

# Numerical Simulation of Structure Evolution of Dust Aggregates Growing in Protoplanetary Disks

Toru Suyama, Koji Wada, Hidekazu Tanaka  
*Institute of Low Temperature Science, Hokkaido University*

# Abstract

**Dust growth:** In protoplanetary disks, dust aggregates grow through mutual collisions. Small dust aggregates have the fluffy structure. However, **dust aggregates are compressed as they grow**. Thus, large dust aggregates can not keep such fluffy structure. Such compression changes the cross section of dust aggregates. Thus, when and how to compression is important on dust growth.

**Method:** In this study, we perform the **N-body simulation of aggregate collisions** of realistic **oblique** collisions. We focused the radius of gyration and the cross section as the property of the structure of dust aggregates. We examine the evolution of the radius of gyration and the cross section during aggregate growth, respectively.

**Result:** We examine the change of the radius of gyration and the cross section at collisions between aggregates which have a variety of the mass and the density. At oblique collisions, dust aggregates are elongated. For the radius of gyration, aggregates are not compressed. However, for the cross section, aggregates are compressed. Applying our compression model based on numerical simulations of head-on collisions (Suyama et al. 2008) to oblique collisions, we succeed the formulation of the evolution of the cross section. As a result of this model, the impact compression make low density. Such aggregates have a large cross section. Thus, dust growth which previous study considered changes quantitatively.

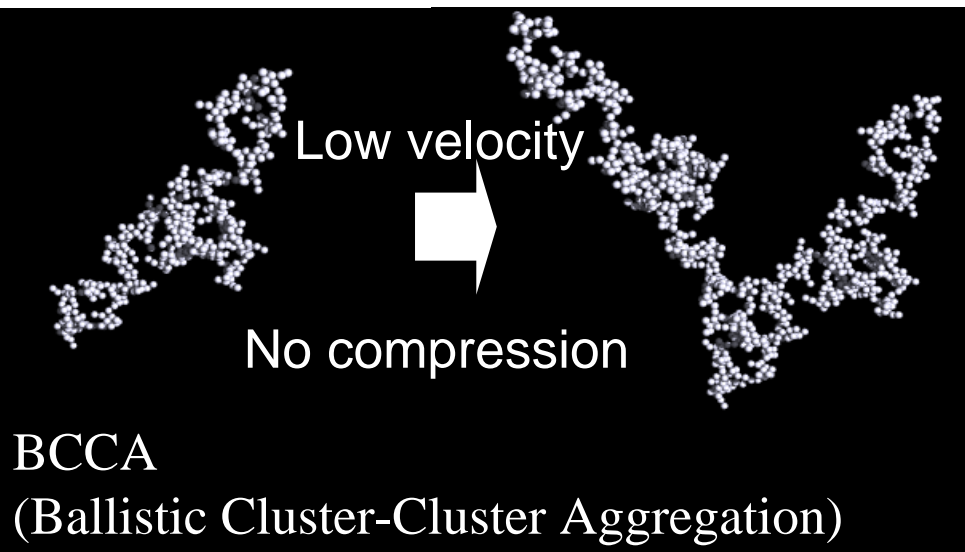
# Structure evolution of the dust aggregates

Dust → Planetesimals → Planets

Initial condition of planet formation

Dust grow through mutual collisions

Impact compression?



The porosity increases and density decreases.

⇒ Dust must be compressed!

The change of the cross section and the density causes the impact velocity change  
⇒ It affects dust growth.



It is important to examine the evolution of density in the course of dust growth.

Simulation of aggregate collision

Dominik and Tielens 1997

Wada et al. 2007, 2008

Suyama et al. 2008

Paszun and Dominik 2008

Only one collision

⇒ Dust must grow through many collisions.

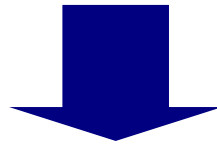
Head-on collision

⇒ Most collisions are oblique ones.

# Aim

For oblique collisions,  
how does the internal structure change?

What is the difference between  
oblique collisions and head-on collisions?

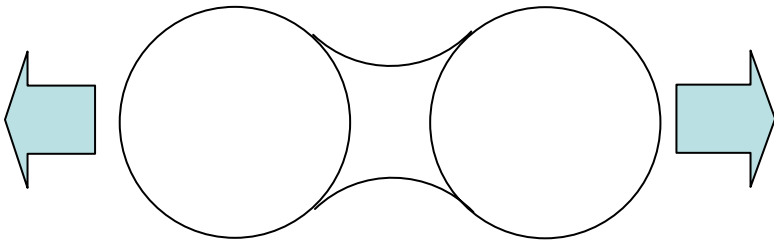


We performed 3D numerical simulation of  
aggregate collisions and examine the density changes  
through dust growth.

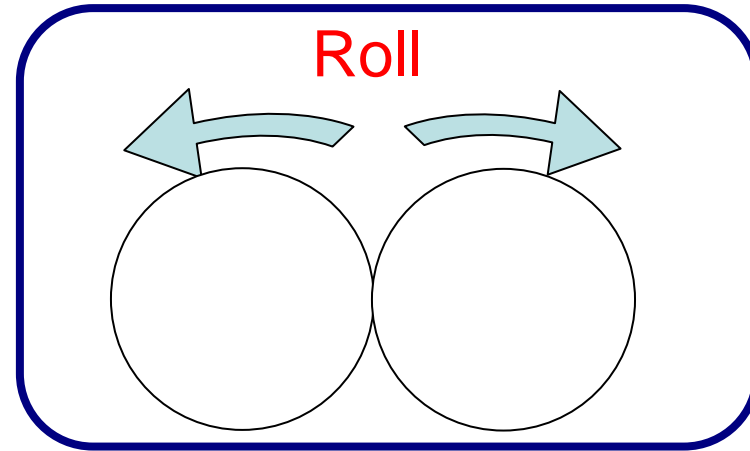
# Interaction model

Johnson et al. (1971)  
Dominik and Tielens (1997)  
Wada et al. (2007)

Tensile

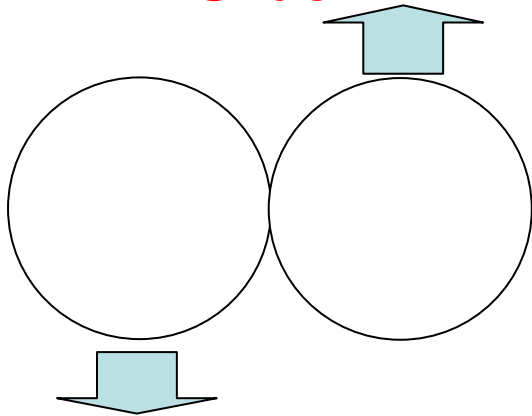


Roll

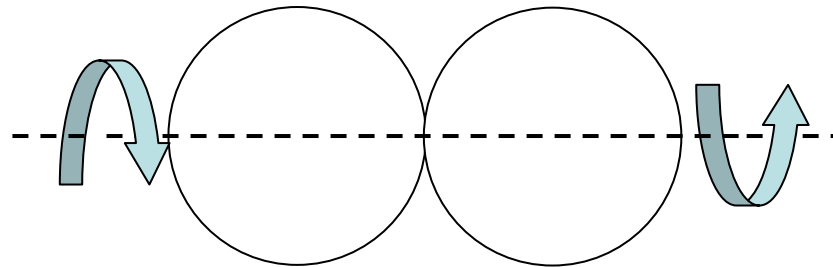


The energy necessary to roll:  $E_{\text{roll}}$

Slide



Twist



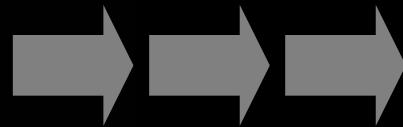
We calculate the force for each degree of freedom.

# N-body simulation of dust collisions

We simulate the **collisional growth** of dust aggregates which consisted of spherical particles (Ice,  $r=0.1\mu\text{m}$ ).

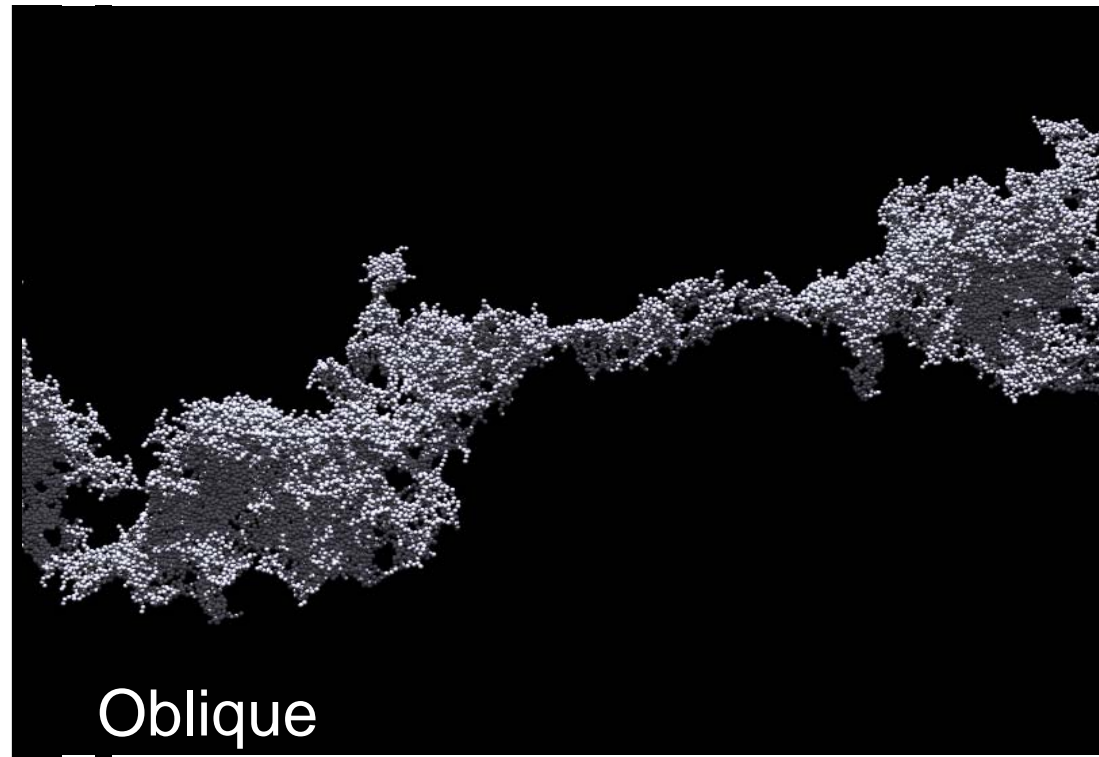
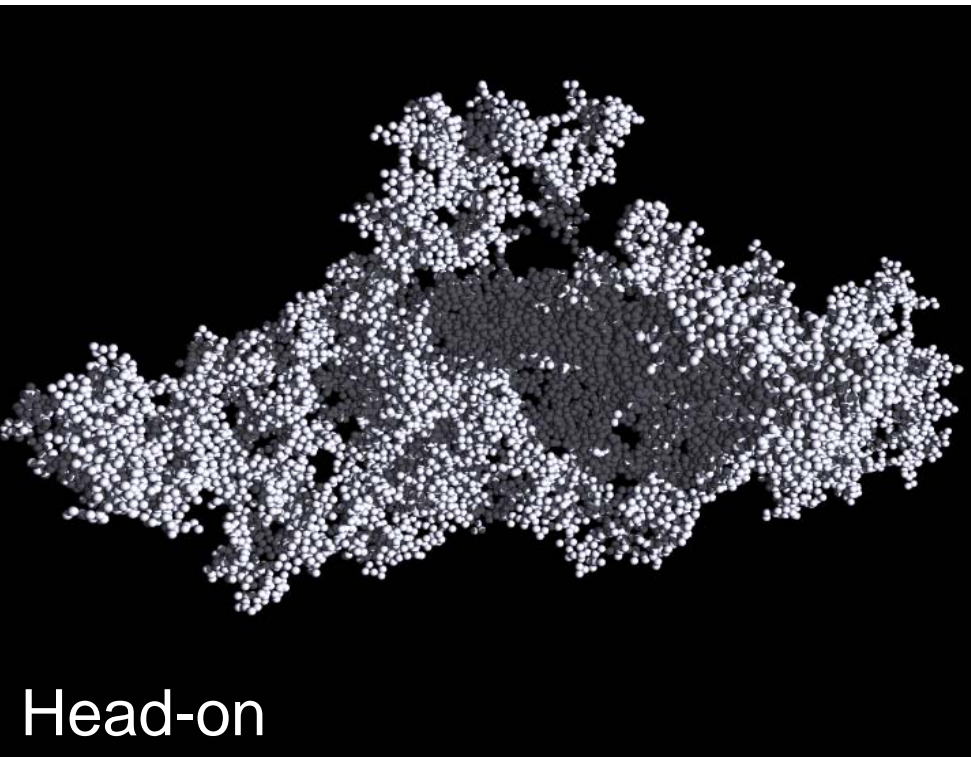
## Method:

- We repeat the collisions between two identical aggregates.  
A sequence of collision: 「 $2 + 2 \Rightarrow 4 + 4 \Rightarrow \dots \Rightarrow 16384 + 16384$ 」
- Oblique collisions (Impact parameter is chosen randomly).
- The impact velocity is constant.
- The estimation of the density:  
We perform the 10 sequences of collisions with each velocity. We obtain the averaged value.



# Effects of oblique collisions

At oblique collisions, dust aggregates are **elongated**.

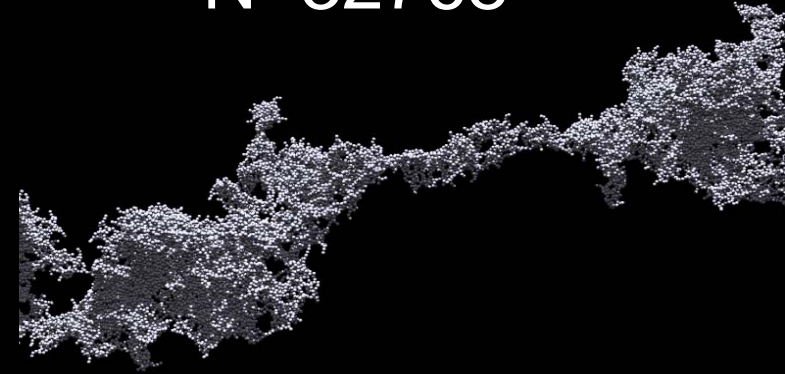
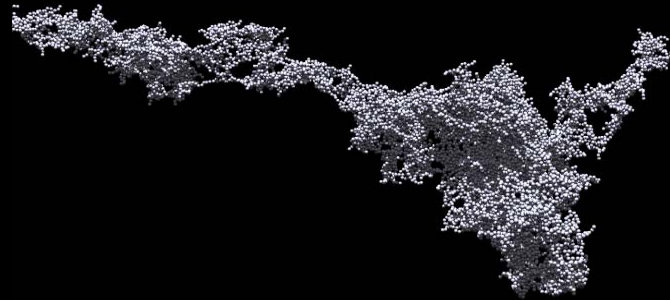


# Resultant aggregates

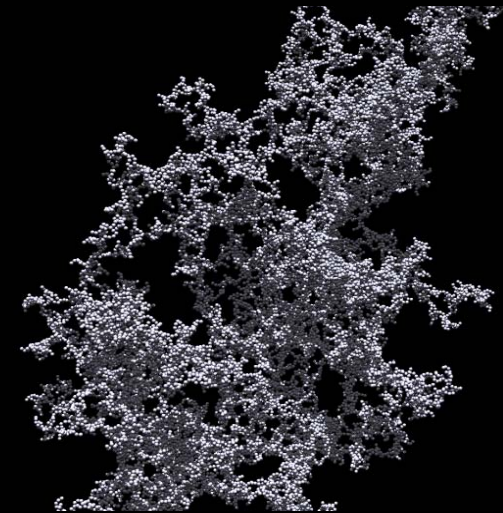
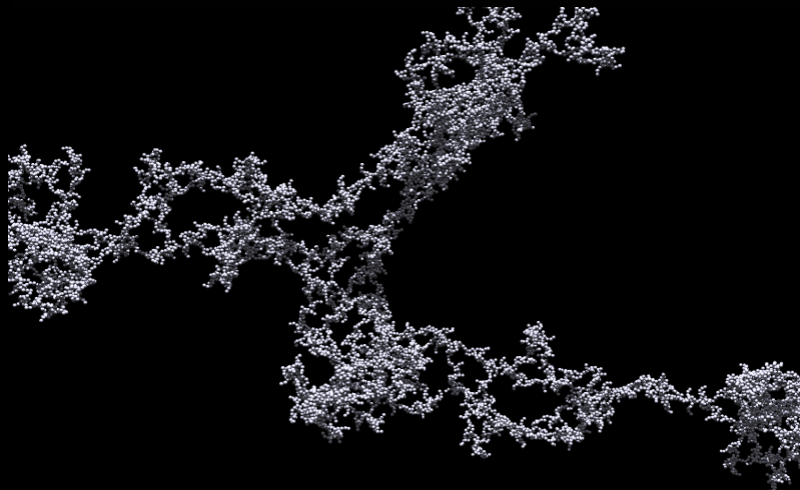
N=16384

N=32768

$V_{\text{imp}}=4.4\text{m/s}$



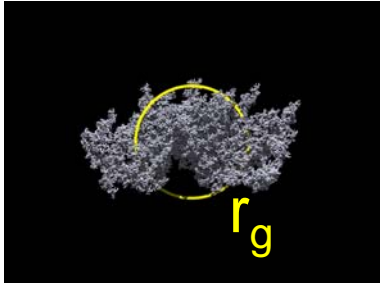
$V_{\text{imp}}=1.1\text{m/s}$



At higher velocity collisions, dust aggregates are **compressed for small scale structure.**

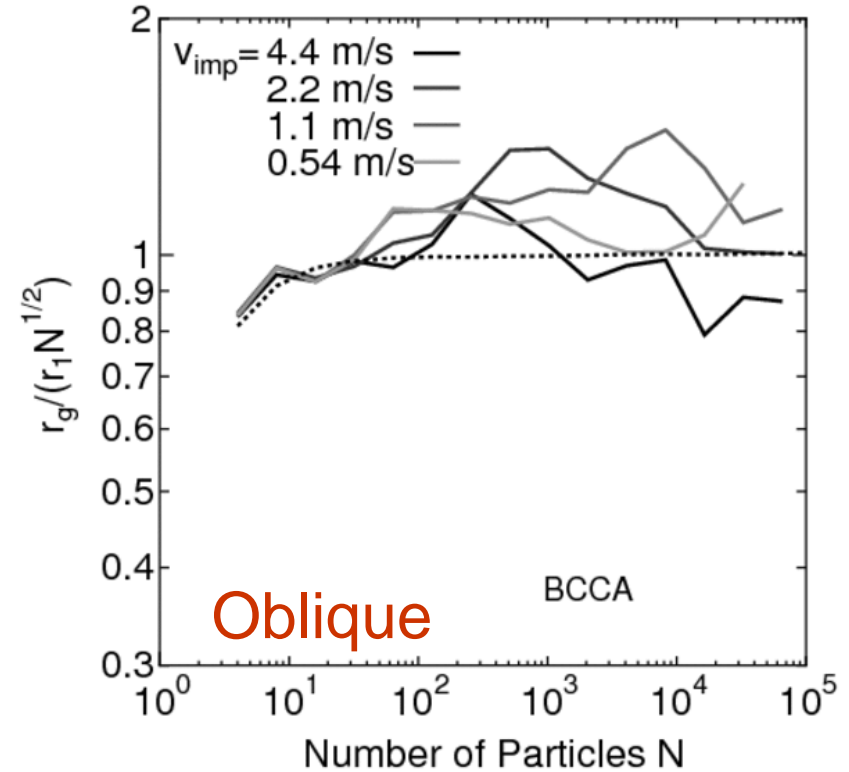
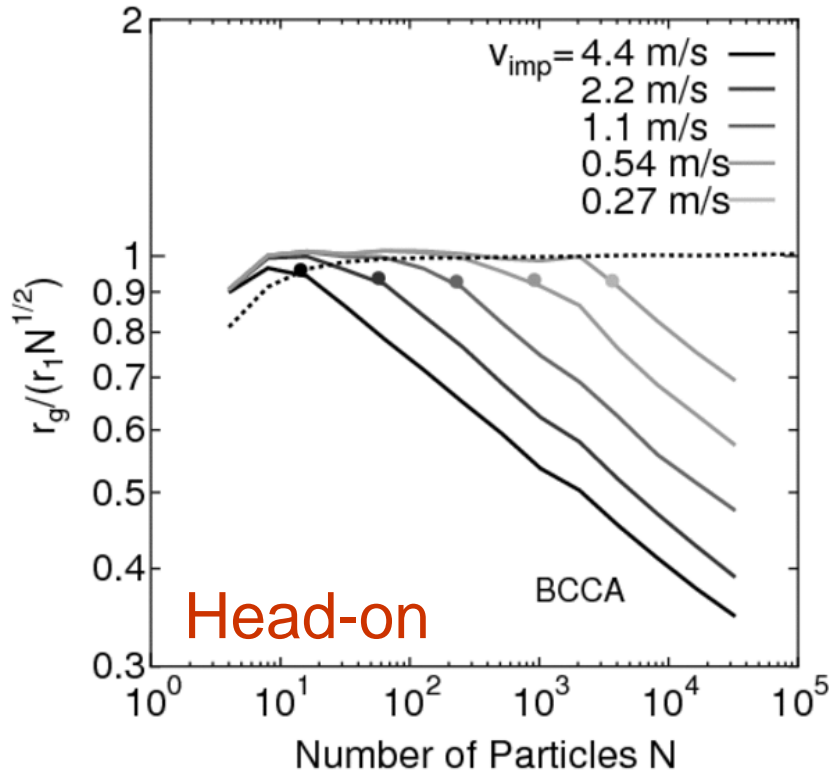
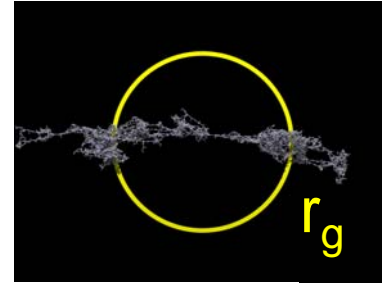


# Evolution of radius of gyration

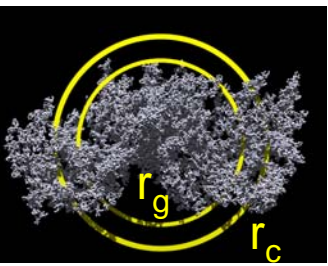


$$r_g = \sqrt{\frac{\sum_{i=1}^N |x_i - x_G|^2}{N}}$$

$x_i$ : the positions of particles  
 $x_G$ : the center of the aggregate



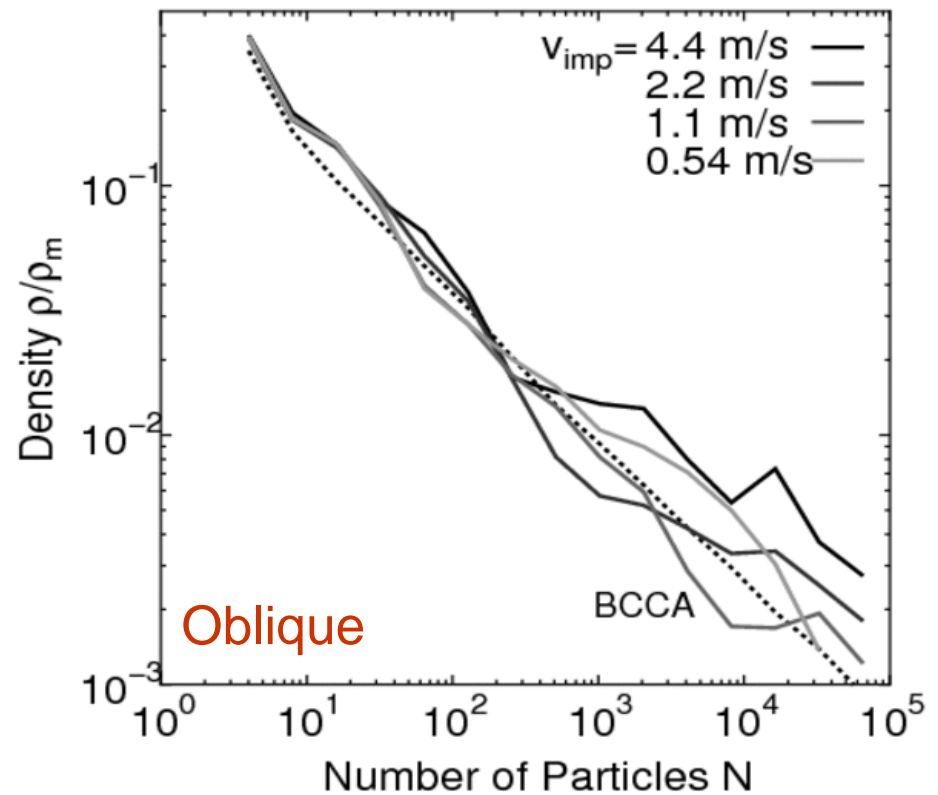
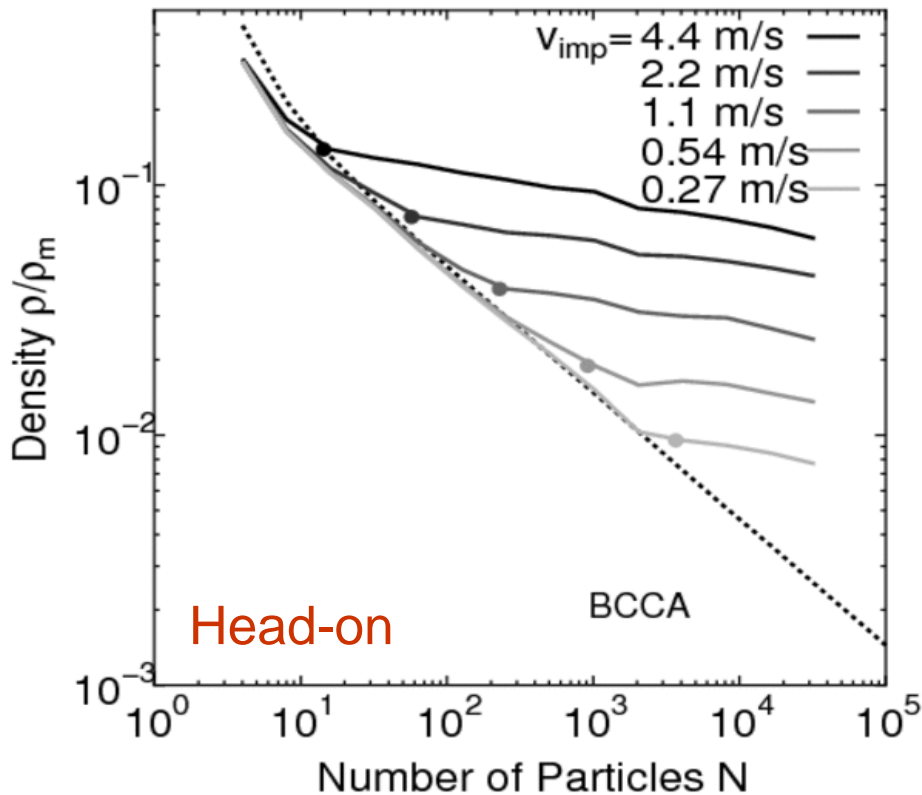
At oblique collisions, aggregates are not compressed because they are elongated.



# Evolution of density $\rho(r_g)$

$$r_c = \sqrt{\frac{5}{3}} r_g$$

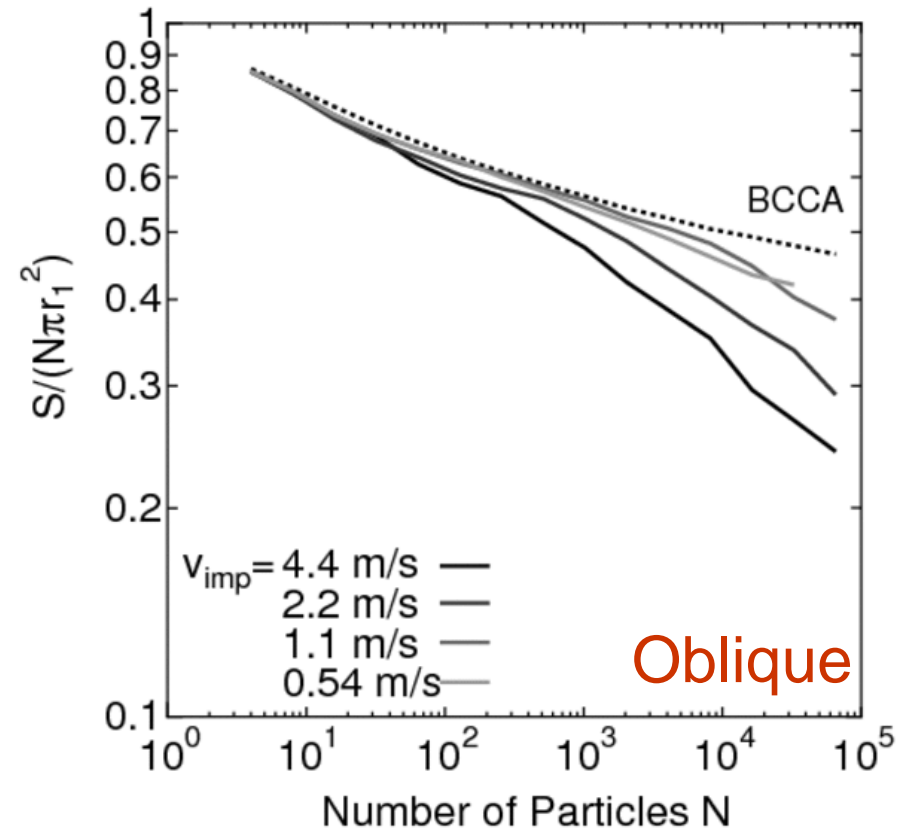
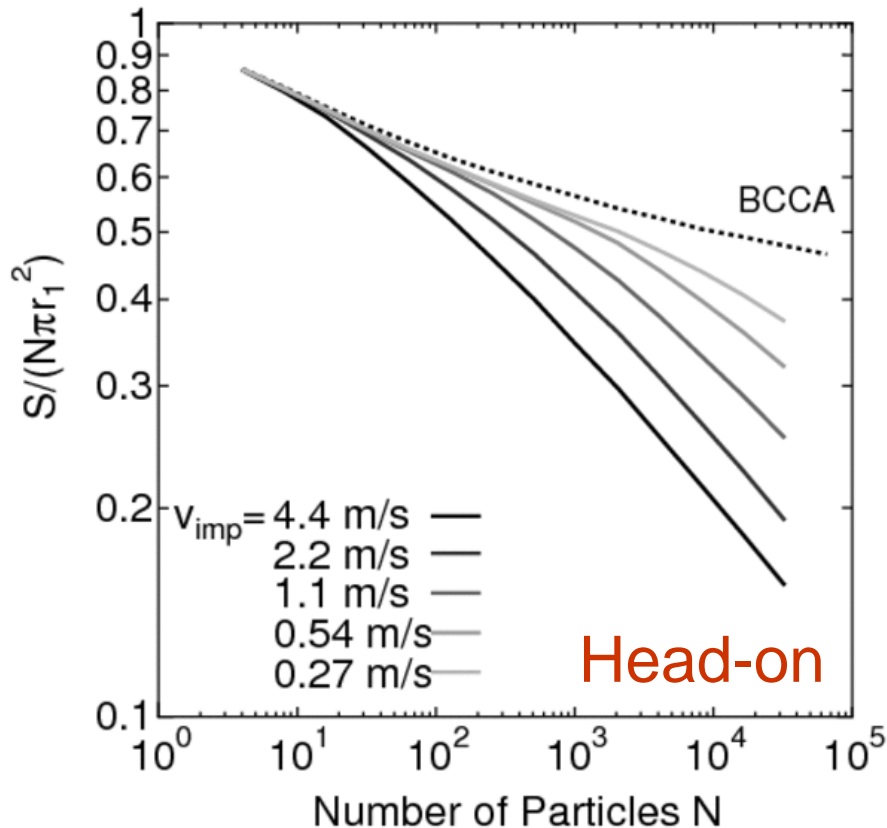
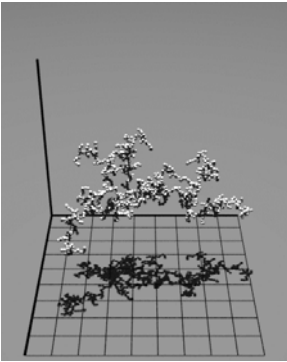
$$\rho(r_g) = \frac{m_1 N}{\frac{4\pi}{3} r_c^3}$$



At oblique collisions, the density decreases as that of BCCA clusters.

# Evolution of cross section

We calculated the cross section of the shadow of dust aggregates (30-directions).



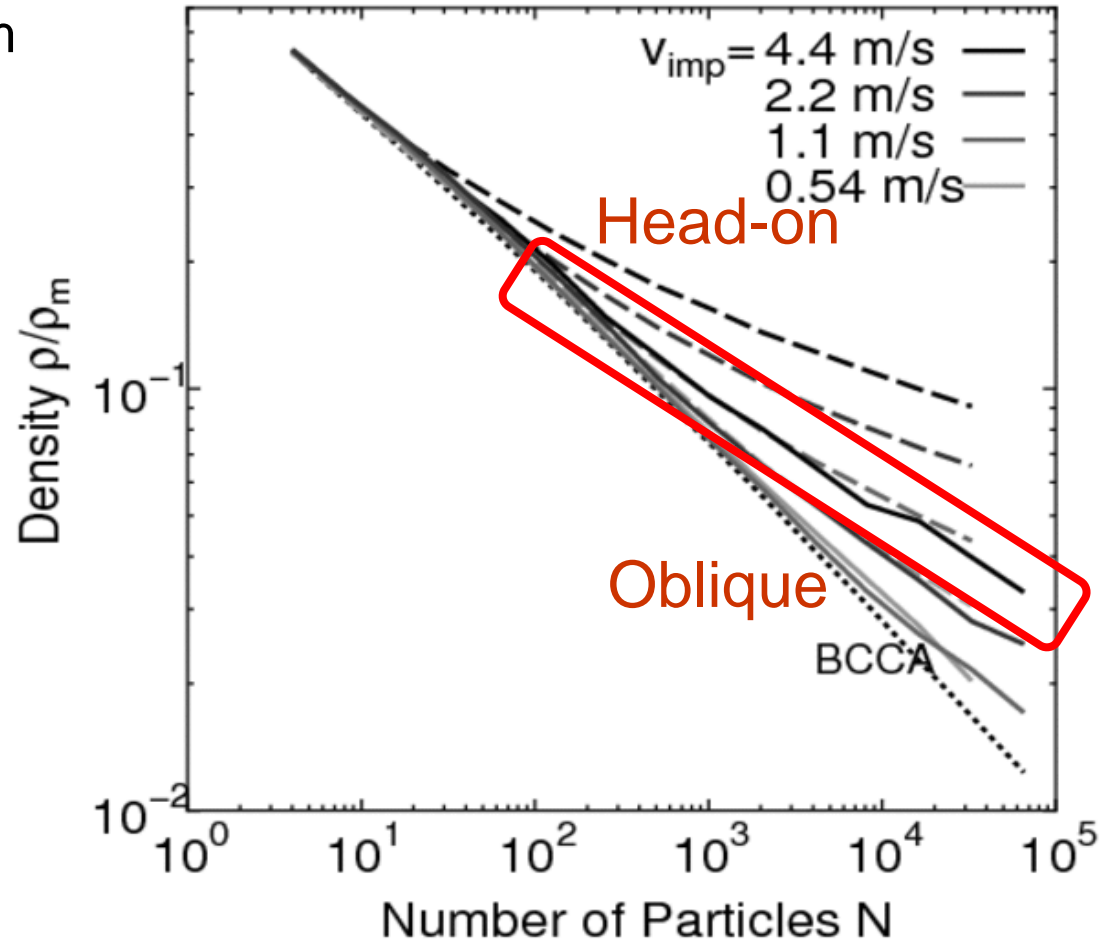
Even at oblique collisions, aggregates are compressed!

# Evolution of density $\rho(S)$

We evaluated the density from  
the cross section.

$$r_s = \sqrt{\frac{S}{\pi}}$$

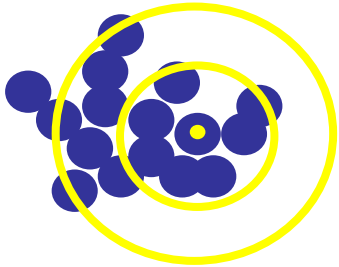
$$\rho(S) = \frac{m_1 N}{\frac{4\pi}{3} r_s^3}$$



High velocity oblique collisions = Low velocity head-on collisions

# Internal structure of compressed dust

We count the number of particles  $N'(r)$  within a distance  $r$ .



Fractal dimension

$$D_f \equiv \frac{d \ln N_{in}(r)}{d \ln r}$$

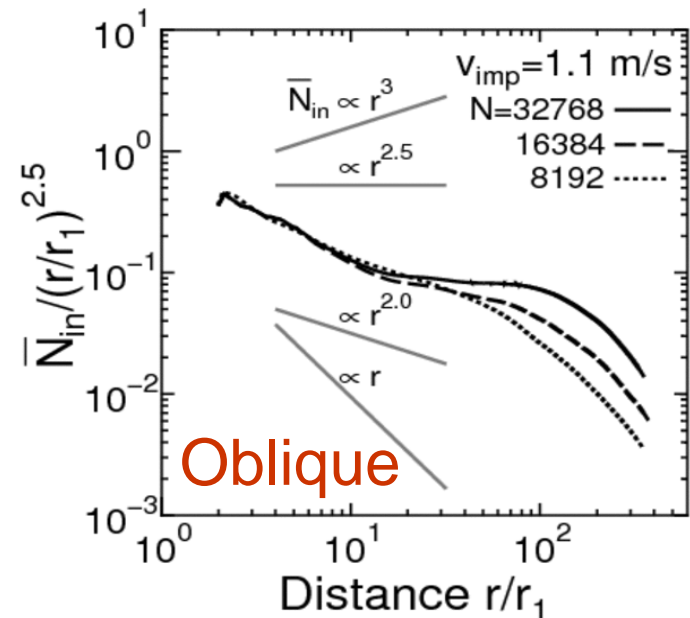
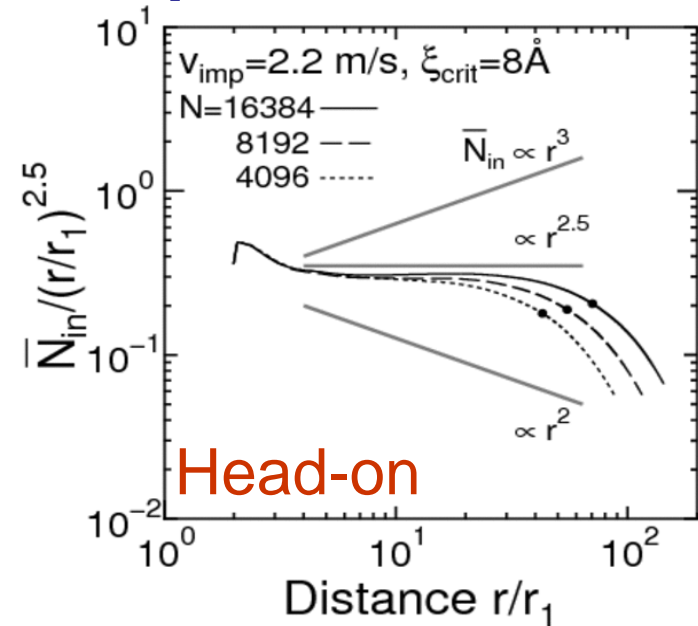
Head-on:  $D_f=2.5$

Oblique:  $D_f=2.5$

Compact:  $D_f=3.0$ , BCCA:  $D_f=2.0$

Fractal dimension is 2.5.

⇒ Even at oblique collisions,  
dust is compressed.



# Compression model: compressive strength

Compression at collisions between BCCA clusters (Wada et al. 2008).

A relation between the volume  $V(r_g)$  and the impact energy  $E_{imp}$ .

$$dE_{imp} = -PdV$$

Compressive strength: 
$$P \equiv -\frac{dE_{imp}}{dV} = 0.5 \frac{E_{roll} \rho_1}{m_1} \left( \frac{\rho(r_g)}{\rho_1} \right)^{\frac{13}{3}} N^{\frac{2}{3}}$$

We obtain the volume change at a collision with  $E_{imp}$ .

# Compression model: $\rho(r_g)$ at head-on

$$E_{imp} = - \int_{V_{initial}}^{V_{final}} P dV$$

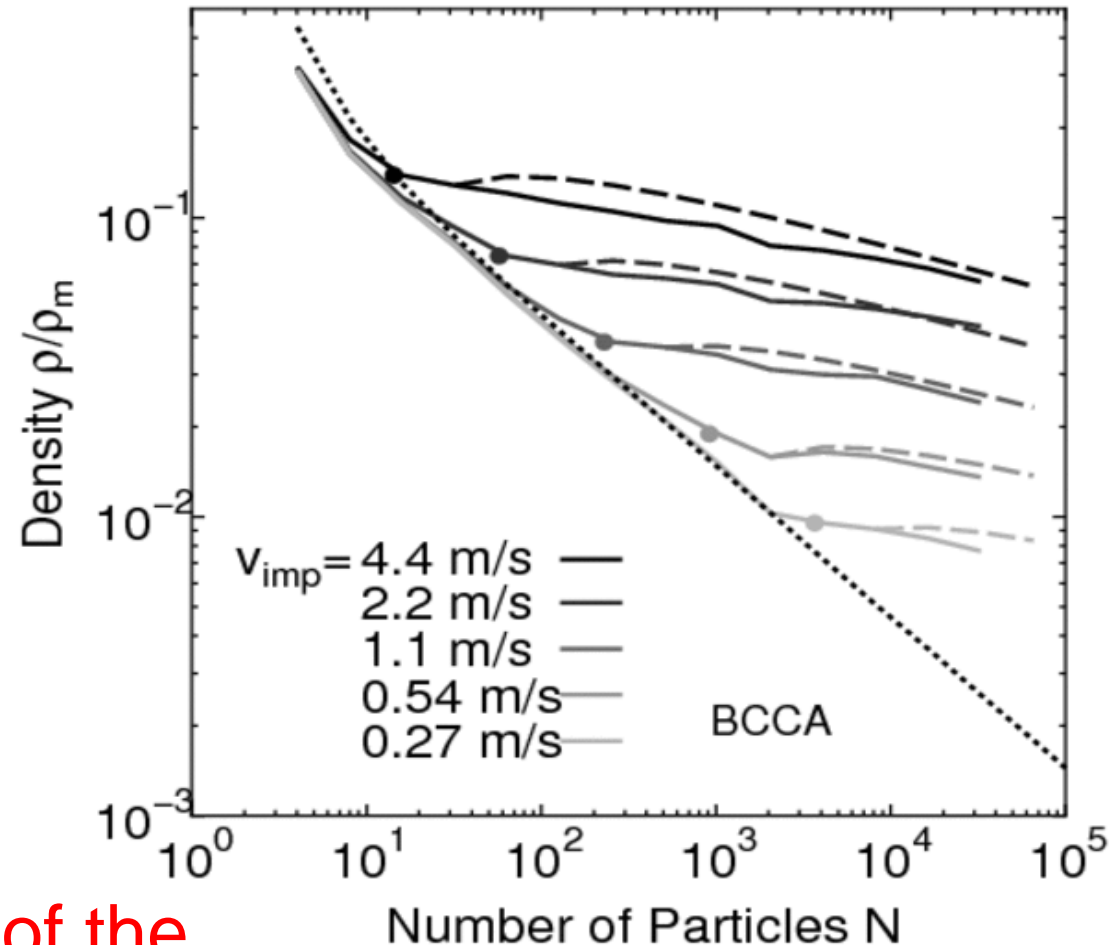


$$\frac{d \ln \rho(r_g)}{d \ln N} = -\frac{1}{5}$$

$$+ 0.23 \left( \frac{\rho(r_g)}{\rho_m} \right)^{-\frac{10}{3}} \left( \frac{M}{m_1} \right)^{-\frac{2}{3}} \frac{E_{imp}}{NE_{roll}}$$

$$r_g = \sqrt{\frac{3}{5}} \left( \frac{m_1 N}{\frac{4\pi}{3} \rho(r_g)} \right)^{1/3}$$

We obtain the evolution of the density and the radius of gyration for given impact velocity.

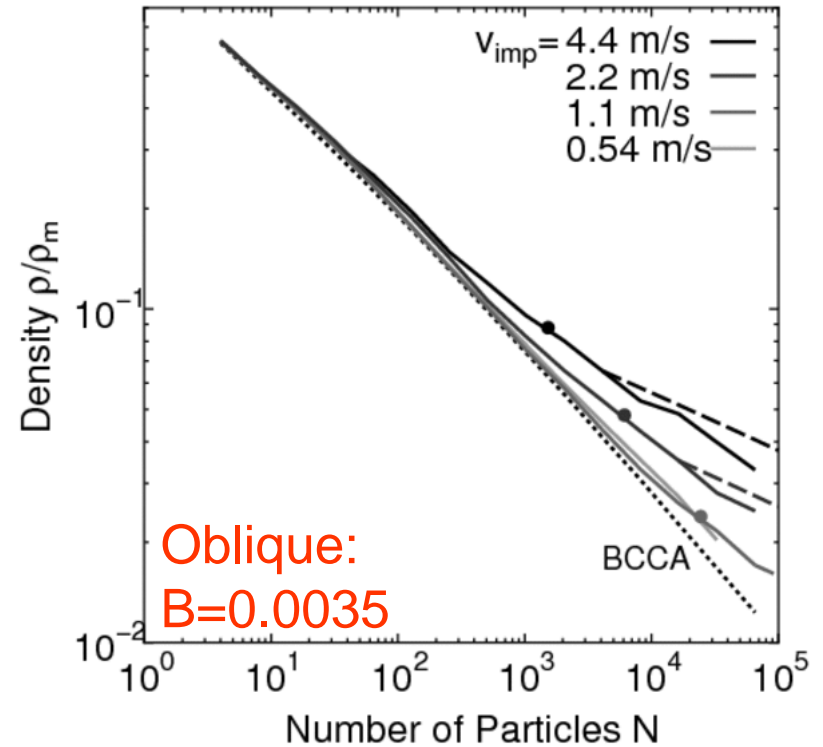
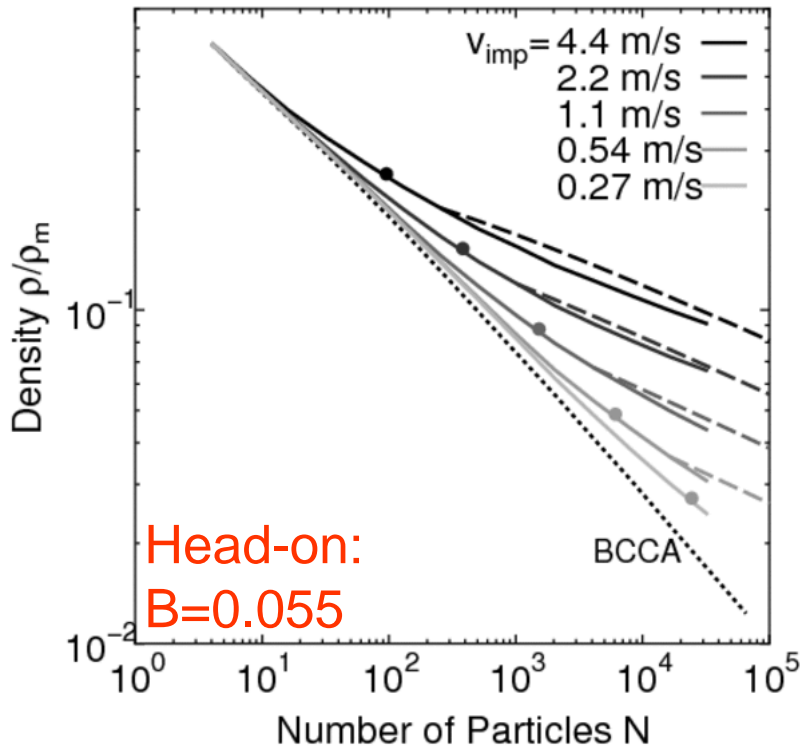


# Compression model: $\rho(S)$

We apply density evolution model about  $\rho(r_g)$  to  $\rho(S)$ .

$$\frac{d \ln \rho(S)}{d \ln N} = -\frac{1}{5} + B \left( \frac{\rho(S)}{\rho_m} \right)^{-\frac{10}{3}} \left( \frac{M}{m_1} \right)^{-\frac{2}{3}} \frac{E_{\text{imp}}}{NE_{\text{roll}}}$$

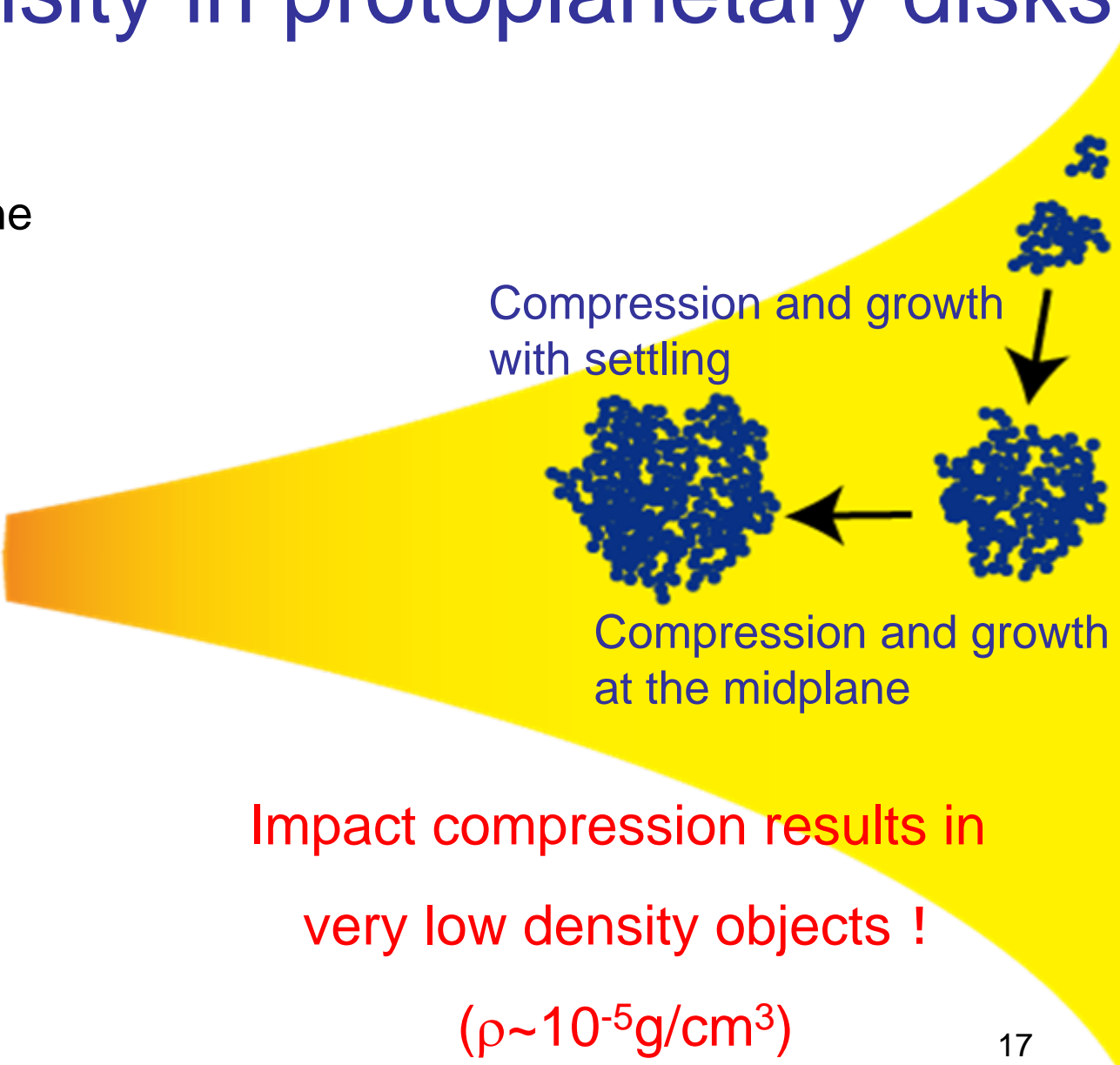
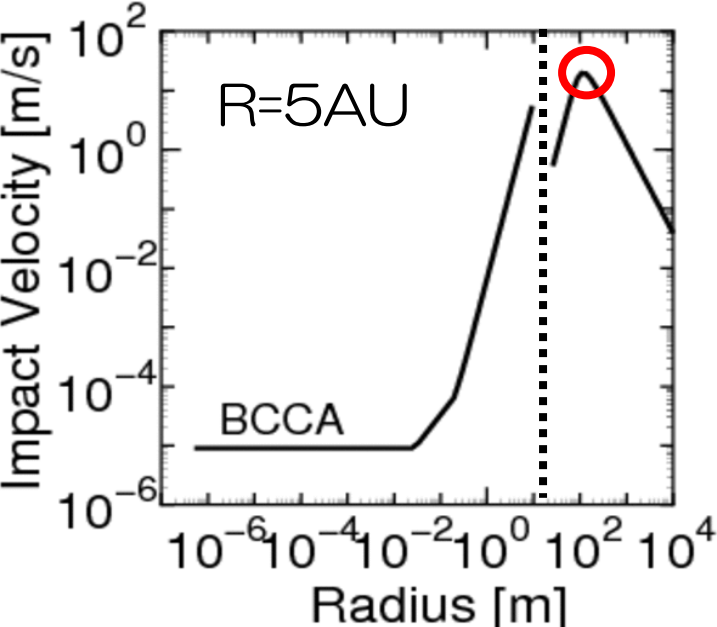
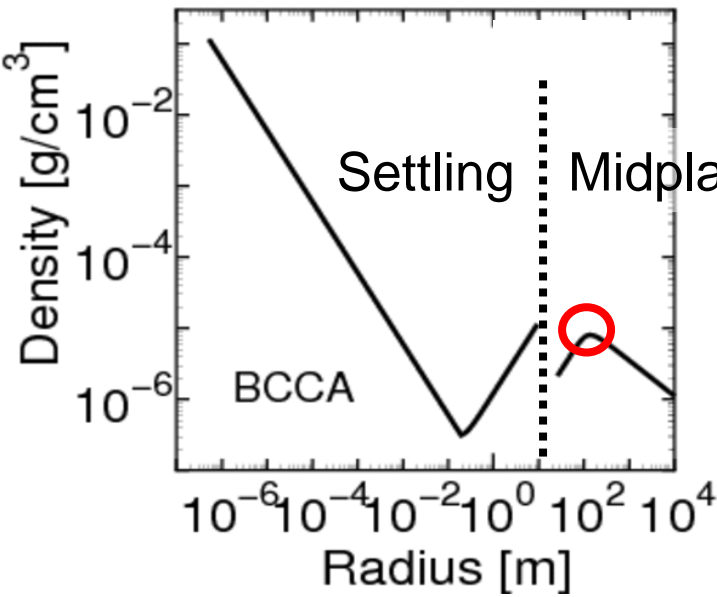
$$S = \pi \left( \frac{m_0 N}{\frac{4\pi}{3} \rho(S)} \right)^{2/3}$$



We obtain the evolution of the density and the cross section.



# Evolution of density in protoplanetary disks



# Summary

- We performed the simulation of **oblique collisions** between dust aggregates and examined the compression process.
- For **the radius of gyration**, aggregates are **not compressed**.
- For **the cross section**, aggregates are **compressed**.
- We formulated the evolution of the density and the cross section.
- Impact compression results in **very low density objects**. ( $\rho \sim 10^{-5} \text{g/cm}^3$ )