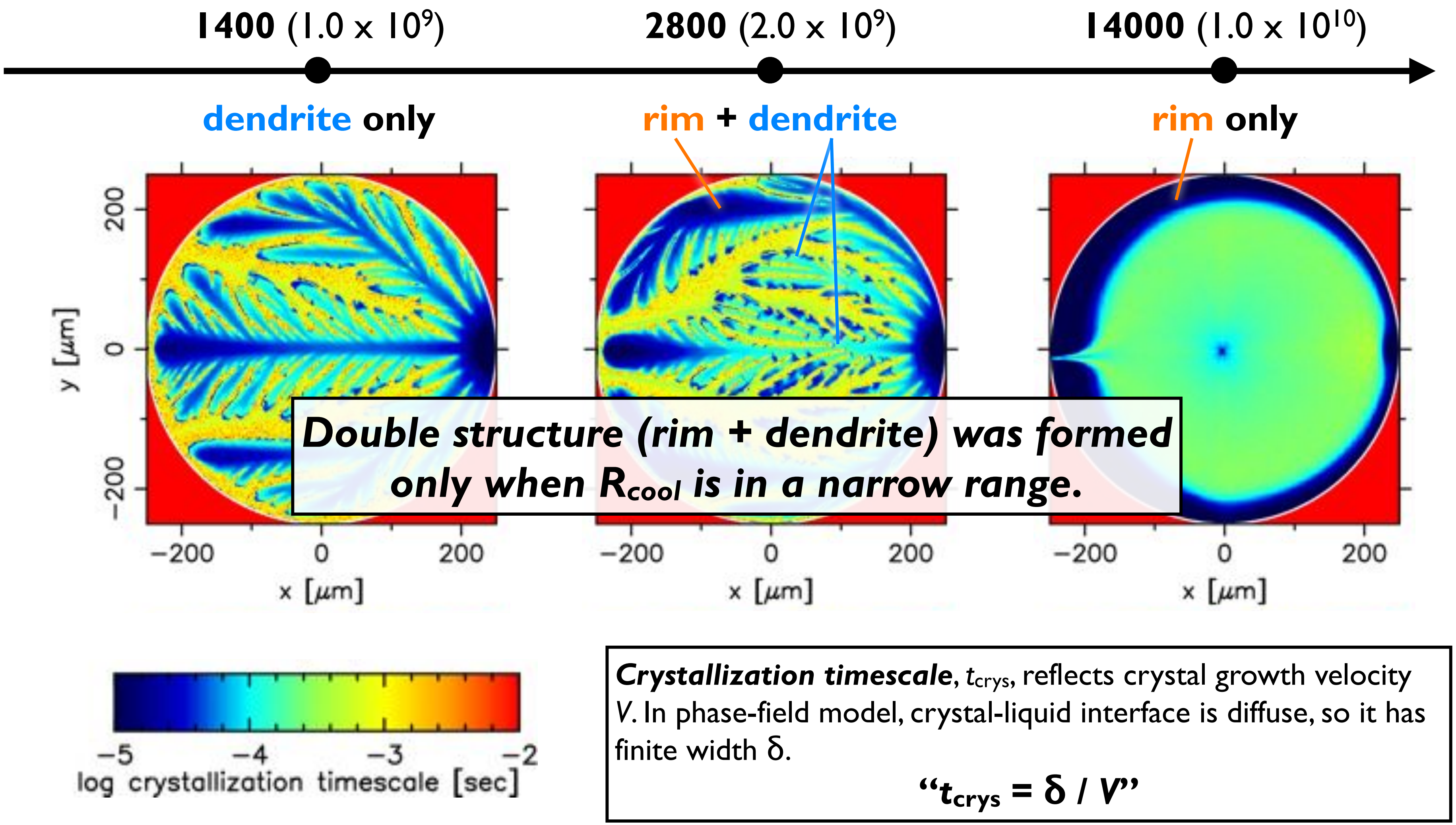
Result:

Effect of surface cooling

Cooling rate R_{cool} [K/s] (heat flux at surface q_s [erg/cm^2/s])

1400 (1.0 x 10^9) — **2800** (2.0 x 10^9) — **14000** (1.0 x 10^{10})

dendrite only — **rim + dendrite** — **rim only**



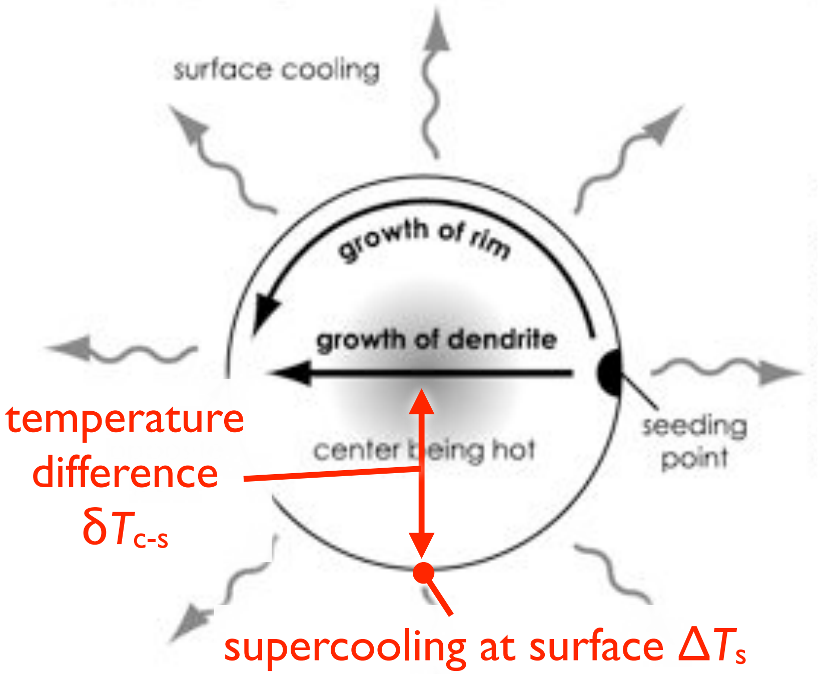
Double structure (rim + dendrite) was formed only when R_{cool} is in a narrow range.

Crystallization timescale, t_{crys}, reflects crystal growth velocity V. In phase-field model, crystal-liquid interface is diffuse, so it has finite width δ.

“$t_{crys} = \delta / V$”

Discussion:

Growth time, rim/dendrite



temperature difference $\delta T_{\text{c-s}}$

supercooling at surface ΔT_{s}

normalized temp. diff. $\alpha = \delta T_{\text{c-s}} / \Delta T_{\text{s}}$

"dependence of growth velocity on undercooling"

$$V(\Delta T) \propto \Delta T^n$$

"temperature profile"

$$\Delta T(r) = \Delta T_{\rm s} - \delta T_{\rm c-s} + \delta T_{\rm c-s}(r/r_{\rm d})^2$$

Growth time t_{growth}:

- **rim** grows fast, but goes the long way

$$t_{\rm rim} = \frac{\pi r_{\rm d}}{V(\Delta T_{\rm s})},$$

droplet radius

growth velocity

- **dendrite** grows slowly, but takes the shortest course

$$t_{\rm den} = 2\int_0^{r_{\rm d}} \frac{dr}{V(\Delta T(r))} = \frac{2r_{\rm d}}{V(\Delta T_{\rm s})} I(n,\alpha)$$

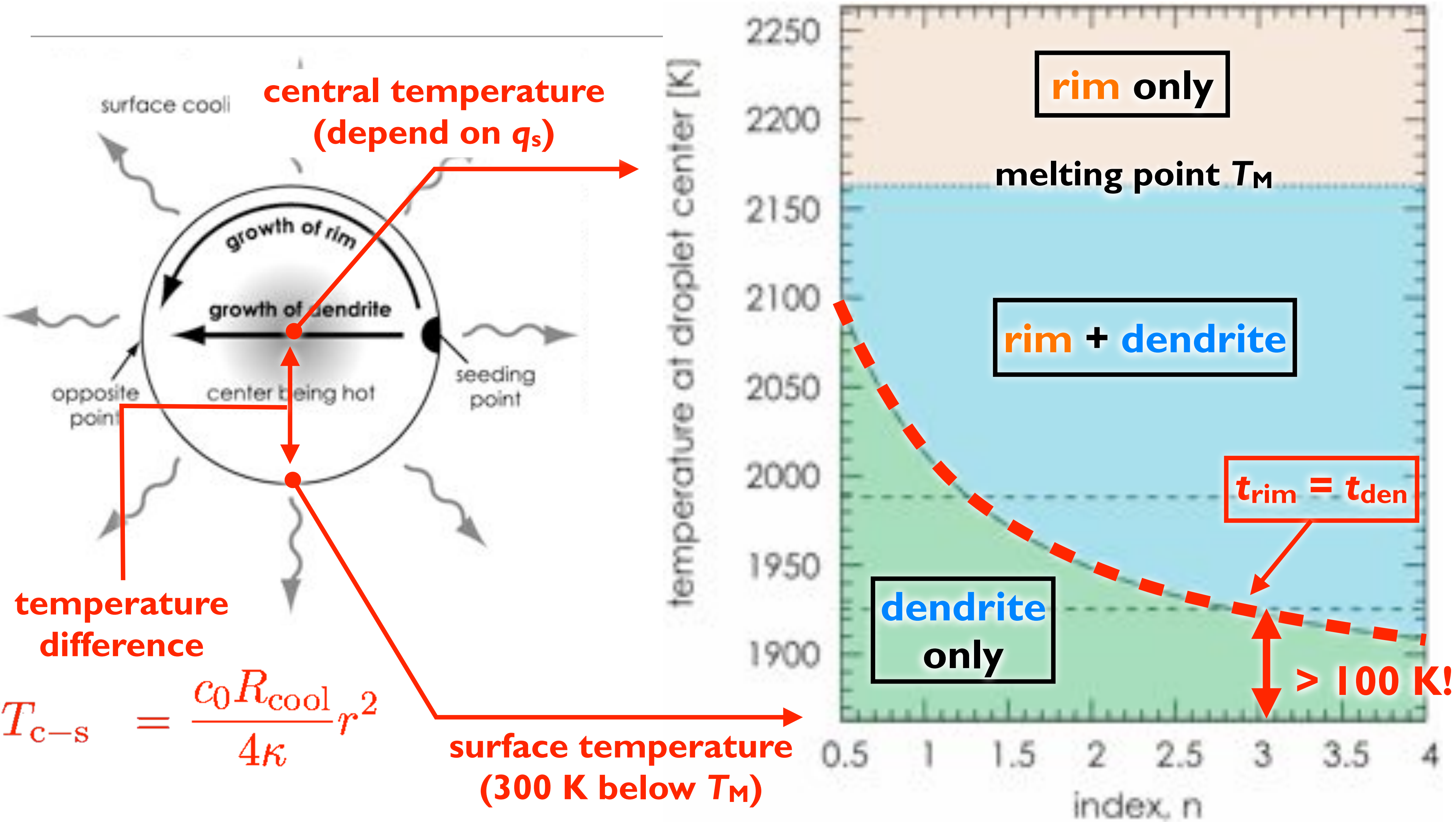
$$I(n,\alpha) \equiv \int_0^1 \frac{dx}{[\alpha(x^2-1)+1]^n}.$$

***n* = 2.5 - 3.5** from theories of dendrite growth

(Langer & Muller-Krumbhaar 1978; Xu 1998)

Discussion:

Condition for rim formation



central temperature (depend on q_s)

temperature difference

$$\delta T_{c-s} = \frac{c_0 R_{cool}}{4\kappa} r^2$$

surface temperature (300 K below T_M)

The rim was formed when R_{cool} **>~ 1300 K/s**
for a forsterite melt droplet of 500 um in diameter.

Conclusions

- We carried out phase-field simulation of crystallization from a highly-supercooled melt droplet.
- We first successfully reproduced double structure by numerical simulation, which is similar to barred olivine texture of chondrules.
- The rim was formed when the cooling rate of the droplet is ~ **10^3 K/s** or larger, which is expected by **radiative cooling**.
  - Astrophysical model predicts a wide range of the cooling rate from **10^{-3} to 10^3 K/s**!
- This is the first step to elucidate the formation mechanism of chondrule solidification texture.