The effect of temporally varying charges of Dusty Plasmas on the linear stage of Magneto-Rotational Instability

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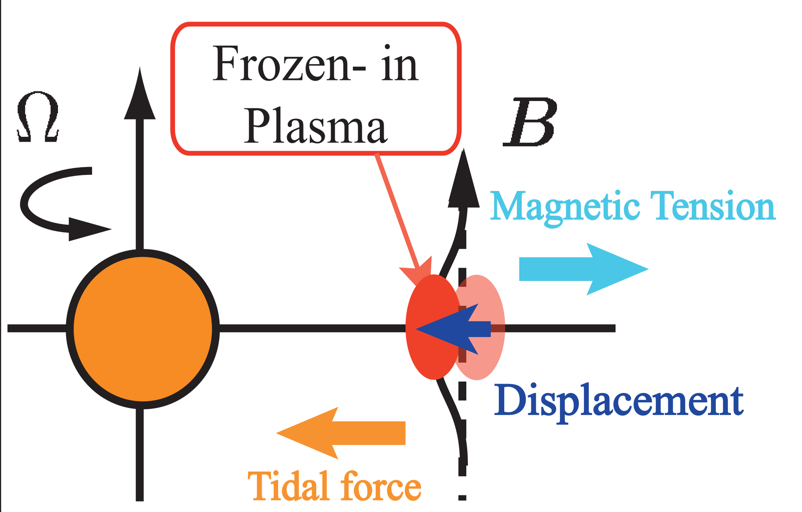
[1] Abstract

We report several results of linear analysis of Magneto-Rotational Instability (MRI) under the effect of Dusty Plasmas. Recent study has shown that dust particles in plasma carry negative charges and can interact with electromagnetic field. These '*charged dusts*' are usually treated as the third component of the plasma in the MHD approximation and several modification on plasma phenomena are found from '*multi-fluid analysis*'. In this study we solved a set of linearized 3 fluid MHD equations including dusty plasma. Results implied that dusty plasma destabilizes MRI. We also assumed that dust particles temporally vary its charges through collisions with ions and considered the effect of fluctuating dust charges. We found that effect of fluctuating dust charges are strongly affected by the equilibrium state of rotating disc.

[2] Introduction

'*Single-fluid*' MRI [Balbus & Hawley, 1991]



Tidal Force > Magnetic Tension

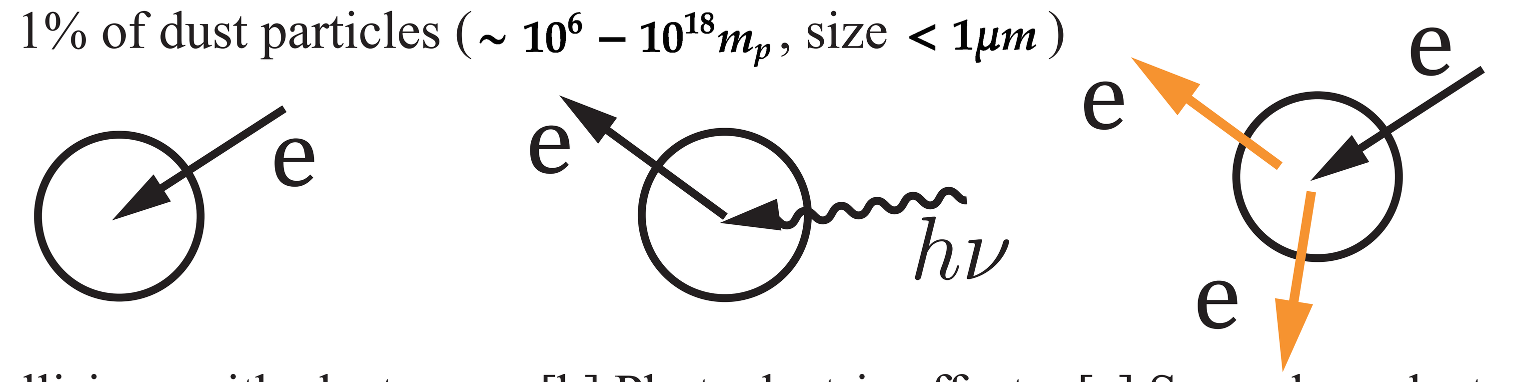
Plasma cannot be dragged back to its equilibrium position → Magneto-Rotational Instability (MRI)

$$\frac{\Omega^2}{k^2 V_A^2} > \frac{1}{3}$$

Ω : Angular frequency
V_A : **Alfvén Velocity**
k : Wavenumber

Dusty Plasmas

- Our space → 99% of electron-ion plasma
- 1% of dust particles ($\sim 10^6 - 10^{18} m_p$, size $< 1\mu m$)



[a] collisions with electrons and ions   [b] Photoelectric effect   [c] Secondary electron emission

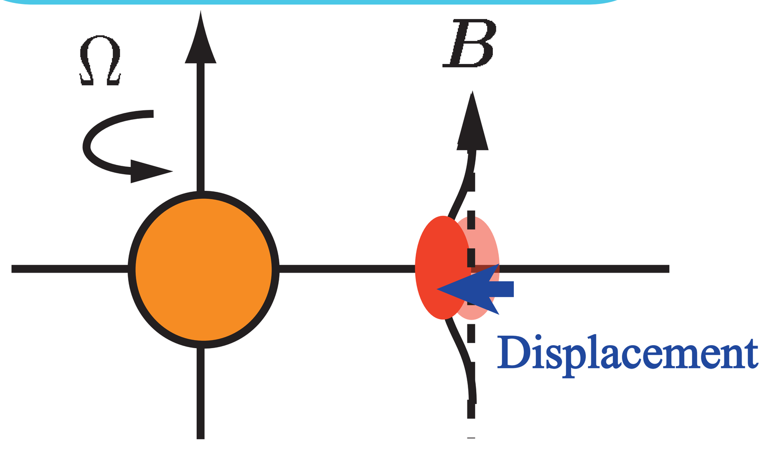
- Collisions with electrons is dominant process
  → dust particles carry negative charges ($> 10^3|e|$)
- Can be treated as the 3rd component of the plasma
- Modifications on plasma phenomena (e.g. Wave propagation, Instabilities)

→ 'Multi-fluid' MHD approach should be performed to understand the effect of dusty plasma on MRI

[3] Linear analysis (1) - fixed dust charge -

- Solve 3 fluid (electron, ion, dusty plasma) MHD equation

▼ '*Local*' approximation



- Linearize 3 fluid MHD equations
  → Fourier Analysis
- Neglect radial gradient by assuming '*Local*' approximation
- Assume axisymmetry

Derived 3fluid MRI dispersion relation

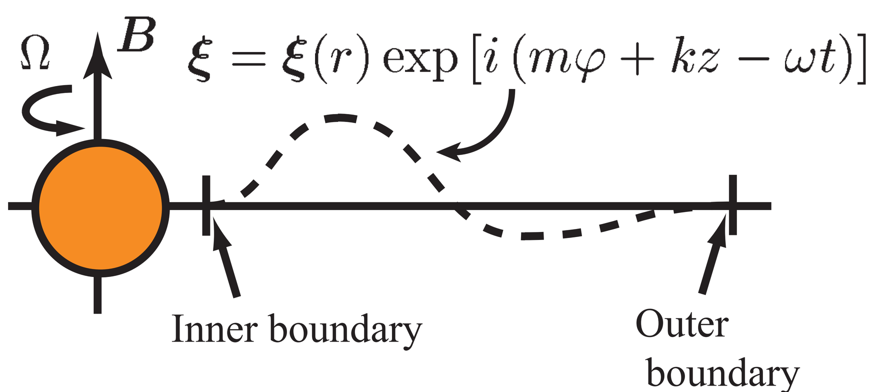
$$\left[\omega^2\left(\sum_\alpha \frac{\Omega_{c\alpha}^2}{k^2 V_{A\alpha}^2 \mathcal{D}_\alpha}\right) - 1\right]^2 - \omega^2\left[\sum_\alpha \frac{\Omega_{c\alpha}^2}{k^2 V_{A\alpha}^2 \mathcal{D}_\alpha}(2\Omega + \Omega_{c\alpha})\right]\left[\sum_\alpha \frac{\Omega_{c\alpha}^2}{k^2 V_{A\alpha}^2 \mathcal{D}_\alpha}\left(\frac{\kappa^2}{2\Omega} + \Omega_{c\alpha}\right)\right] - \frac{d\Omega}{d\log r}\sum_\alpha \frac{\Omega_{c\alpha}^2}{k^2 V_{A\alpha} \mathcal{D}_\alpha}\left(\frac{\kappa^2}{2\Omega} + \Omega_{c\alpha}\right) = 0$$

Onset condition of MRI

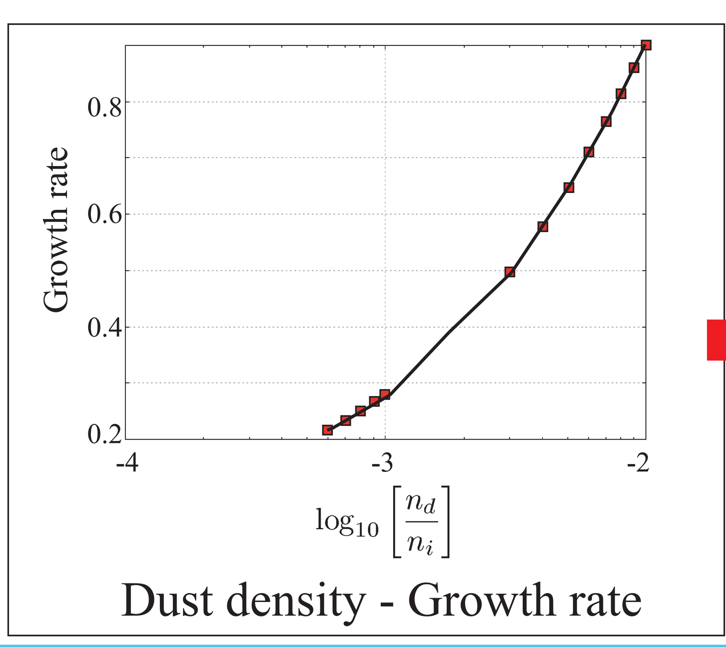
$$\frac{\Omega^2}{k^2}\sum_\alpha\left(\frac{1}{V_{A\alpha}^2}\right) > \frac{1}{3}$$

- Shorter wavelength can be unstable compared to ordinary 1 fluid MRI

▼ '*Non-Local*' Analysis



- Consider radial gradient and numerically calculate fluctuation satisfying boundary conditions
- Solve eigen value-eigen function problem by changing ω
- Calculate growth rate from derived eigen value ω



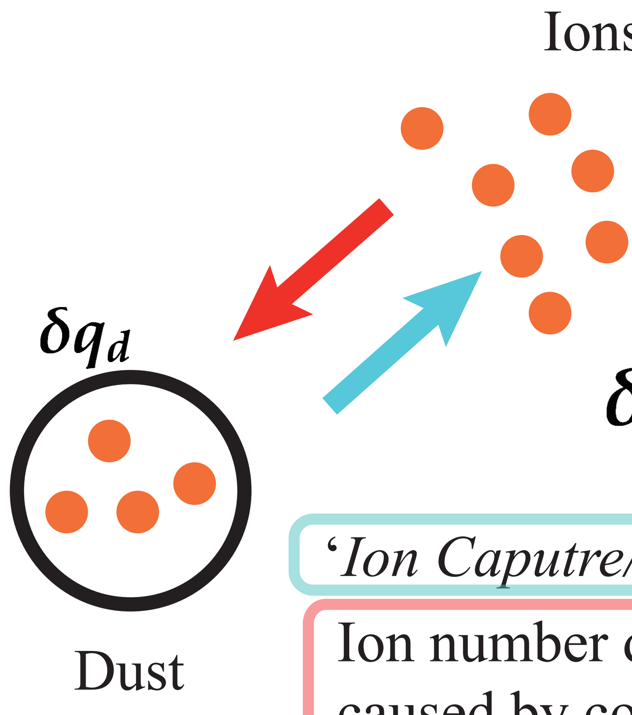
Dust density - Growth rate

- Dusty Plasma can destabilize MRI
  → Tidal force is enhanced by dust (massive component). Magnetic tension is unchanged by increase of dust number density.

[4] Linear analysis (2) - temporally varying dust charges -

Model of Charge fluctuation

- Consider two component (ion,dust) plasma
- Temporal variation of dust chrage is caused by capture/release of ions



Ion-Rich region → inflow of ion exceeds → dust gains positive charge
Ion-Less region → outflow of ion exceeds → dust loses positive charge

$$\delta q_d = \frac{q_i}{n_d}\Gamma\delta n_i \qquad \frac{Dn_i}{Dt} + n_i\nabla\cdot v = S$$

'*Ion Caputre/release*' rate (Γ)

Ion number density fluctuation caused by convective effect (δn_i)

Include sink/source term in ion continuity equation for consistency (S)

Equilibrium State

Linearized Lorentz force term in dusts' equation of motion

$$\frac{q_d}{m_d}(E + v_d \times B) \rightarrow \frac{\delta q_d}{m_d}(E + v_d \times B) + \frac{q_d}{m_d}(\delta E + \delta v_d \times B + v_d \times \delta B)$$

Effect of fluctuating of dust charge cannot be determined unless the equilibrium state of rotating disk is known

↕

The equilibrium state (electric field, angular velocity) in rotating disk is not known (cannot be determined without an *appropriate* assumption).
[Nekrazov 2007, Phys. of Plasmas]

Results

$v_{\alpha 0} \propto r^{-\frac{1}{2}}$ All plasma components are rotating with Keplerian angular velocity

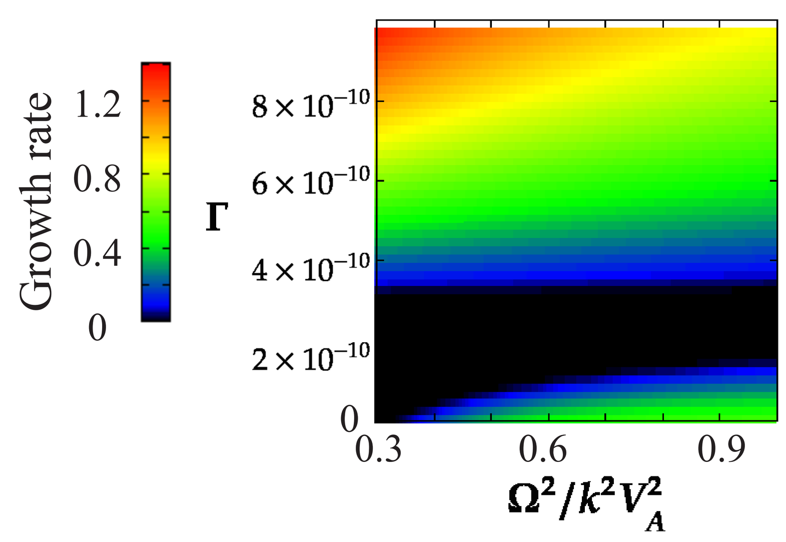


Fig.1 The case $E_0 = 0$

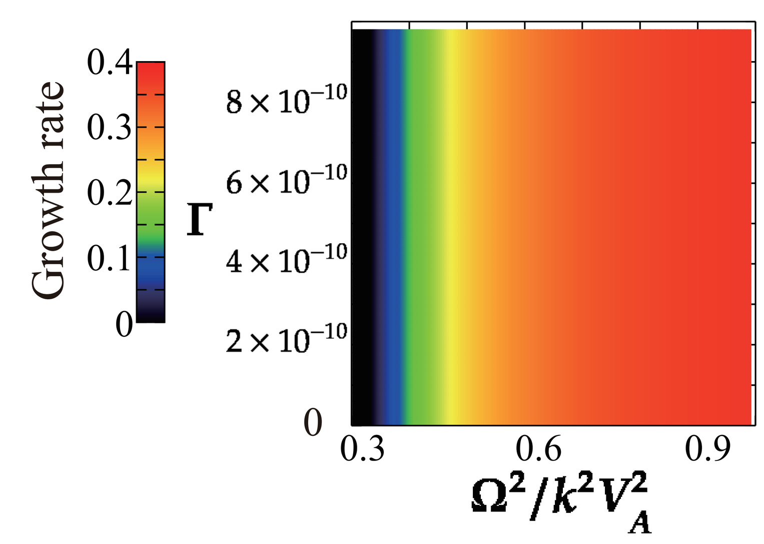


Fig.2 The case $E_0 + v_d \times B_0 = 0$

Effect of dusts' charge fluctuation is significant when $E_0 + v_{d0} \times B_0 \neq 0$

[5] Summary and future works

- Result of linear analysis implied that existence of dust would destabilize MRI
- Effect of charge fluctuation is strongly affected by equilibrium state of rotating disk. In particular condition this effect stabilizes MRI
- Linear analysis considering radial, azimuthal wave propagation should be done to consider interaction with new modes arose by dusty plasma
- Modeling of caputre/release process, estimation of capture/release rate should be done
- 'Multi fluid' simulation to study nonlinear stage as a future study