Generalized Nonlinear Subcritical Moist Symmetric Instability

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1.   Introduction

- **Emanuel(1983),Xu(1986),Cho et al.(1993) and Mu et al.(1999) investigated the problem of nonlinear symmetric stability by different means.**
- **Lu(2003) developed a new kind of generalized energy as the Lyapunov function, and proposed Subcritical Symmetric Instability. However, the moist process did not be considered.**

2. Subcritical Moist Symmetric Instability(GNSMSI)

Starting from nonlinear equations on the f-plane containing frictional dissipation and condensation heating under the Boussinesq approximation, let $u=u', v=\overline{v}+v', w=w', \theta=\overline{\theta}_e+\theta'$, **and we get the nonlinear disturbance equations.**

Starting Equations

$$\begin{cases} \dfrac{Du}{Dt} = fv - \dfrac{1}{\rho_0}\dfrac{\partial p'}{\partial x} + \eta_F \nabla_3^2 u \\ \dfrac{Dv}{Dt} = -fu - \dfrac{1}{\rho_0}\dfrac{\partial p'}{\partial y} + \eta_F \nabla_3^2 v \\ \dfrac{Dw}{Dt} = \dfrac{\theta'}{\theta_0} g - \dfrac{1}{\rho_0}\dfrac{\partial p'}{\partial z} + \eta_F \nabla_3^2 w \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0 \\ \dfrac{D\theta}{Dt} = \eta_\theta \nabla_3^2 \theta + kw \end{cases}$$

η_θ, η_F denote **frictional, thermal diffusion coefficient separately.**

$$k=(\theta_0/2T_0)(\operatorname{sgn} w+1)(\gamma_d-\gamma_m)$$

γ_d, γ_m denote **dry, moist adiabatic temperature lapse rate separately.**

From (1)-(5), we can obtain the equation of generalized disturbance energy as

$$dE/dt = -\eta D + I \qquad (6)$$

Where, $E=E_a+\alpha E_b, D=D_a+\alpha D_b, I=I_a+\alpha I_b$

$$E_a = \frac{1}{2}\left[\|u\|^2+\|v\|^2+\|w\|^2+R_0^2\|\theta\|^2+\lambda R_0^2\|\partial\theta/\partial z\|^2\right]$$

$$E_b = \frac{1}{2}\left[\|\nabla u\|^2+\|\nabla v\|^2+\|\nabla w\|^2+R_0^2\|\nabla\theta\|^2\right]$$

$$\langle(\ )\rangle = \int_0^L\int_0^H (\ )dz\,dx, \|(\ )\|^2 = \langle(\ )^2\rangle,$$

$$\nabla = \vec{i}\,\partial/\partial x + \vec{k}\,\partial/\partial z, R_0 = g/(\theta_0 N_0)$$

$$\begin{cases} \dfrac{\partial u}{\partial t} = -u\dfrac{\partial u}{\partial x} - w\dfrac{\partial u}{\partial z} + fv - \dfrac{\partial P}{\partial x} + \eta\nabla^2 u & (1) \\ \dfrac{\partial v}{\partial t} = -u\dfrac{\partial v}{\partial x} - w\dfrac{\partial v}{\partial z} - \dfrac{F^2}{f}u - \dfrac{S_w^2}{f}w + \eta\nabla^2 v & (2) \\ \dfrac{\partial w}{\partial t} = -u\dfrac{\partial w}{\partial x} - w\dfrac{\partial w}{\partial z} + \dfrac{g}{\theta_0}\theta - \dfrac{\partial P}{\partial z} + \eta\nabla^2 w & (3) \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0 & (4) \\ \dfrac{\partial \theta}{\partial t} = -u\dfrac{\partial \theta}{\partial x} - w\dfrac{\partial \theta}{\partial z} - \dfrac{\theta_0 S_w^2}{g}u - \dfrac{\theta_0 N_w^2}{g}w + \eta\nabla^2\theta + kw & (5) \end{cases}$$

The Brunt-Vaisala frequency *N*, baroclinic frequency *S*, and interial frequency *F* are defined as follows

$$N_w^2 = (g/\theta_0)(\partial\overline{\theta}_e/\partial z)$$

$$S_w^2 = (g/\theta_0)(\partial\overline{\theta}_e/\partial x) = f(\partial\overline{v}/\partial z)$$

$$F^2 = f(f+\partial\overline{v}/\partial x)$$

3. Criterion of GNSMSI

From (6), we can get

$$\left.\begin{aligned} &\eta > \eta^*, \eta^* = \max(\eta_a, \eta_b) && \textbf{(7)} \\ &E(0) < B^{-2} = E_c(0) && \textbf{(8)} \end{aligned}\right\} \longrightarrow \frac{dE}{dt} < 0$$

(7)-(8) hold simultaneously is a sufficient condition of generalized nonlinear symmetric stability. If not, a necessary condition of instability would occur. Especially, as a linearly stable disturbance only meets the needs of (7), but doesn't satisfy (8), GNSMSI is likely to occur. After meso-scale typical values are given, it is concluded that GNSMSI may occur if the original disturbance wind is larger than 4m/s.

4. Relationship with precipitation

Data: WRF output every 1Hr
Method: Barnes Band Filter to separate disturbance and basic flow

$S = \dfrac{\overline{\zeta}_a}{f} - \dfrac{1}{Ri}$ ***S*>0 denotes linear symmetric stability.**

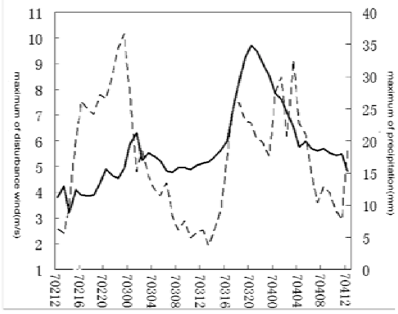

Fig1.Relationship of the maximum of disturbance wind (solid line, m/s) and precipitation (dashed line, mm).

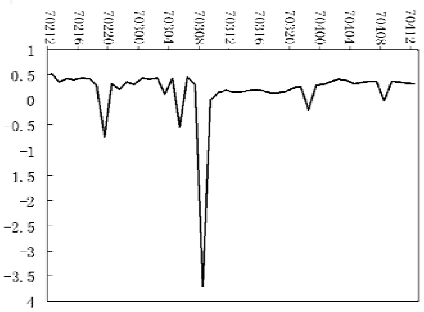

Fig2.Index of the linear symmetric stability .

During the precipitation, disturbance is of linear symmetrical stable while the uwind is larger than critical value. Therefore, GNSMSI is possible to occur, and induce this precipitation process.

5. Summary

◆Generalized nonlinear subcritical moist symmetric instability is proposed, and its criterion is derived through the equation of generalized energy .

◆ The relationship between GNSMSI and precipitation in a case is analysed. GNSMSI is the probable triggering mechanism in this precipitation process.

Reference:

Lu Weisong, Shao Haiyan. 2003, Generalized nonlinear subcritical Symmetric instability. Advances in Atmospheric Sciences, 20(4): 623-630.
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