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Orbital Evolution of a Particle Interacting with a Single Planet in a Protoplanetary Disk

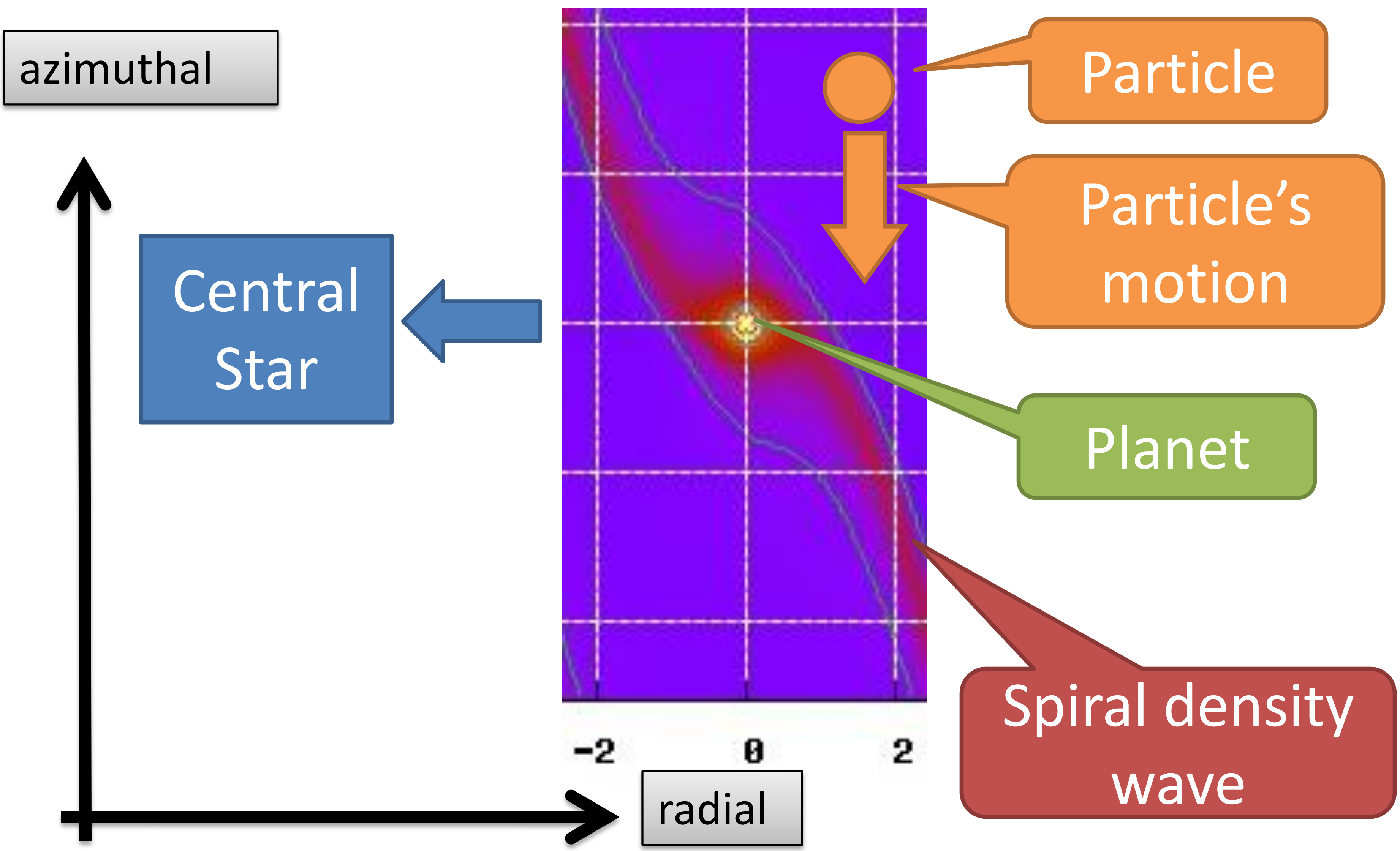
Takayuki Muto and Shu-ichiro Inutsuka
(Kyoto University)

Abstract

- **Is it possible to observe an Earth mass planet embedded in a disk?**
- Analytic calculation of dust motion in the vicinity of a planet embedded in a gas disk
  - Fundamental physical processes are made clear
- Effects considered:
  - Global pressure gradient
  - Steady accretion flow
  - Planet gravity
  - Spiral density wave produced by the planet
- **Gas effects are (almost) fully taken into account**

Investigation of Dust Motion

Setup: Local Approximation



Basic Equations: Hill's eqn

- Dust motion in the vicinity of the Planet

Tidal force

Gas friction

Planet gravity

$$\ddot{x} - 2\Omega_{\mathrm{p}}\dot{y} = 3\Omega_{\mathrm{p}}^2 x - \nu(\dot{x} - v_{x,\mathrm{gas}}) - \frac{\partial}{\partial x}\psi_{\mathrm{p}}$$

$$\ddot{y} + 2\Omega_{\mathrm{p}}\dot{x} = -\nu(\dot{y} - v_{y,\mathrm{gas}}) - \frac{\partial}{\partial y}\psi_{\mathrm{p}}$$

Corioris force

$$\psi_{\mathrm{p}} = -\frac{GM_{\mathrm{p}}}{\sqrt{x^2+y^2}}$$

ν : friction rate is assumed to be constant (related to dust size)

Gas Effects Considered

- Global pressure gradient

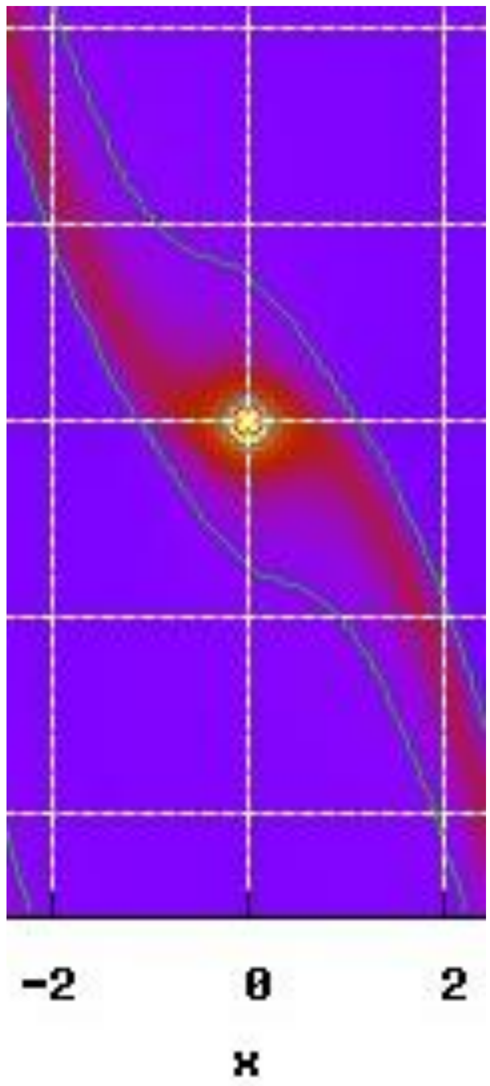
$$\delta \boldsymbol{v}_{\mathrm{g}} = \eta v_{\mathrm{p}} \boldsymbol{e}_y = \mathrm{const},$$

- Mass accretion onto central star

$$\delta \boldsymbol{v}_{\mathrm{g}} = \zeta v_{\mathrm{p}} \boldsymbol{e}_x = \mathrm{const}.$$

- Spiral density wave
  - Obtained by perturbative approach upto 2nd order

$$\delta \boldsymbol{v}_g = \delta \boldsymbol{v}^{(1)} + \delta \boldsymbol{v}^{(2)},$$



Approximations

- Impulse approximation
- Initially circular orbit
- Calculate one encounter between the planet and the particle

Resonance effects and Close encounter are not calculated!

Average Rate of the Change of Semi-major Axis

Pressure gradient

Mass accretion

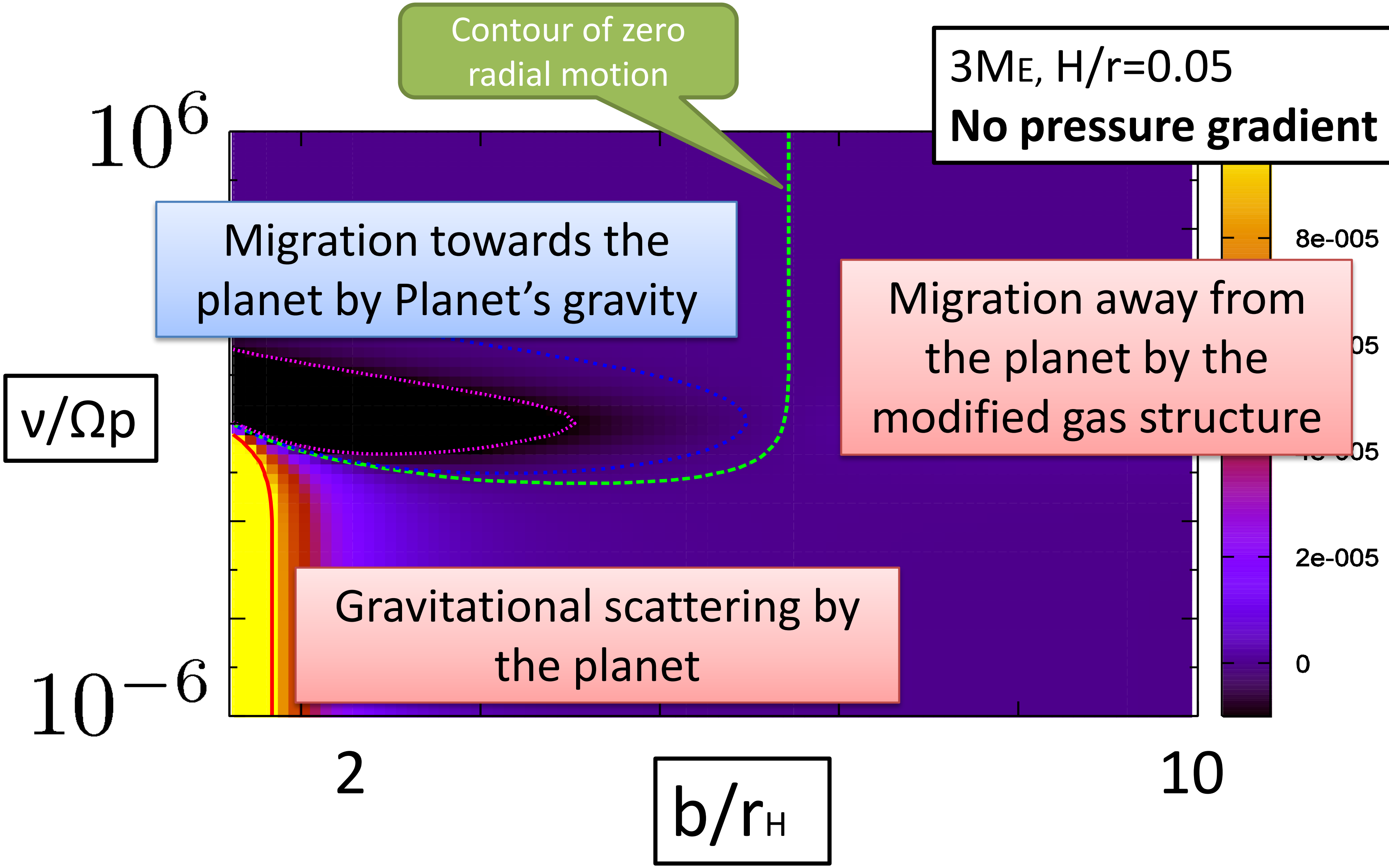
Gravitational scattering and attraction

$$\frac{\Delta b}{T} = 2\eta v_{\rm p}\frac{\nu\Omega_{\rm p}}{\nu^2+\Omega_{\rm p}^2} + \zeta v_{\rm p}\frac{\nu^2}{\nu^2+\Omega_{\rm p}^2} - {\rm sgn}(b)\frac{4}{T}\frac{r_{\rm H}^3}{b^2}\frac{\nu\Omega_{\rm p}}{\nu^2+\Omega_{\rm p}^2} + \frac{\alpha}{T}\frac{r_{\rm H}^6}{b^5}\frac{\Omega_{\rm p}^2}{\nu^2+\Omega_{\rm p}^2} + {\rm sgn}(b)\frac{2}{T}\frac{r_{\rm H}^3}{bH}\left[e^{-(b/H)}{\rm Ei}\left(\frac{b}{H}\right) - e^{b/H}{\rm Ei}\left(-\frac{b}{H}\right)\right]\frac{\nu\Omega_{\rm p}}{\nu^2+\Omega_{\rm p}^2},$$

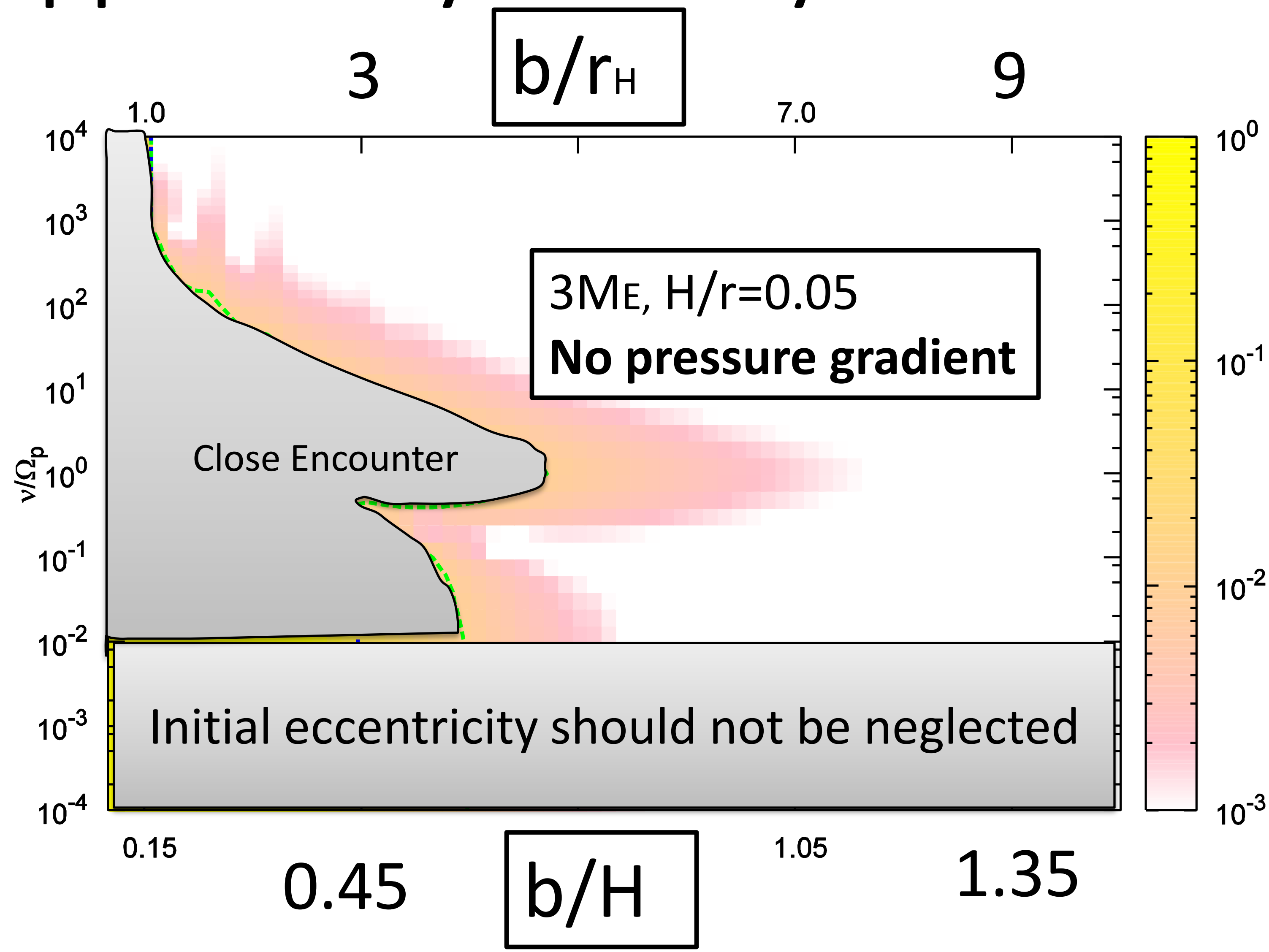
Spiral density wave

$$\alpha \equiv \frac{128}{27}\left[K_1\left(\frac{2}{3}\right) + 2K_0\left(\frac{2}{3}\right)\right]^2 = 30.094$$

Average Rate of Semi-major Axis Evolution



Applicability of Analytic Results



Application:
Dust Gap Opening and Observability of an Earth-mass Planet embedded in a Disl

Model for Evolution of Dust Distribution in a Protoplanetary Disk

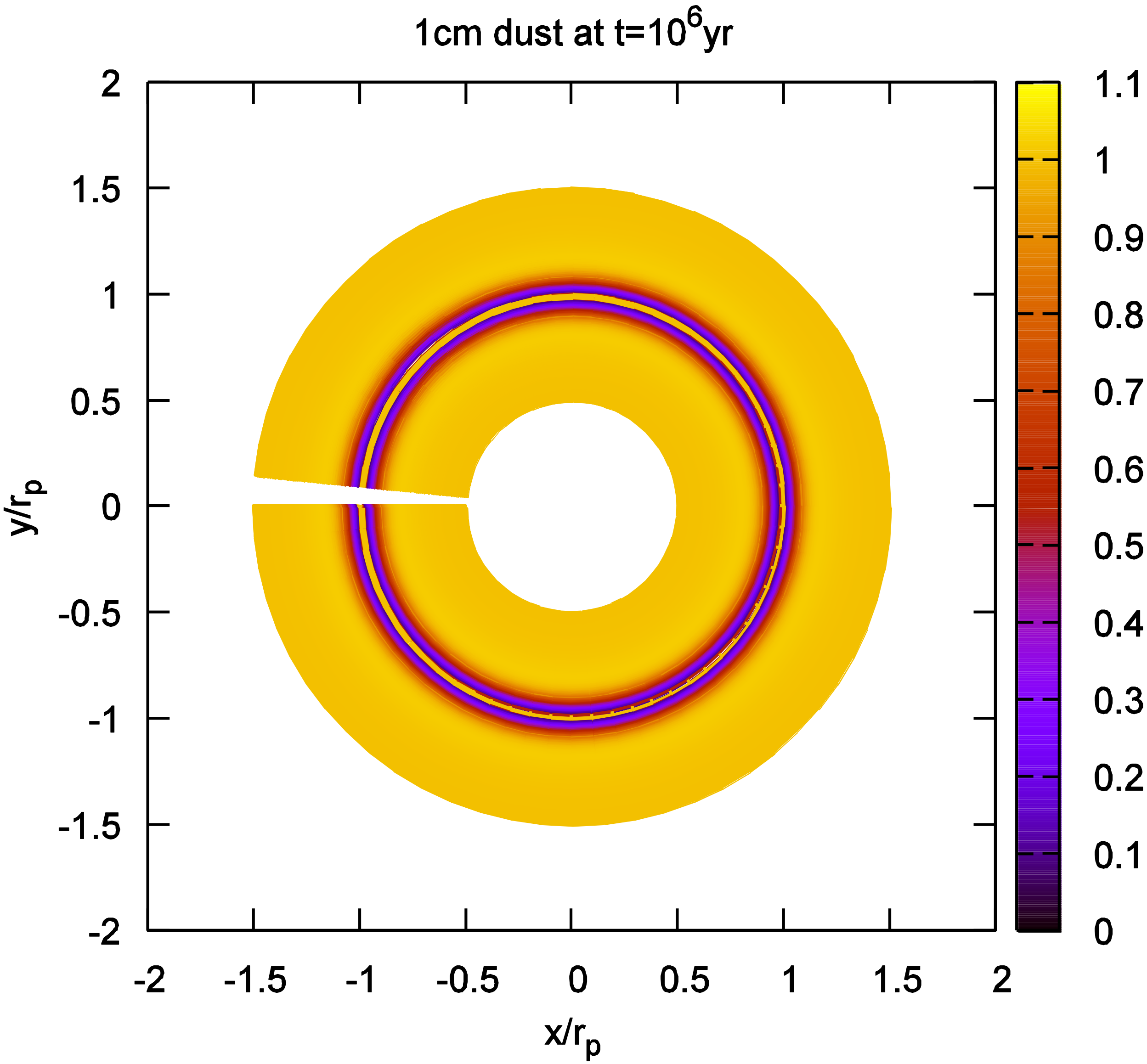
1-D axisymmetric model:

$$\frac{\partial N(t,b)}{\partial t} + \frac{\partial}{\partial b}\left[v_b(b) N(t,b)\right] = 0$$

We use radial velocity of dust analytically obtained

Possible to reach 10^6 years!

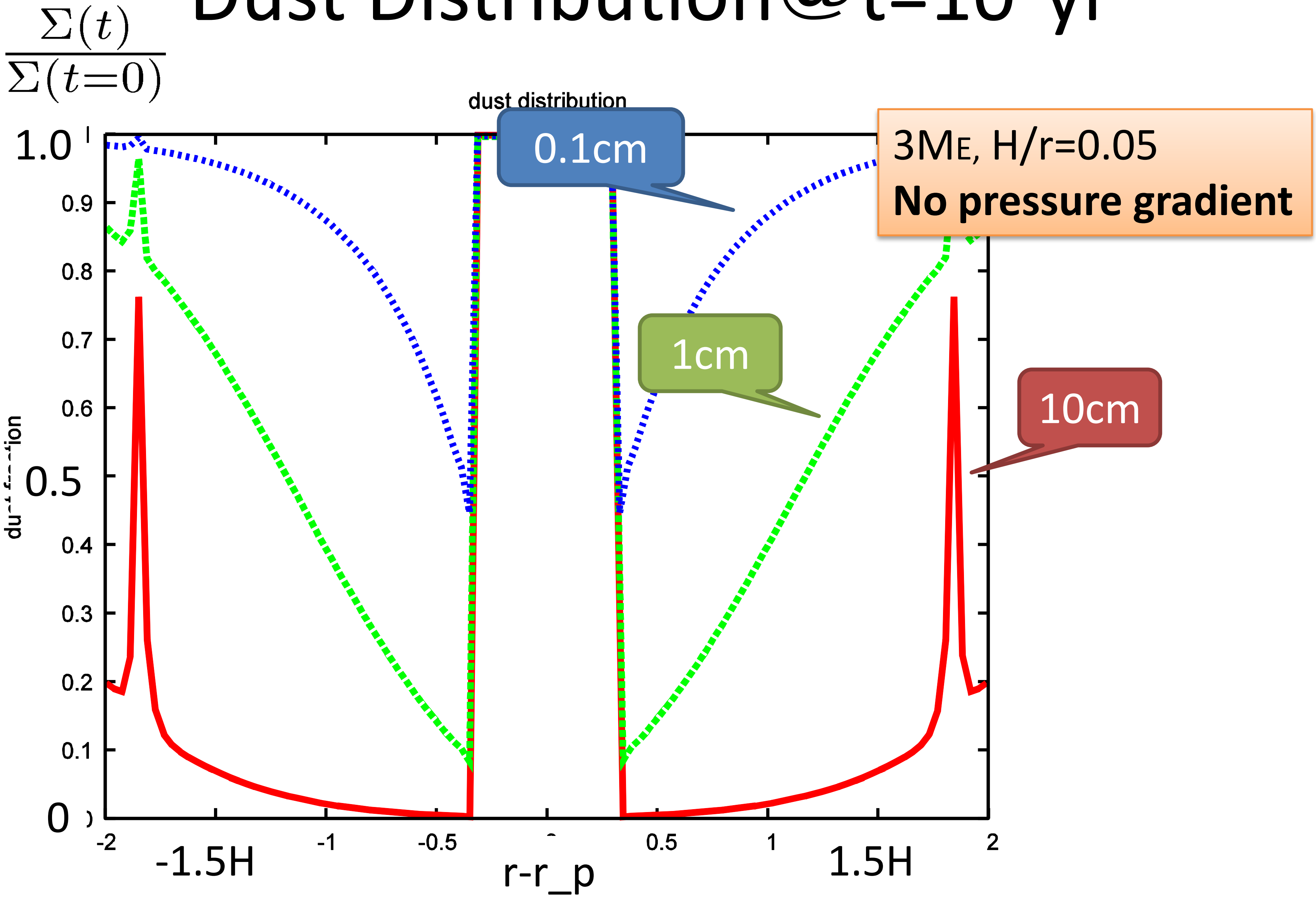
Distribution of 1cm dust@t=10^6yr



3ME, H/r=0.05

No pressure gradient

Dust Distribution@t=10^6yr



Dust Gap Opening and Observability

- If a disk with eta=0 is considered:
  - Gap of particles larger than ~1cm, width ~H, may be formed, ***even around an Earth mass planet***
- If the planet is at ~100AU and H/R~0.1, gap width is ~10AU
  - May be possible with ALMA, better on SKA?
  - Possibly VLBI, if sensitivity allows.
  - High-resolution imaging study at long wavelengths may provide a good tool in finding low-mass planet in a disk