Standard Accretion Disks Driven by MRI Stress

— comparison with the α-viscosity model —

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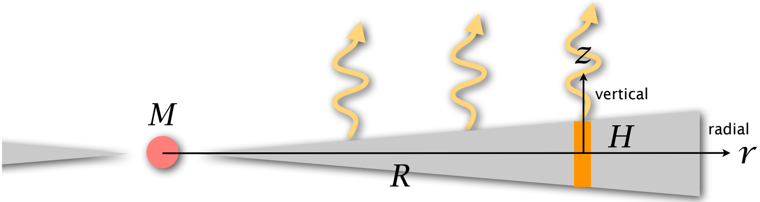
Workshop on MRI in Protoplanetary Disks
Center for Planetary Science, Kobe University

Standard Accretion Disks (Shakura & Sunyaev 1973)

definition

- optically thick
- geometrically thin: $H \ll R$ (nearly Keplerian: $v_{\mathrm{sound}} \ll R\Omega_{\mathrm{K}}$)
- vertical hydrostatic balance
- local thermal balance: $Q^{+}_{\mathrm{diss}}(r) = Q^{-}_{\mathrm{rad}}(r)$



Timescales in Standard Accretion Disks

local structure

- dynamical time: $t_{\text{dynamical}} \equiv H / v_{\text{sound}}$
- thermal time: $t_{\text{thermal}} \equiv \mathcal{E}_{\text{thermal}} / Q^{\pm}$

global structure

- inflow time: $t_{\text{inflow}} \equiv R / v_r$

sharp difference in the timescales

$$t_{\text{orbital}} \sim t_{\text{dynamical}} < t_{\text{thermal}} \ll t_{\text{inflow}}$$

Basic Equations of the α Model (Shakura & Sunyaev 1973)

local structure (one zone approximation)

$H = \dfrac{2P_c}{\Sigma\Omega_K^2}$	hydrostatic balance
$-\dfrac{3}{4}T_{r\phi}\Omega_K = \dfrac{2acT_c^4}{3\kappa\Sigma}$	thermal balance
$P_c = \dfrac{a}{3}T_c^4 + \dfrac{\Sigma k_B T_c}{2\mu H}$	equation of state
$T_{r\phi} = -2H\alpha P_c$	α prescription
$\Sigma =$ constant	$t_{\mathrm{dynamical}}, t_{\mathrm{thermal}} \ll t_{\mathrm{inflow}}$

Basic Equations of the α Model (Shakura & Sunyaev 1973) (continued)

local solution

$$
\begin{aligned}
H &= H(\Sigma, \Omega_{\mathrm{K}}, \alpha) \\
P_{\mathrm{c}} &= P_{\mathrm{c}}(\Sigma, \Omega_{\mathrm{K}}, \alpha) \\
T_{\mathrm{c}} &= T_{\mathrm{c}}(\Sigma, \Omega_{\mathrm{K}}, \alpha) \\
T_{r\phi} &= T_{r\phi}(\Sigma, \Omega_{\mathrm{K}}, \alpha)
\end{aligned}
$$

global structure

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r \Sigma v_r) = 0 \qquad \text{mass conservation}$$

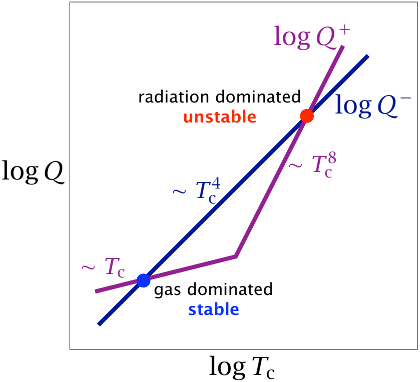
$$\Sigma v_r \Omega_{\mathrm{K}} r^2 = -2 \frac{\partial}{\partial r}(r^2 T_{r\phi}) \qquad \text{angular momentum conservation}$$

Thermal Stability of the α Model (Shakura & Sunyaev 1976)

equation for $\delta T_c (\equiv T_c - T_c|_{Q^+=Q^-})$

$$\frac{\partial \delta T_c}{\partial t} \propto \left(\left.\frac{\partial \log Q^+}{\partial \log T_c}\right|_\Sigma - \left.\frac{\partial \log Q^-}{\partial \log T_c}\right|_\Sigma \right) \delta T_c$$

note: Σ is assumed to be constant since $t_{\mathrm{thermal}} (\ll t_{\mathrm{inflow}})$.

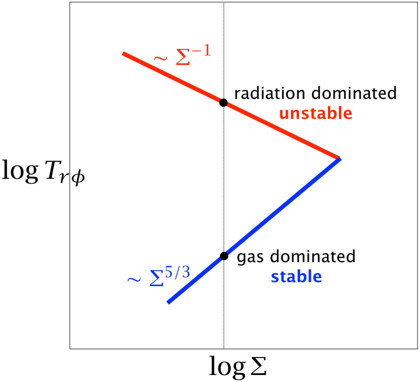


Inflow Stability of the α Model (Lightman & Eardley 1974)

diffusion equation for $\delta\Sigma (\equiv \Sigma - \Sigma_{\text{steady state}})$

$$\frac{\partial \delta\Sigma}{\partial t} \propto \left.\frac{\partial \log T_{r\phi}}{\partial \log \Sigma}\right|_{Q^+=Q^-} \frac{\partial \delta\Sigma}{\partial r^2}$$

note: $Q^+ = Q^-$ is assumed since $t_{\text{inflow}} (\gg t_{\text{thermal}})$.



Outline of This Work

modern view of stress in accretion disks

- MHD turbulence driven by magneto-rotational instability (MRI)

modern model of standard accretion disks

- vertical structure with local dissipation of turbulence and radiative transport
- 3D radiation MHD simulations in a stratified local shearing box
- local equilibrium solution in an averaged sense

$$H = H(\Sigma, \Omega_{\rm K})$$
$$P_{\rm c} = P_{\rm c}(\Sigma, \Omega_{\rm K})$$
$$T_{\rm c} = T_{\rm c}(\Sigma, \Omega_{\rm K})$$
$$T_{r\phi} = T_{r\phi}(\Sigma, \Omega_{\rm K})$$ ⇐ thermal equilibrium curve

Related Studies (stratified local shearing box simulations)

- Brandenburg et al.(1995)
- Stone et al.(1996)
- Miller & Stone (2000)
- Turner (2004)
- Hirose et al. (2006)
- Krolik et al. (2007)
- Blaes et al. (2007)
- Johansen & Levin (2008)
- Suzuki & Inutsuka (2009)
- Hirose et al. (2009)
- ...

Basic Equations

radiation MHD equations with FLD approximation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v}) = -\nabla (p + q) + \frac{1}{4\pi}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \frac{(\bar{\kappa}_{\mathrm{ff}}^{\mathrm{R}} + \kappa_{\mathrm{es}})\rho}{c}\boldsymbol{F} + \boldsymbol{f}_{\text{shearing box}}$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \boldsymbol{v}) = -(\nabla \cdot \boldsymbol{v})(p + q) - (4\pi B - cE)\bar{\kappa}_{\mathrm{ff}}^{\mathrm{P}}\rho - cE\kappa_{\mathrm{es}}\rho \frac{4k_{\mathrm{B}}(T - T_{\mathrm{rad}})}{m_{\mathrm{e}}c^2}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (E \boldsymbol{v}) = -\nabla \boldsymbol{v} : \mathsf{P} + (4\pi B - cE)\bar{\kappa}_{\mathrm{ff}}^{\mathrm{P}}\rho + cE\kappa_{\mathrm{es}}\rho \frac{4k_{\mathrm{B}}(T - T_{\mathrm{rad}})}{m_{\mathrm{e}}c^2} - \nabla \cdot \boldsymbol{F}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0$$

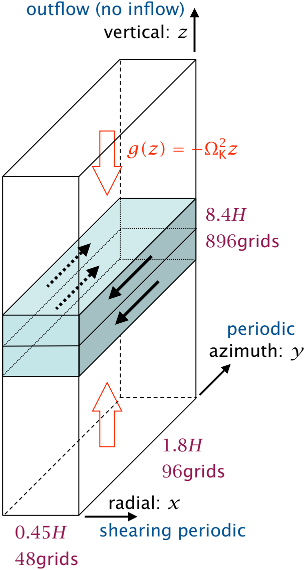
$$\boldsymbol{F} = -\frac{c\lambda}{(\bar{\kappa}_{\mathrm{ff}}^{\mathrm{R}} + \kappa_{\mathrm{es}})\rho}\nabla E$$

no explicit resistivity and viscosity

numerical method

- hydro part: ZEUS
- magnetic part: MOC+CT
- radiation diffusion part (implicit): multigrid SOR

Simulation Setup



simulation box

- stratfied shearing box
- $\Omega_K = 190 s^{-1}$
  $(M/M_\odot = 6.62, r/r_g = 30)$

initial condition

- gas and radiation
  - hydrostatic in z without $\boldsymbol{B}$
- magnetic field
  - twisted flux tube in y of $\beta \simeq 20$

parameters

- surface density Σ
- initial guess of Q^+ (, or thermal energy content) $\Longrightarrow$ gas/radiation-dominated

Parameter Space

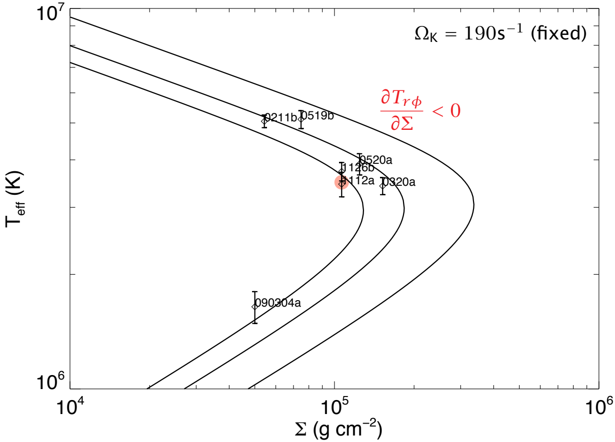
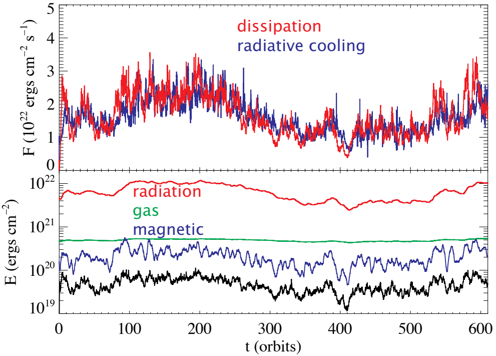


Fig. 2.— Time averaged effective temperature of the radiation leaving each vertical face of the box, as a function of surface mass density for each simulation. From right to left, the solid curves show the predictions of alpha disk models with $\alpha = 0.01$, 0.02, and 0.03, respectively. (See the Appendix for the equations used to define these alpha parameters.)

Radiation-dominated Disk Solution

- parameters
  - $\Sigma = 1.1 \times 10^5 \mathrm{gcm}^{-2}$
  - guessed $Q^+ = 9.4 \times 10^{21} \mathrm{ergcm}^{-2}\mathrm{s}^{-1}$



- radiation-dominated: $E_{\mathrm{rad}} \sim 20 E_{\mathrm{gas}}$
- stable for $600 t_{\mathrm{orbit}} \sim 40 t_{\mathrm{thermal}}$
- time variations (quasi-steady state)
  - MHD turbulence driven by MRI
  - magnetic buoyancy (Parker instability)
  - vertical oscillation (epicyclic mode, breathing mode)

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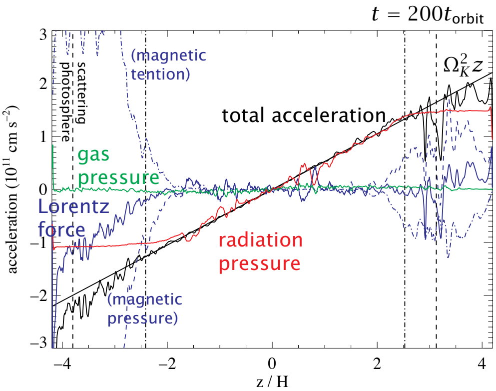
Local Structure: Hydrostatic Balance



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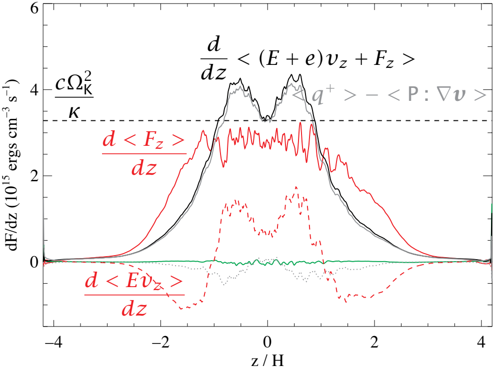
- $|z| < 2H$: radiation pressure
- $|z| > 2H$: magnetic pressure (+ magnetic tention)
  - magnetic field is supplied to the upper (subphotospheric) layers by magnetic buoyancy (Parker instability)

Local Structure: Thermal Balance



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$$< q^+ > - < \mathsf{P} : \nabla \boldsymbol{v} > = \frac{d}{dz} < (E+e)v_z + F_z >$$

- dissipation: extended with double peaks
- radiation diffusion: $d < F_z > /dz$
  - $\simeq c\Omega_K^2/\kappa$ where radiation pressure competes the gravity ($|z| < H$)
- radiation advection: $d < Ev_z > /dz$
  - transports the excess energy
  - associated with vertical oscillation, not buoyancy

Thermal Stability of MRI Disks

thermal instability in the α model

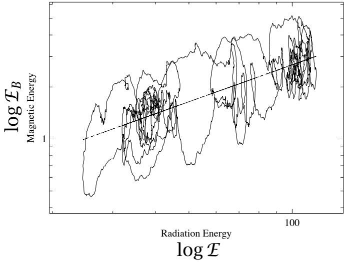
$$\frac{d\mathcal{E}(t)}{dt} = \frac{\alpha\Omega}{4}\mathcal{E}(t) - \frac{c\Omega}{\kappa\sqrt{3\Sigma}}\sqrt{\mathcal{E}(t)}$$

$$\begin{cases} \dfrac{d\mathcal{E}(t)}{dt} = -\dfrac{3}{4}T_{r\phi}(t)\Omega_{\mathrm{K}} - \dfrac{2acT_{\mathrm{c}}^4(t)}{3\kappa\Sigma} \\ T_{r\phi}(t) = -\alpha P(t) \end{cases}$$

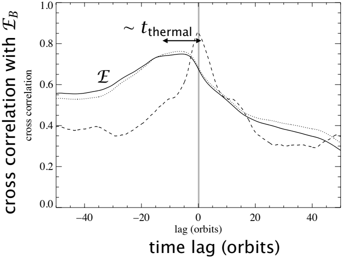
$T_{r\phi}$ synchronized with P

- $\mathcal{E}_B$ – $\mathcal{E}$ relation in the simulation (in place of $T_{r\phi}$ – P relation)

$\mathcal{E}_B \sim \mathcal{E}^{0.71}$



$\mathcal{E}_B(t) \sim \mathcal{E}(t + t_{\mathrm{thermal}})$



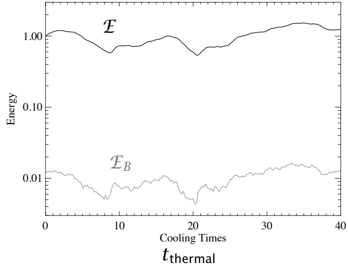
Thermal Stability of MRI Disks (continued)

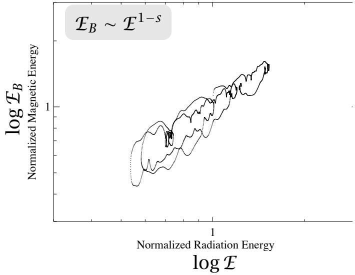
a toy model that allows a time lag between $\mathcal{E}_B$ and $\mathcal{E}$

$$\left.\begin{aligned}\frac{d\mathcal{E}(t)}{dt} &= \frac{\mathcal{E}_B(t)}{t_{\text{diss}}} - \frac{\mathcal{E}(t)}{t_{\text{cool}}(\mathcal{E}(t_0))\,(\mathcal{E}(t)/\mathcal{E}(t_0))^s} \\ \frac{d\mathcal{E}_B(t)}{dt} &= R(t)\,\frac{\mathcal{E}_B(t_0)}{t_{\text{grow}}}\left(\frac{\mathcal{E}(t)}{\mathcal{E}(t_0)}\right)^n - \frac{\mathcal{E}_B(t)}{t_{\text{diss}}}\end{aligned}\right\}$$

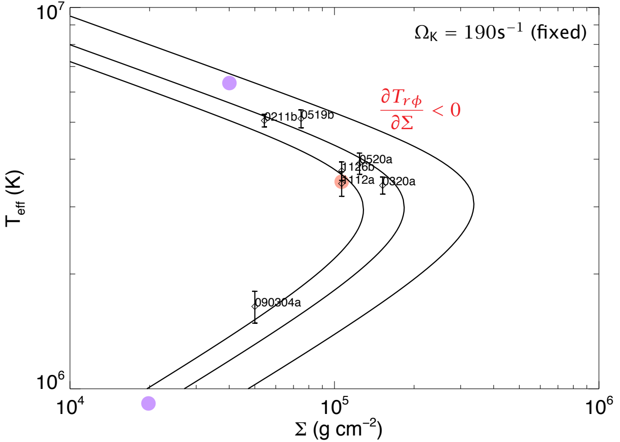
instability criterion
$(1-s) < n$

- thermally stable solution: $(1-s) = 1, n = 0$





Thermal Equilibrium Curve



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Summary

Comparison between the α disks and MRI disks

	α disks	MRI disks
hydrostatic pressure	thermal	thermal magnetic[a]
energy transport	radiation diffusion	radiation diffusion radiation advection[b]
stress-pressure correlation	yes	yes[c]
thermal stability	rad: unstable gas: stable	rad: stable[d] gas: stable

a) important in the upper *subphotospheric* layers

b) important in the radiation dominated regime

c) on timescales longer than t_{thermal}

d) – time lag between stress and pressureis necessary
– intrinsic fluctuation of turbulence is longer than t_{cool}

Future Works

- construction of a new standard accretion disk model
  - thermal equilibrium curves at different radii

$$\dot{M} = \dot{M}(\Sigma; \Omega_{\mathrm{K}}(r))$$
$$H = H(\Sigma; \Omega_{\mathrm{K}}(r))$$
$$P_{\mathrm{c}} = P_{\mathrm{c}}(\Sigma; \Omega_{\mathrm{K}}(r))$$
$$T_{\mathrm{c}} = T_{\mathrm{c}}(\Sigma; \Omega_{\mathrm{K}}(r))$$

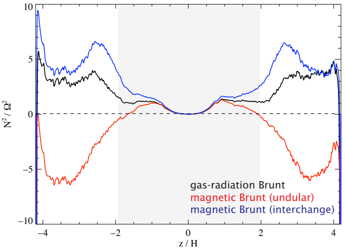
  - radial distriubutions of Σ with different mass accretion rates

$$\Sigma = \Sigma(r; \dot{M})$$
$$H = H(r; \dot{M})$$
$$P_{\mathrm{c}} = P_{\mathrm{c}}(r; \dot{M})$$
$$T_{\mathrm{c}} = T_{\mathrm{c}}(r; \dot{M})$$

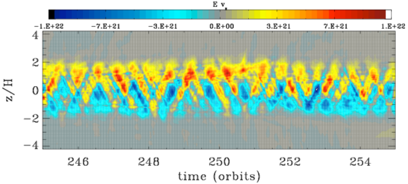
- application of our method to construct a protoplanetary disk model
  - MRI in weakly ionized plasma
  - dead zone
  - complicated thermodynamics
    - heating sources other than the turbulent dissipation
    - cooling mechanisms other than the thermal radiation
  - dust grains
  - ...

Origin of the (Vertical) Radiation Advection Ev_z

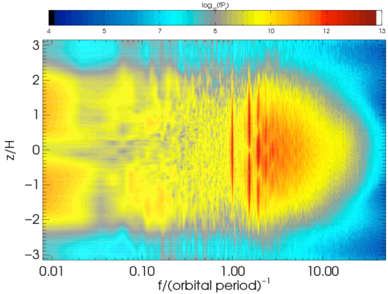
- Energy transport in the core is not associated with convection or buoyancy.



- Spacial and temporal behavior of Ev_z

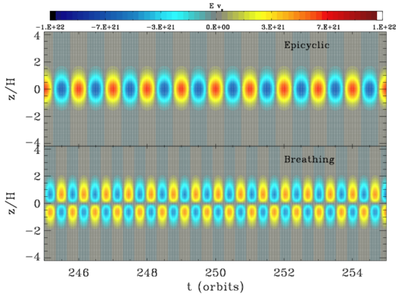


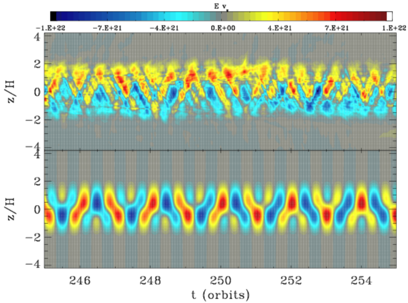
- Vertical profile of Ev_z power spectrum



Origin of the (Vertical) Radiation Advection Ev_z (continued)

- radiation advection patterns for n=3 adiabatic polytropic modes
- comparison between the simulation and adiabatic polytropic mode

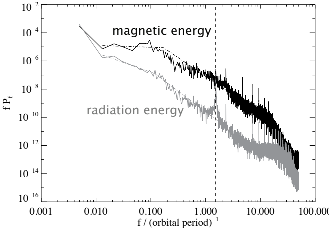


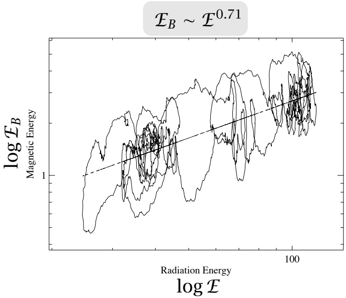


- Radiation advection pattern in the simulation can be reproduced by epicyclic + breathing mode + radiative diffusion.

Thermal Stability of MRI Disks (continued)

- Why MRI disks can be thermally stable?
  1. time lag between stress and pressure relaxes the instability criterion $(1 - s) < n$
  2. timescale of the large-amplitude turbulence fluctuations is longer than t_{cool}
- On the α prescription
  - When time-averaged over many thermal times, pressure is correlated with stress as the α model predicts.
  - Causality is critical: $T_{r\phi} \rightarrow P$, not vice versa. Stress fluctuations drive pressure fluctuations, creating a correlation between the two.





Expectations of Thermal Balance

- thermal energy equation

$$\frac{\partial}{\partial t}(E+e)+\nabla\cdot((e+E)\boldsymbol{v})$$
$$=-\nabla\cdot\boldsymbol{F}-p\,(\nabla\cdot\boldsymbol{v})+q^{+}-\mathsf{P}:\nabla\boldsymbol{v}$$

- averaged thermal balance equation

$$\underbrace{<q^{+}>}_{\text{dissipation rate}}\ \underbrace{-<p\,(\nabla\cdot\boldsymbol{v})>-<\mathsf{P}:\nabla\boldsymbol{v}>}_{\text{compression work}}$$
$$=\frac{d}{dz}\underbrace{\langle(E+e)\boldsymbol{v}+\boldsymbol{F}\rangle}_{\text{thermal energy flux}}$$

"magic" dissipation rate in radiation-dominated regime

Amount of dissipated energy that radiative diffusion flux can transport is vertically fixed constant (Shakura & Sunyaev 1976).

$$q^{+}_{\mathrm{magic}}(z)=\frac{c\Omega_{\mathrm{K}}^{2}}{\kappa_{\mathrm{es}}}\quad\text{(constant)}$$

- hydrostatic balance

$$\frac{\kappa_{\mathrm{es}}F_z(z)}{c}=\Omega_{\mathrm{K}}^{2}z$$

- thermal balance

$$q^{+}(z)=\frac{dF_z(z)}{dz}$$