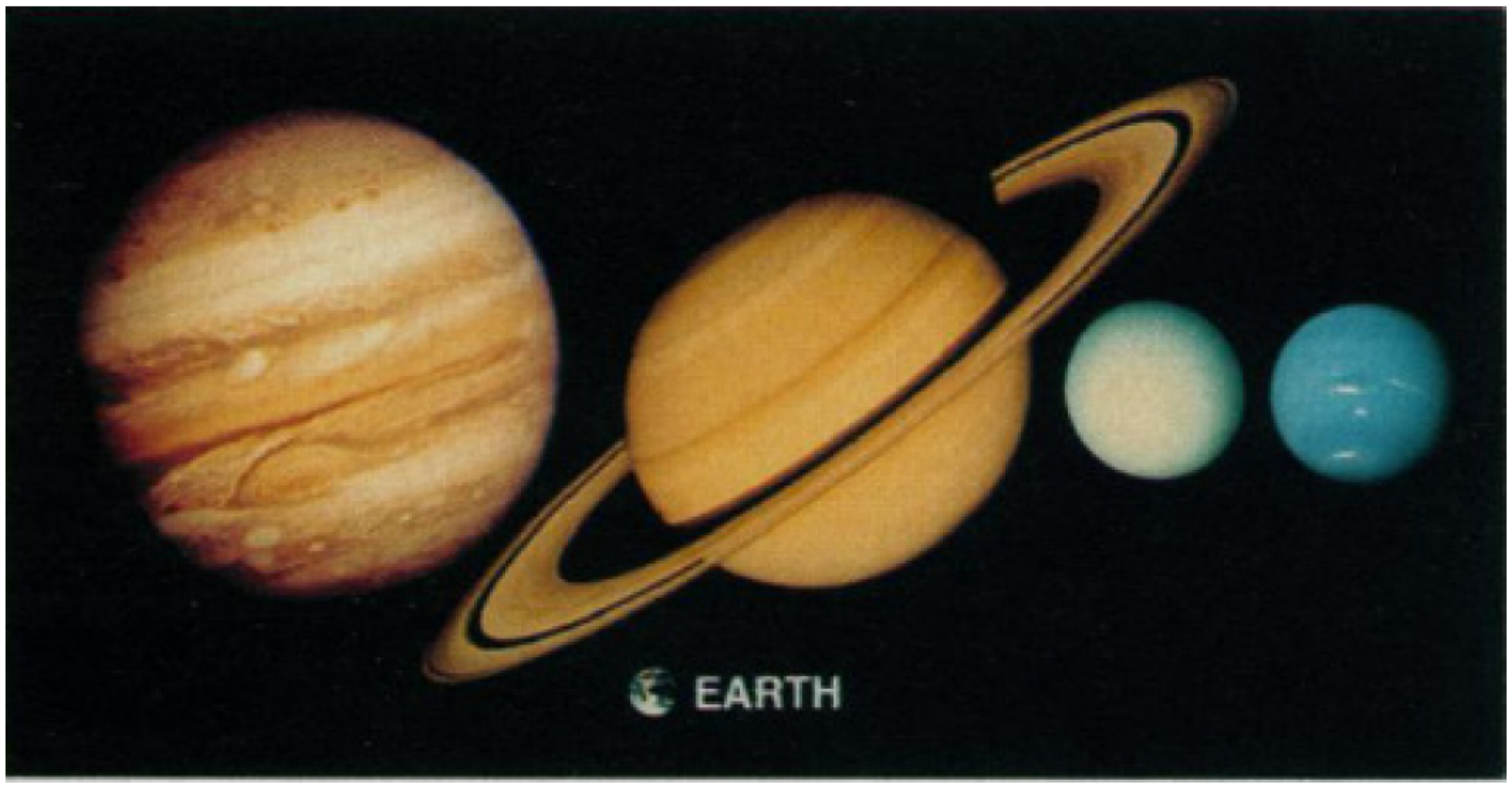
# Angular momentum balance in planetary atmospheres




Tapio Schneider

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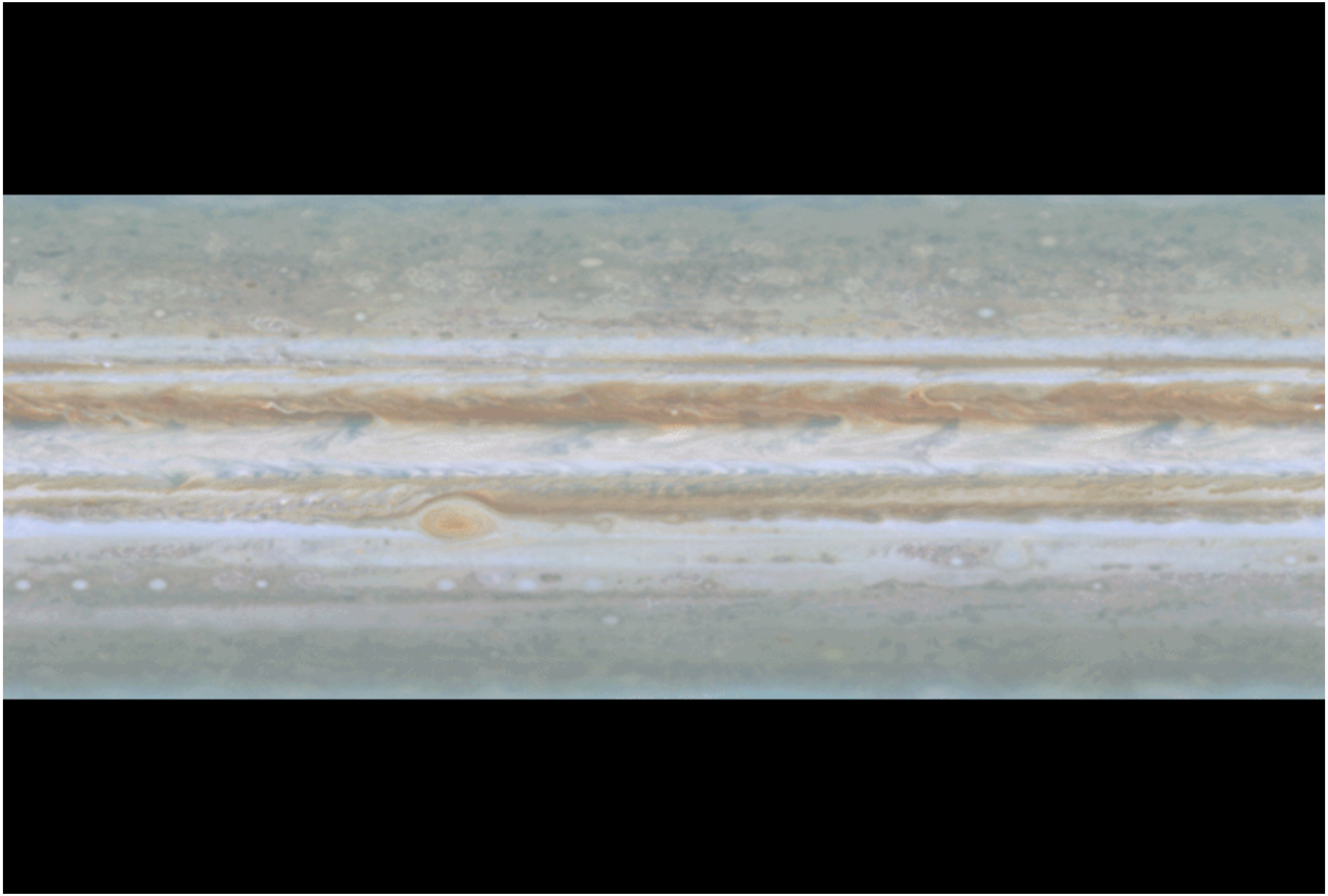
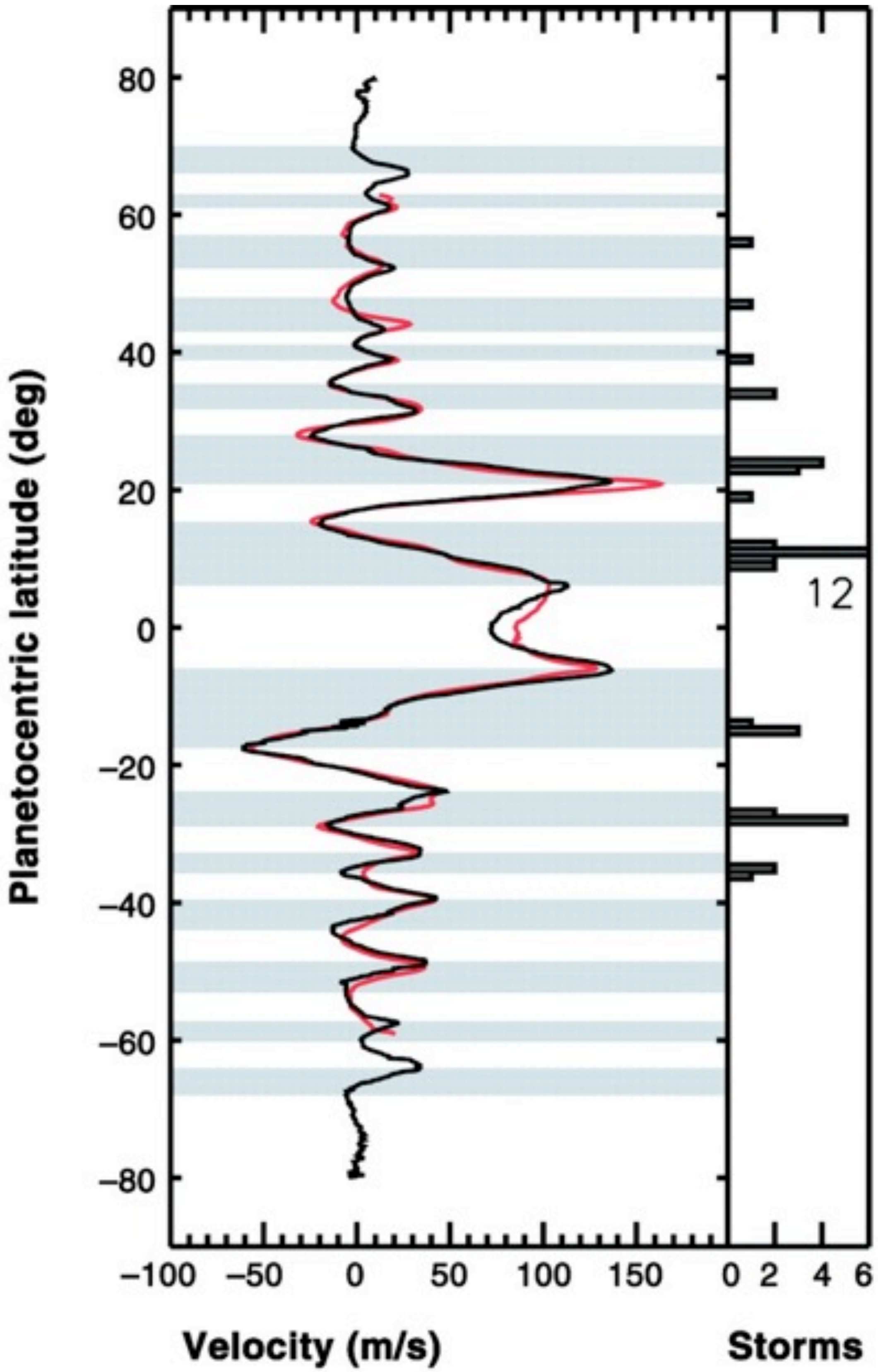
# Jupiter



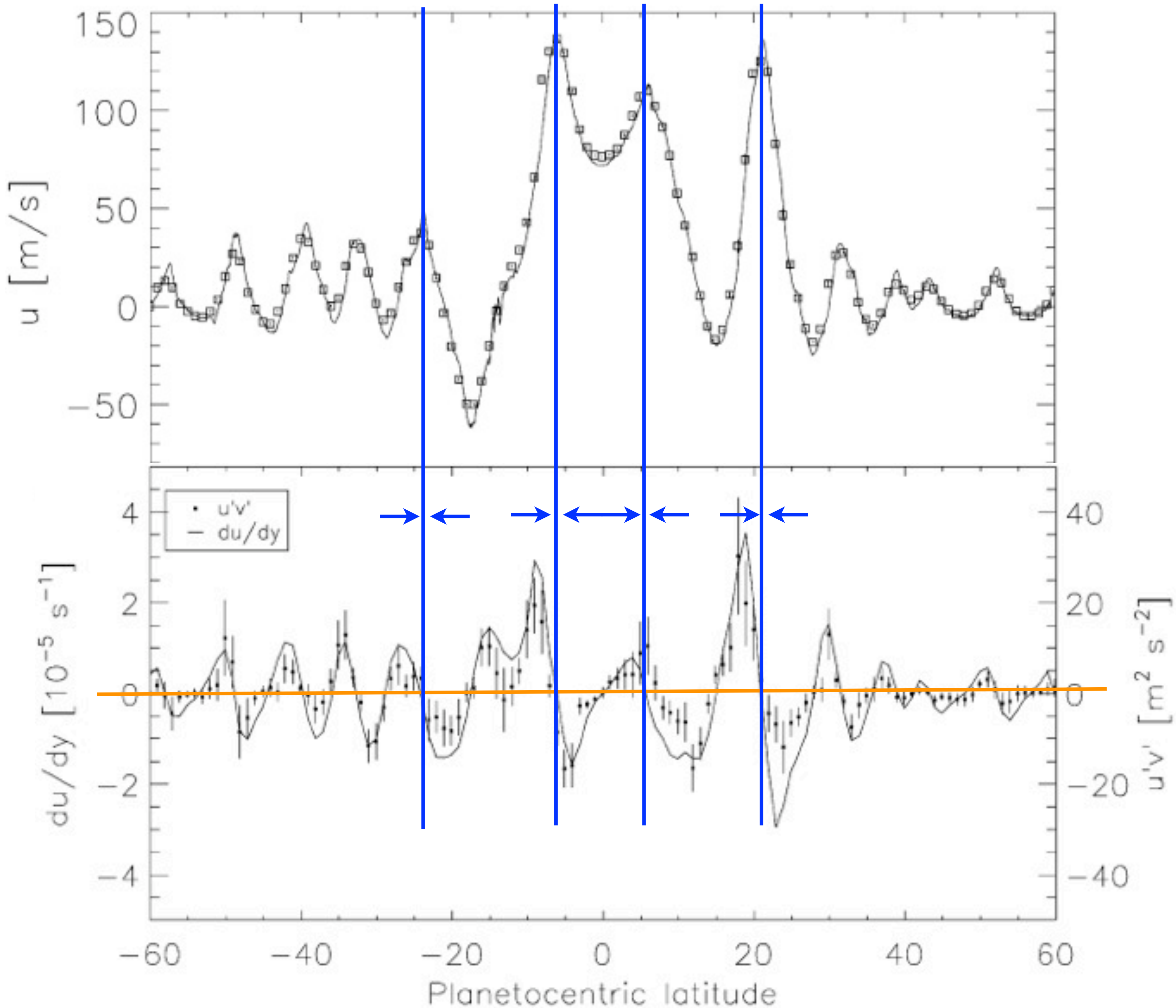
(Cassini Imaging Team 2000)

# Jupiter zonal wind




(Porco et al. 2003)

# Eddy angular momentum flux on Jupiter




(Salyk et al. 2006)

- *Eddies transport angular momentum from retrograde into prograde jets on Jupiter (and on Earth).*

- *Why? Is this a general property of planetary flows? What does it imply about the circulation?*

# Angular momentum balance I

Angular momentum about planet's spin axis:

$$M = M_\Omega + M_u = \Omega r_\perp^2 + u r_\perp$$

$$r_\perp = r\cos\phi$$

Balance equation in pressure coordinates:

$$\partial_t M + \nabla_p \cdot (\mathbf{u} M) = -\partial_\lambda \Phi + r_\perp D$$

Averaged zonally (azimuthally) along isobars:

$$\partial_t \bar{M} + \boldsymbol{\nabla}_p \cdot \left(\overline{\mathbf{u} M}\right) = \overline{r_\perp D}$$

In statistically steady state, separating mean and eddies:

$$\bar{\mathbf{u}} \cdot \boldsymbol{\nabla}_p M_\Omega + \boldsymbol{\nabla}_p \cdot \left(\bar{\mathbf{u}} \bar{M}_u\right) = -\boldsymbol{\nabla}_p \cdot \left(\overline{\mathbf{u}' M_u'}\right) + \overline{r_\perp D}$$

# Angular momentum balance II

In thin-shell approximation,

$$r_\perp = a\cos\phi, \qquad a = \text{const}$$

$$\bar{\mathbf{u}} \cdot \boldsymbol{\nabla}_p M_\Omega = -r_\perp f v, \qquad f = 2\Omega \sin\phi$$

Ratio of mean advection of relative to planetary AM:

$$\frac{\bar{\mathbf{u}} \cdot \boldsymbol{\nabla}_p \bar{M}_u}{\bar{\mathbf{u}} \cdot \boldsymbol{\nabla}_p M_\Omega} \sim \frac{U}{fL} \equiv \text{Ro}$$

For $\text{Ro} \ll 1$ , AM balance becomes

$$\bar{\mathbf{u}} \cdot \boldsymbol{\nabla}_p M_\Omega = -\boldsymbol{\nabla}_p \cdot \left(\overline{\mathbf{u}' M_u'}\right) + \overline{r_\perp D}$$

# Angular momentum and meridional circulation

In free atmosphere, *D=0*,

$$\bar{\mathbf{u}} \cdot \nabla_p M_\Omega = -\nabla_p \cdot (\overline{\mathbf{u}' M_u'})$$

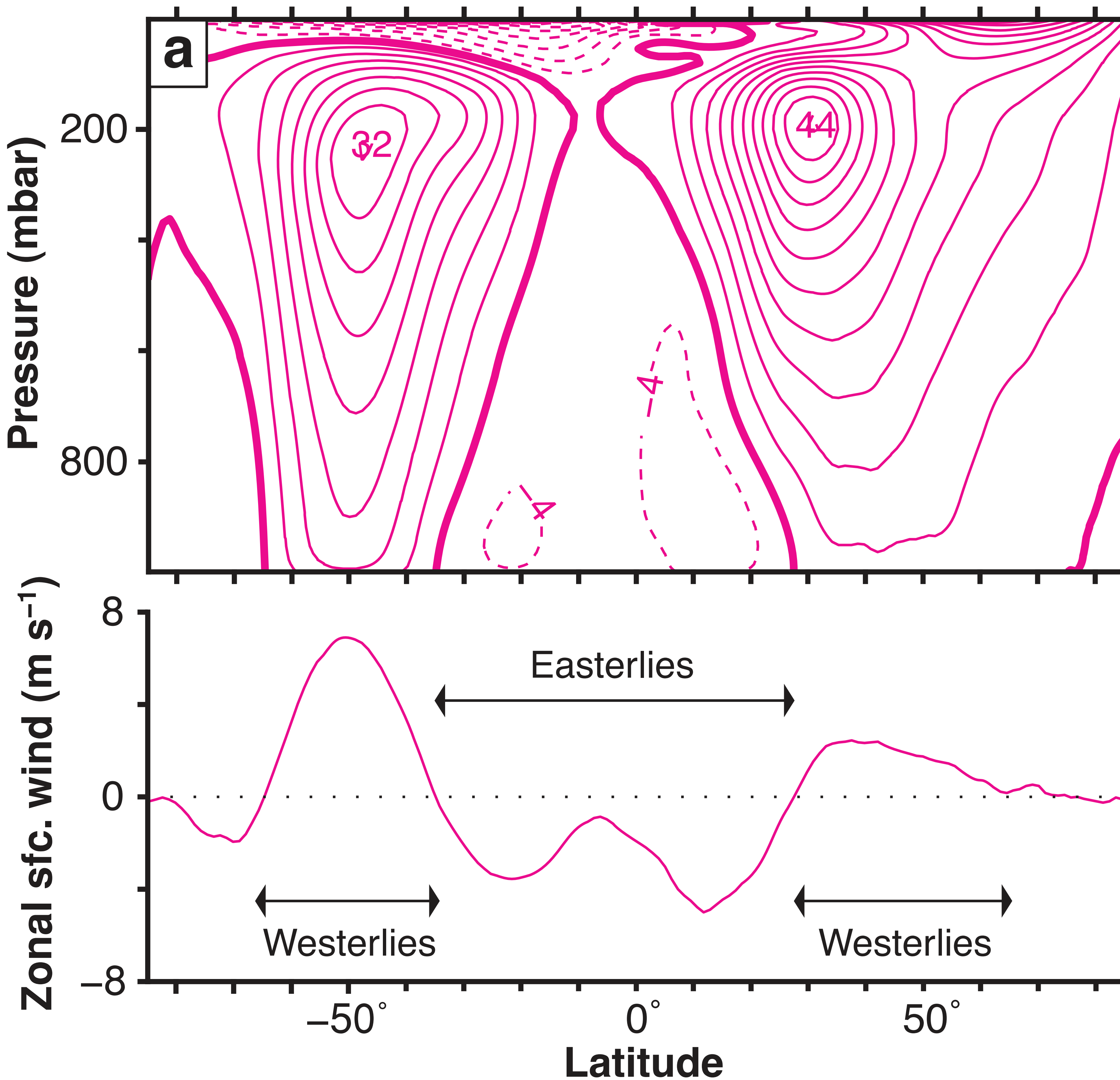
- Flow *toward* spin axis where there is eddy AM flux divergence: $\nabla_p \cdot (\overline{\mathbf{u}' M_u'}) > 0$
- Flow *away from* spin axis where there is eddy AM flux convergence: $\nabla_p \cdot (\overline{\mathbf{u}' M_u'}) < 0$

Where drag dominates (Ekman balance), $\nabla_p \cdot (\overline{\mathbf{u}' M_u'}) \approx 0$

$$\bar{\mathbf{u}} \cdot \nabla_p M_\Omega = r_\perp \bar{D}$$

- Flow *toward/away from* spin axis where there is prograde/retrograde zonal flow if $\mathrm{sgn}(D) = -\mathrm{sgn}(\bar{u})$

# Eastward wind (January)




(Schneider, *Ann. Rev. Earth Planet. Sci.*, 2006)

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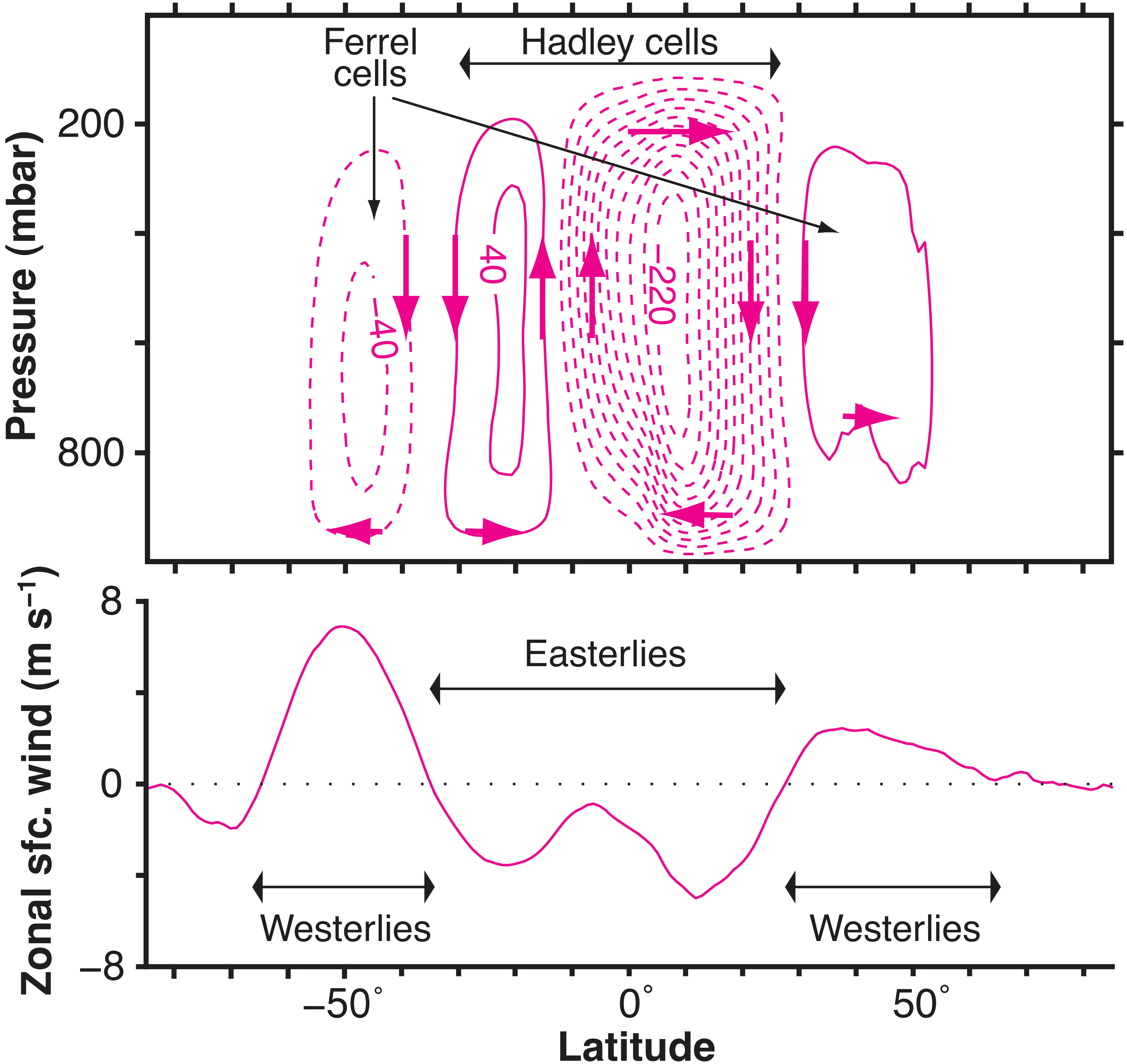


(Schneider, *Ann. Rev. Earth Planet. Sci.*, 2006)

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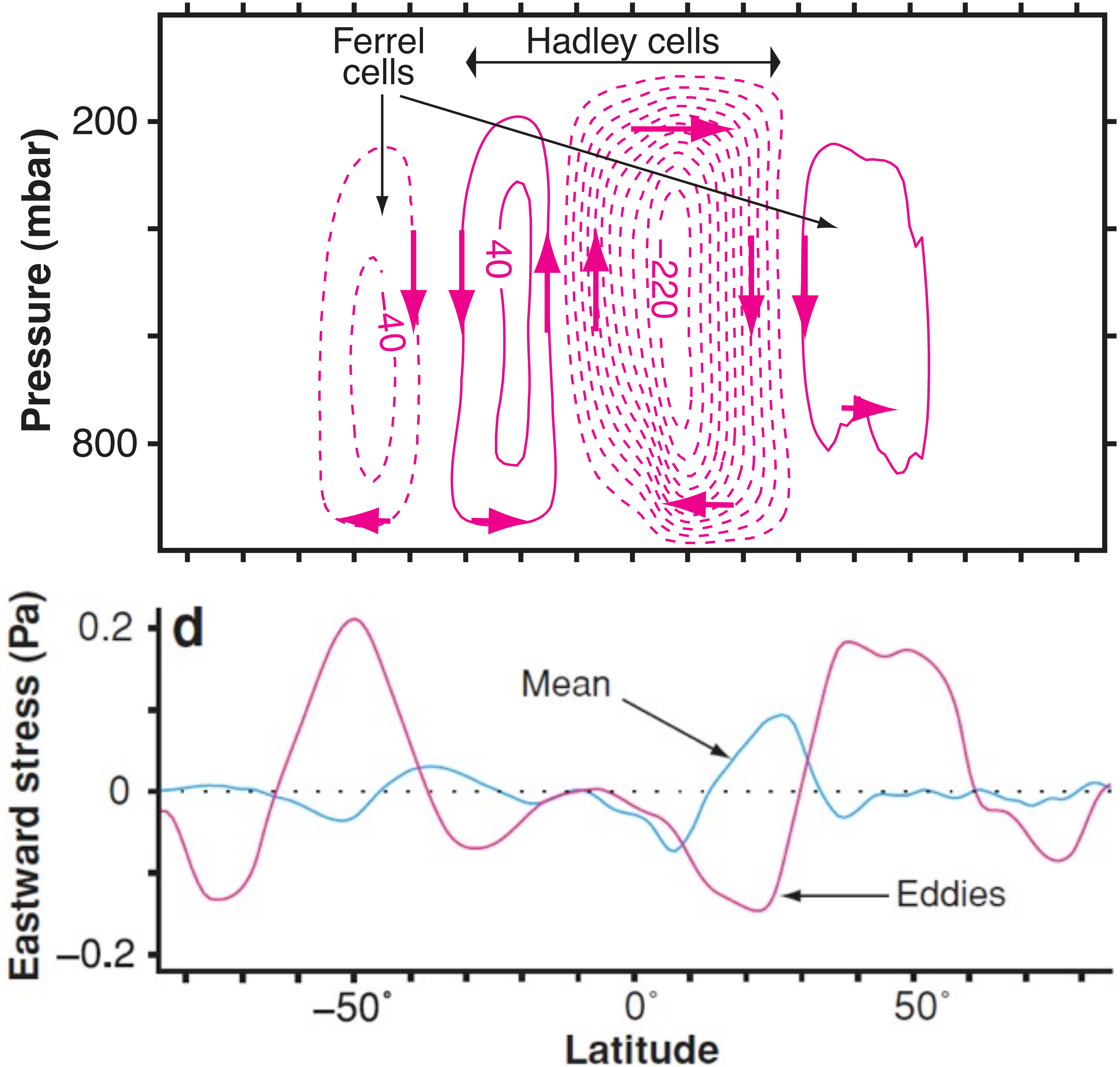
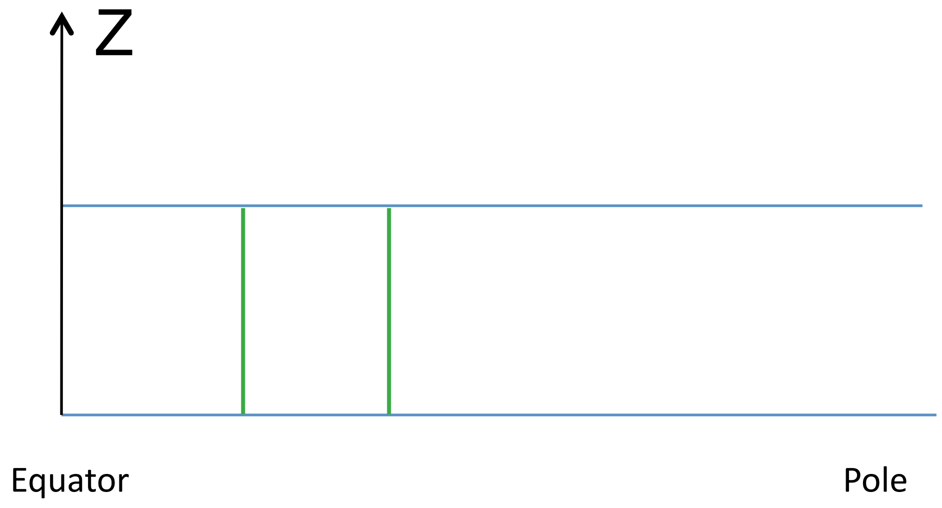



(Schneider, *Ann. Rev. Earth Planet. Sci.*, 2006)

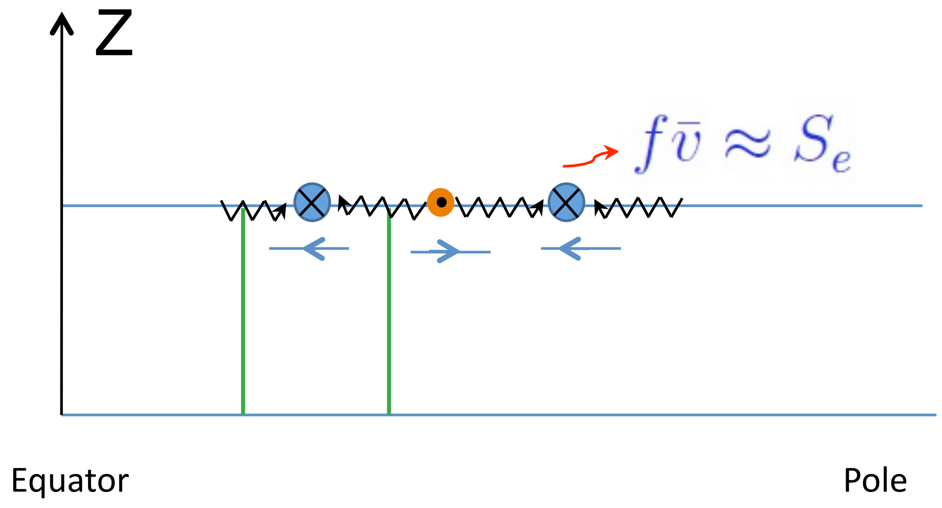
# Angular momentum in thin atmospheres




Away from equator, $\mathrm{Ro} \ll 1$ :

$$f\bar{v} \approx r_\perp^{-1} \boldsymbol{\nabla}_p \cdot \left(\overline{\mathbf{u}' M_u'}\right) - \bar{D}$$

# Angular momentum in thin atmospheres

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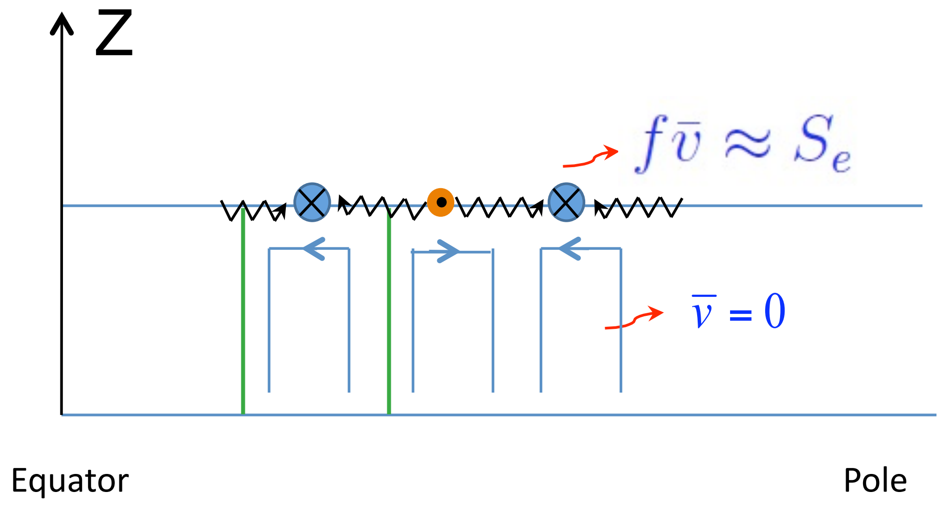
# Angular momentum in thin atmospheres

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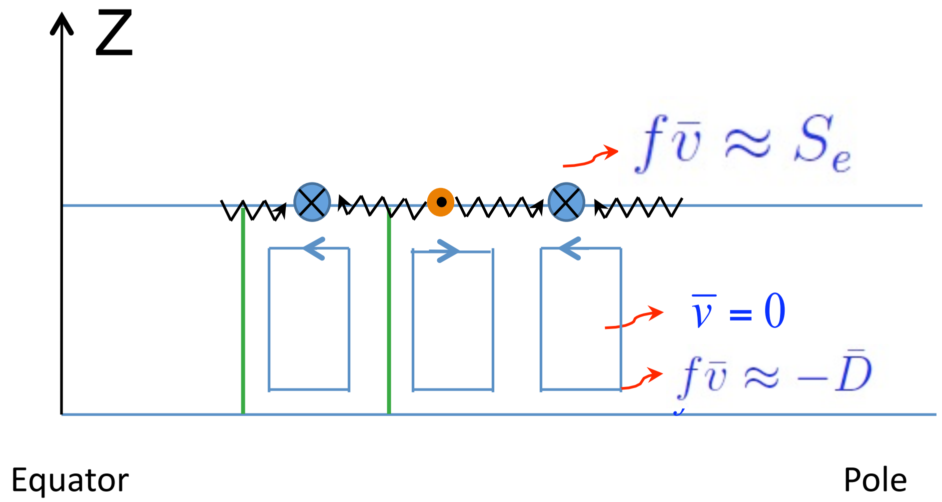
Where eddy AM flux convergence vanishes, no meridional flow (across AM contours): $\bar{v} \approx 0$

# Angular momentum in thin atmospheres

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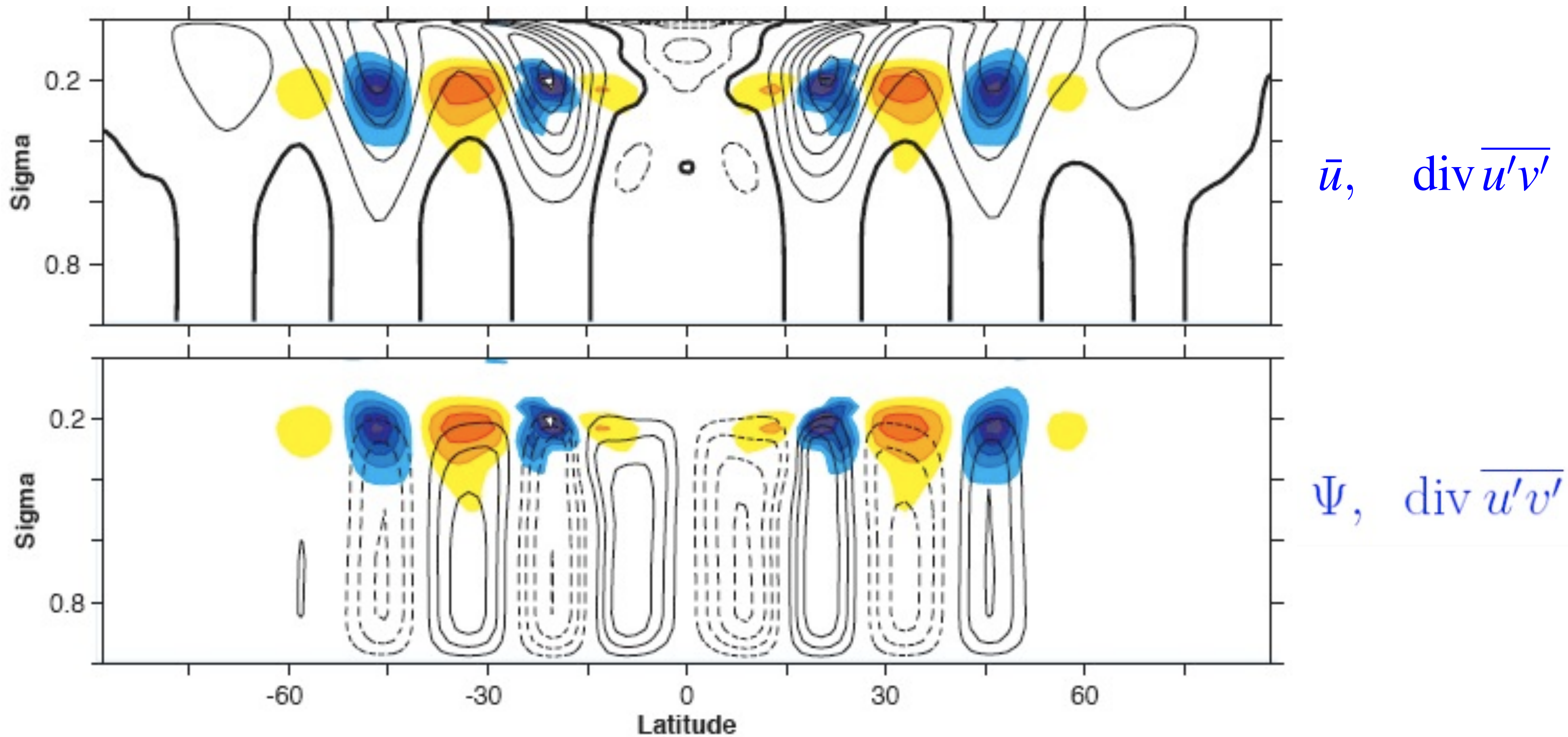
In free atmosphere, eddy AM flux convergence balances Coriolis torque on mean meridional flow: $f\bar{v} \approx S_e$

Where eddy AM flux convergence vanishes, no meridional flow (across AM contours): $\bar{v} \approx 0$

Circulation must close where there is drag (or opposite AM flux convergence): $f\bar{v} \approx -\bar{D}$

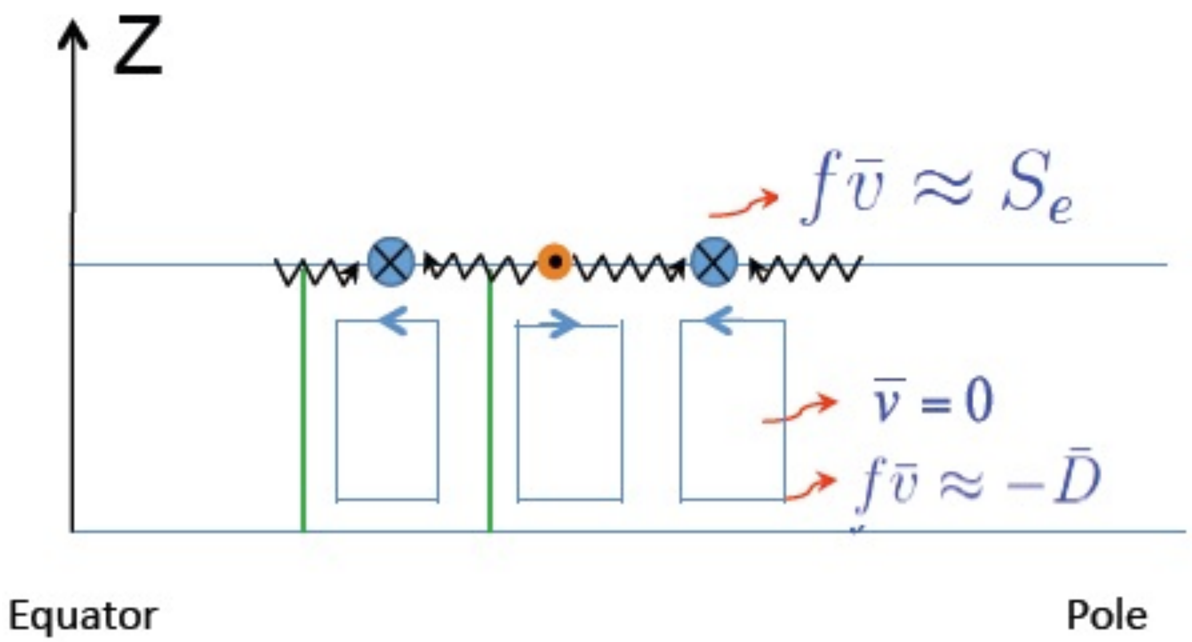
# AM fluxes and mean meridional circulations




*Ageostrophic circulation transmits momentum downward, along angular momentum contours, to wherever dissipation occurs*

(Haynes et al. 1991; O'Gorman & Schneider 2008)

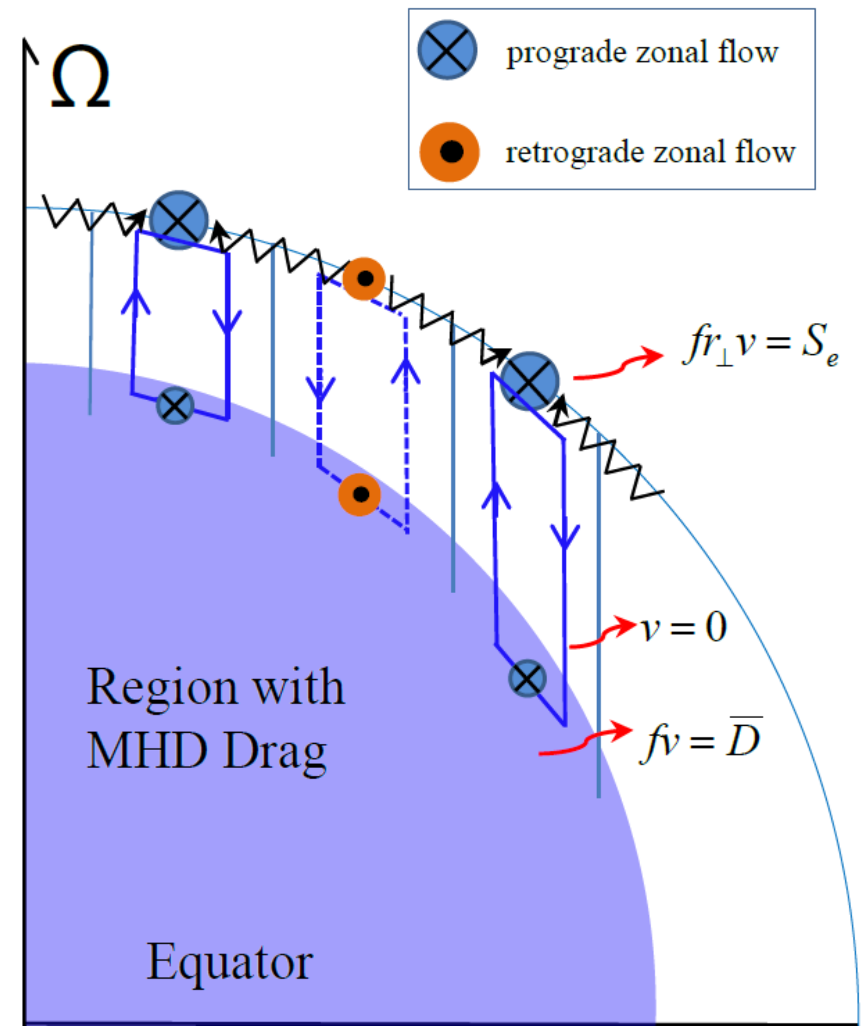
# Angular momentum in deep atmospheres




Thin atmosphere

$$r_\perp = a \cos\phi, \qquad a = \text{const}$$

Planetary AM surfaces: vertical




Deep atmosphere

$$r_\perp = r \cos\phi$$

Planetary AM surfaces: cylinders

# Integrated angular momentum balance

Integrate AM balance for small Rossby number

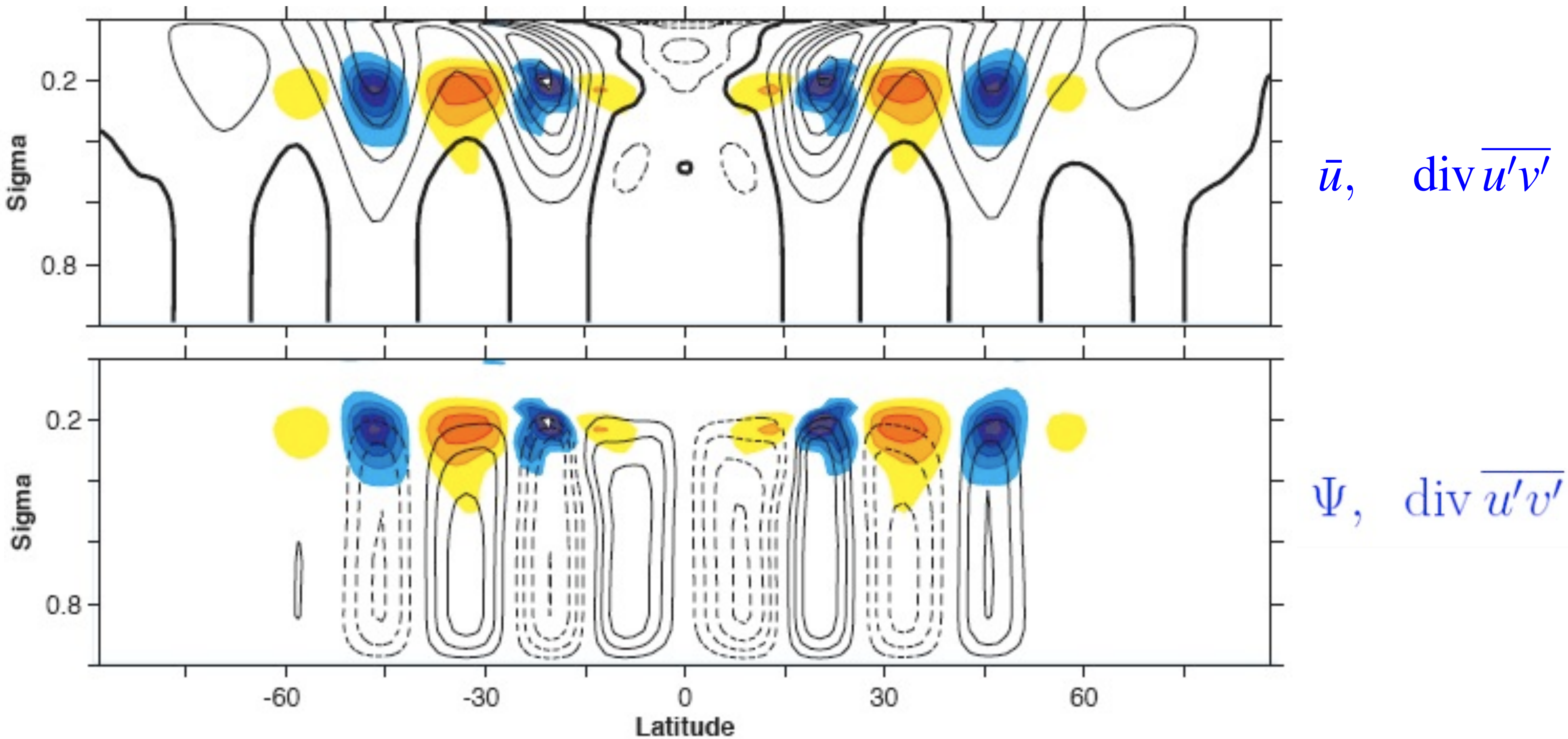
$$\bar{\mathbf{u}} \cdot \boldsymbol{\nabla}_p M_\Omega = -\boldsymbol{\nabla}_p \cdot \left(\overline{\mathbf{u}' M_u'}\right) + \overline{r_\perp D}$$

along surface of constant $M_\Omega$:

$$\boldsymbol{\nabla}_p \cdot \left\{\overline{\mathbf{u}' M_u'}\right\} = r_\perp \{\bar{D}\}$$

*That is, any eddy AM flux convergence on an $M_\Omega$ surface must be balanced by drag on zonal flow on same $M_\Omega$ surface.*

# AM fluxes and mean meridional circulations



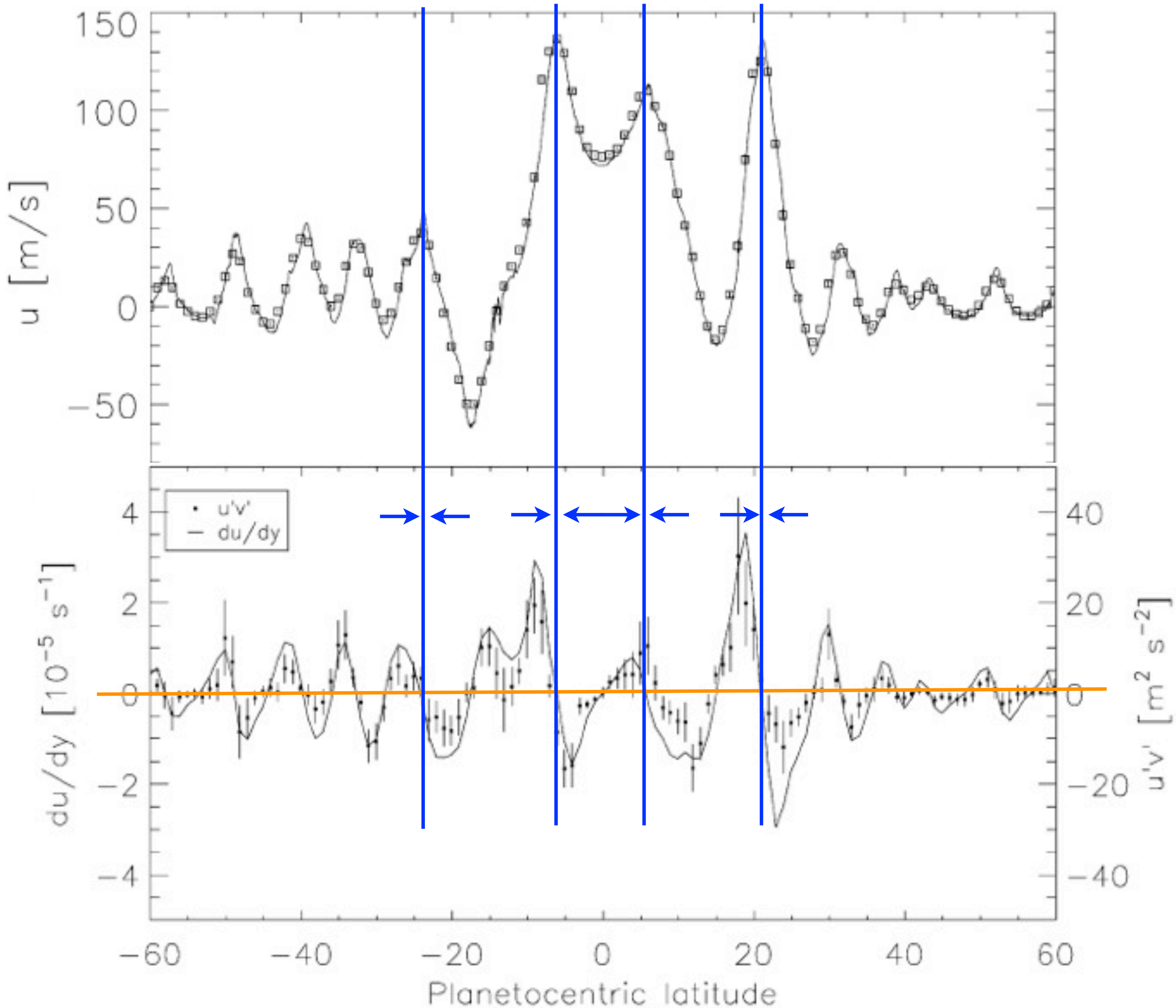

*Ageostrophic circulation transmits momentum downward, along angular momentum contours, to wherever dissipation occurs*

(Haynes et al. 1991; O'Gorman & Schneider 2008)

# Downward control (Haynes et al. 1991)

- Angular momentum flux convergence somewhere in the atmosphere implies that there must be MMC.
- MMC generally closes in region of dissipation (unless there is compensating AM flux); hence, it generally closes at depth.
- Where the dissipation occurs is irrelevant; MMC will reach it, and will adjust thermal structure and wind shear such that AM balance is satisfied.

# Back to Jupiter...




(Salyk et al. 2006)

# Implications of AM balance for Jupiter flow

- Eddy AM flux convergence in prograde and divergence in retrograde off-equatorial jets must be balanced by drag somewhere (unless there is serendipitous compensating flux at depth).

- Drag must occur at depth (otherwise would have been observable).

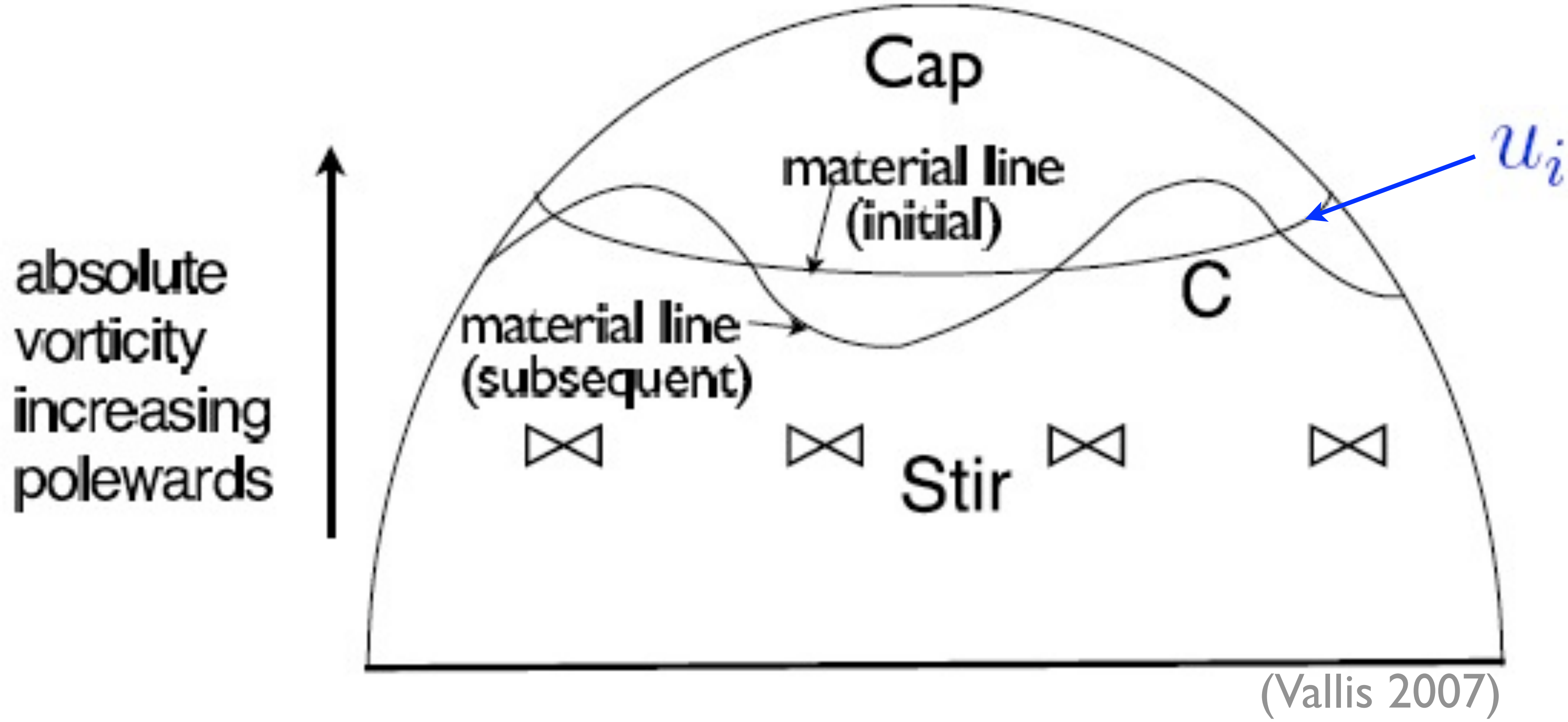
- AM fluxes imply energy transfer from eddies to mean flow: $O(10^{-4}$ W m$^{-3})$.

- AM fluxes cannot extend deeply: at most $O(1$ W m$^{-2})$ available to maintain flows, so AM fluxes cannot be deeper than ~10 km.

- AM fluxes on Jupiter must have baroclinic structure.

*So why is there AM flux convergence in prograde jets and divergence in retrograde jets?*

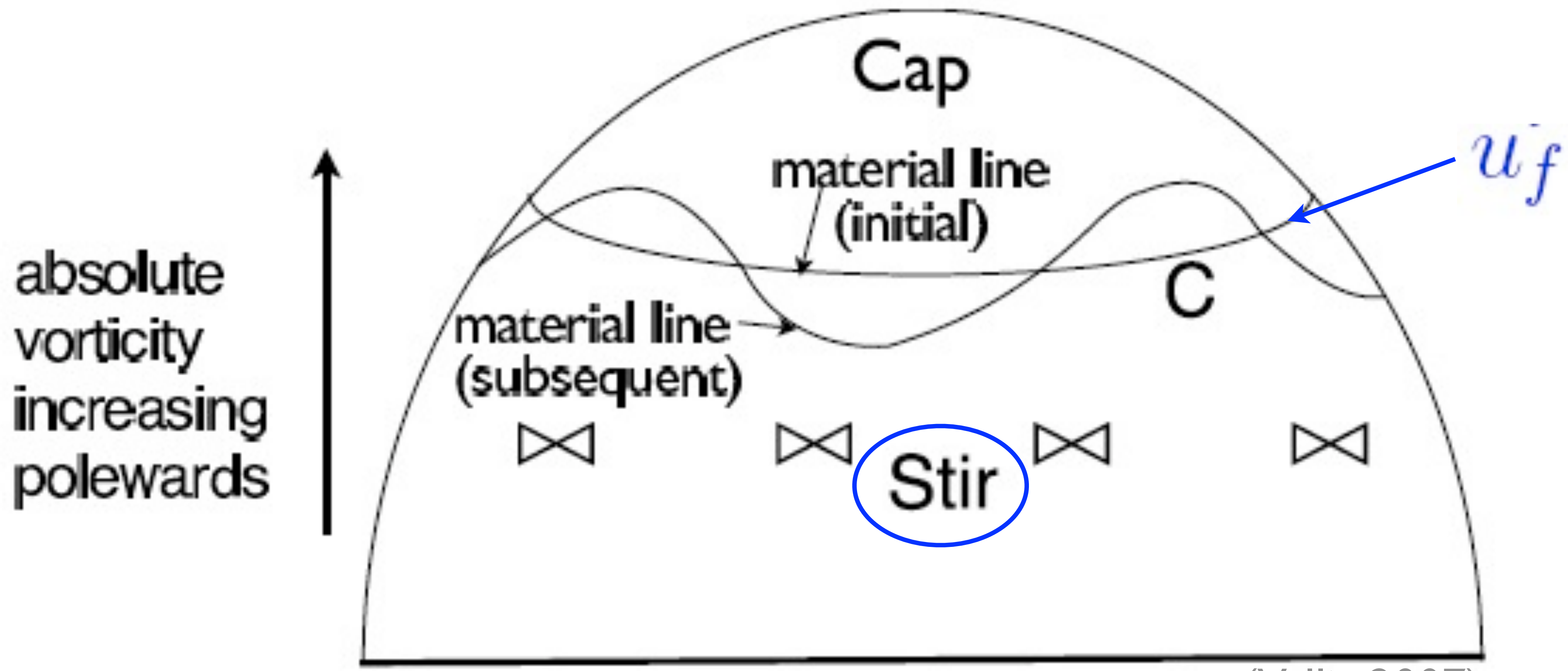
# Direction of eddy angular momentum flux




Initial circulation around polar cap:

$$C_i = \int_{\text{Cap}} \underbrace{(\nabla \times \mathbf{u}_{ai})}_{\omega_{ai}} \cdot d\mathbf{A} = \oint_C u_{ai}\, dl = \oint_C (u_i + \Omega a \cos\phi)\, dl$$

# Direction of eddy angular momentum flux




(Vallis 2007)

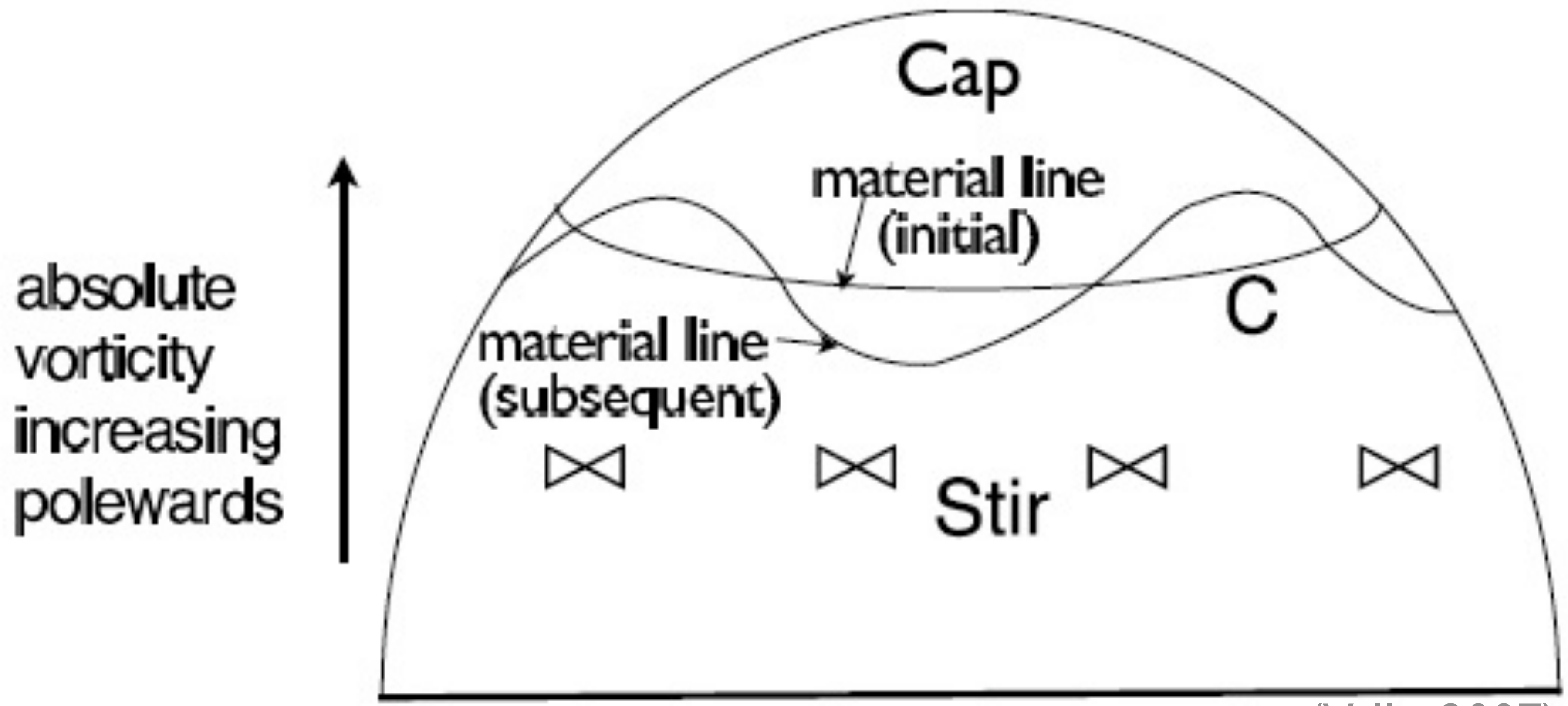
Conservative stirring reduces vorticity in polar cap:

$$C_f = \int_{\mathrm{Cap}} \boldsymbol{\omega}_{af} \cdot d\mathbf{A} = \oint_C (u_f + \Omega a \cos\phi)\, dl < C_i$$

$$\Rightarrow u_f < u_i$$

(Kuo 1951; Held 1975, 2000; Rhines 1994)

# Direction of eddy angular momentum flux




(Vallis 2007)

*Reduction of angular momentum outside stirring region*
*Momentum converges into stirring region*

(Kuo 1951; Held 1975, 2000; Rhines 1994)

*So there is AM flux convergence in "stirring" region (generation region of eddies) and divergence in dissipation region.*

*Eddies are preferentially generated in prograde jets on Jupiter (also true on Earth).*

*Coming next: how are these eddies generated, on Jupiter and in giant planet atmospheres generally?*

# references


- Haynes, P., M. McIntyre, T. Shepherd, C. Marks, and K. Shine (1991). "On the Downward Control of Extratropical Diabatic Circulations by Eddy-Induced Mean Zonal Forces". Journal of the Atmospheric Sciences, 48(4) pp.651-678.
- Held, I. (1975). "Momentum transport by quasi-geostrophic eddies". J. Atmos. Sci, 32(1494.1497) pp.1494-1497.
- Held, I. (2000). "The General circulation of the atmosphere". Woods Hole Program in Geophysical Fluid Dynamics, p. 66.
- Kuo, H. (1951). "Vorticity transfer as related to the development of the general circulation". Journal of the Atmospheric Sciences, 8(5) pp.307-315.
- O'Gorman, P. and T. Schneider (2008). "Weather Layer Dynamics of Baroclinic Eddies and Multiple Jets in an Idealized General Circulation Model". Journal of the Atmospheric Sciences, 65(2) pp.524-535.
- Porco, C., R. West, A. McEwen, A. Del Genio, A. Ingersoll, P. Thomas, S. Squyres, L. Dones, C. Murray, T. Johnson, et al. (2003). "Cassini imaging of Jupiter's atmosphere, satellites, and rings". Science, 299(5612) pp.1541-1547.
- Rhines, P. (1994). "Jets". Chaos: An Interdisciplinary Journal of Nonlinear Science, 4 pp.313-339.
- Salyk, C., A. Ingersoll, J. Lorre, A. Vasavada, and A. Del Genio (2006). "Interaction between eddies and mean flow in Jupiter's atmosphere: Analysis of Cassini imaging data". Icarus, 185(2) pp.430-442.
- Schneider, T. (2006). "The general circulation of the atmosphere". Annual Reviews of Earth and Planetary Sciences, 34 pp.655-688.
- Vallis, G.K. (2007). "Atmospheric and oceanic Fluid dynamics: fundamentals and large-scale circulation". Cambridge University Press, p. 745.