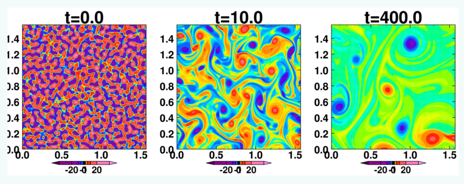
The effect of filaments on the axisymmetrization process of the 2D elliptic vortex with non-uniform vorticity

Shin'ya MURAKAMI[1] and Takahiro IWAYAMA[2]
1 Graduate School of Science and Technology, Kobe University, Japan, 2 Graduate School of Science, Kobe University, Japan.

1 Background

Two-dimensional (2D) turbulence has been of considerable interest because it is believed to be an apt paradigm for geophysical flows.
Moreover, vortex motions are fundamental elements of the 2D turbulence in the physical space, because



- 2D turbulence is full of vortices,
- such vortices mutually advect,
- two like-signed vortices merge into a single vortex,
- and a single elliptic vortex approaches circular one.

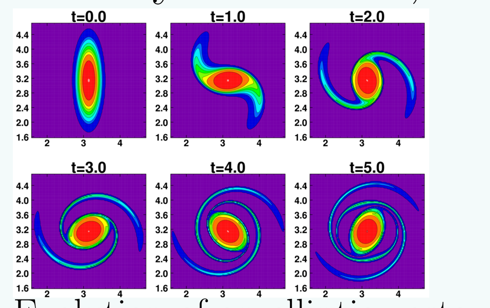
In this study, we reconsider a research on the axisymmetrization process of the elliptic vortex of the incompressible 2D fluid by Melander *et al.*(1987), numerically.
Thus the governing equation is the following vorticity equation:

$$\frac{\partial \omega}{\partial t}+\frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y}-\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x}=\nu\nabla^2\omega$$

where $\omega=\nabla^2\psi$.

2 Axisymmetrization of the elliptic vortex

In this section, we briefly review the kinematics of the axisymmetrization of the elliptic vortex proposed by Melander, *et al.*(1987).

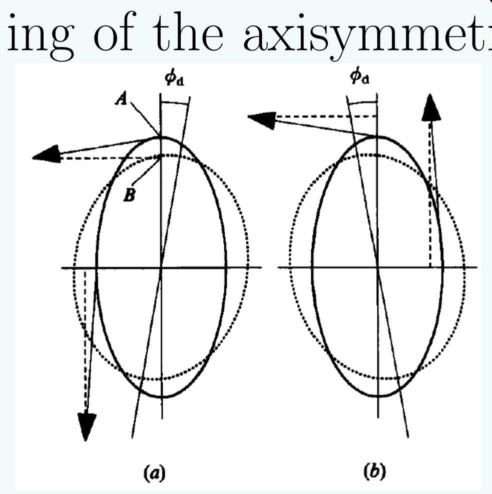


Evolution of an elliptic vortex obtained by a numerical simulation.

- An elliptic vortex evolves to an axisymmetric vortex.
- The vortex ejects filaments.

2.1 The relationship between the axisymmetrization and ϕ_d

Melander, *et al.*(1987) introduced the difference angle ϕ_d between the orientation of the ellipse with a vorticity contour and that with a streamline contour as a diagnostic tool for understanding of the axisymmetrization of an elliptic vortex.



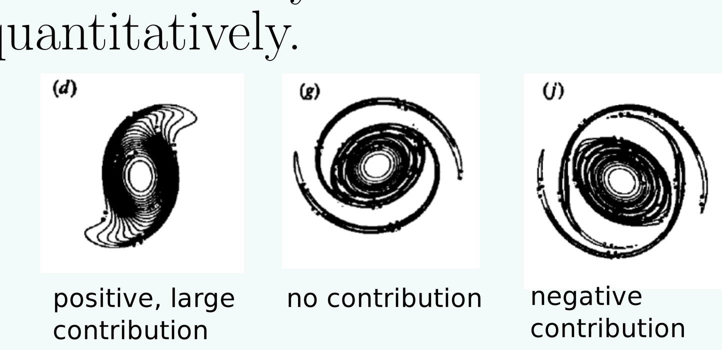
Melander, *et al.*(1987) Fig.2

- solid line: vorticity contour
- broken line: streamline contour
- $\phi_d := \phi_\omega - \phi_\psi$
- ϕ_ω: orientation of the ellipse with vorticity contour
- ϕ_ψ: orientation of the ellipse with streamline contour

- (a)
  - The inward (outward) velocity is induced near the maximum (minimum) curvature of the ellipse of the vorticity contour. These induced velocities axisymmetrize the elliptic vortex.
- (b)
  - Opposite sense to (a).

2.2 Estimate of the contribution of the filaments to ϕ_d

Melander, *et al.*(1987) devoted themselves to study the contributions of the vortex core to ϕ_d. They didn't investigate the contribution of the filaments to ϕ_d, although they mention the effect of filaments to the axisymmetrization of the elliptic vortex quantitatively.



Melander, *et al.*(1987) Fig.8(excerpt).

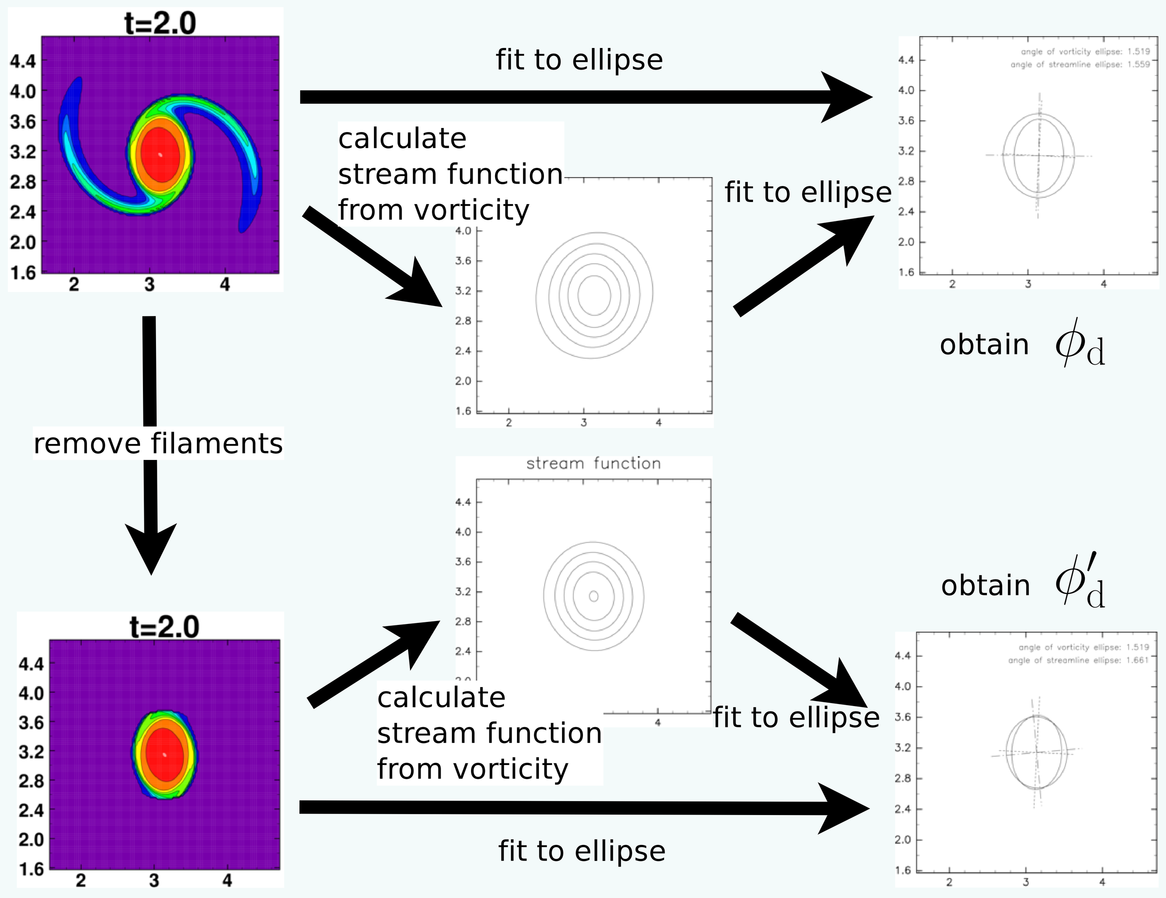
- A natural question is raised:
  How large do the filaments contribute to ϕ_d?
  $\Rightarrow$ We examine this point numerically.

3 Numerical estimate of ϕ_d

3.1 Procedure

We investigate the contribution of the filaments to the ϕ_d with the following procedure:

1. Divide the vorticity field into the vortex core and the filaments.
   - Here, the filaments are determined by the morphology of the vorticity field.
2. Calculate the stream function induced by the vortex core.
3. Fit ellipse to the vorticity contour and the streamline, then calculate the difference angle of the principal axes of the ellipses.
   - We use the fitting method proposed by Fitzgibbon, *et al.*(1999).

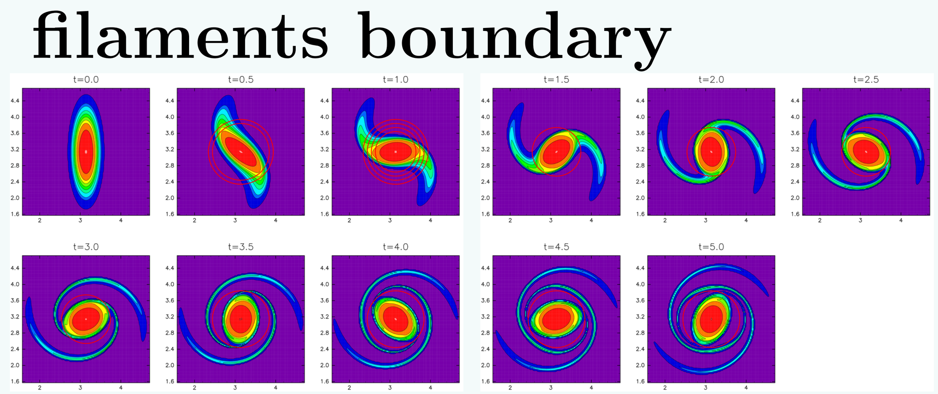


ϕ'_d is calculated from the vorticity field without filaments.
We focus on the vorticity contour with $\omega=6$ the streamline contour with $\psi=-1.2$ to determine the vorticity and streamline ellipses.

3.2 Settings for the simulation

- dealiased pseudospectral method with 2/3 rule
- time stepping scheme: the 3rd order Adams-Bashforth
- number of grid points: 3072
- truncated wave number: 1023
- Δt: 2.5×10^{-4}
- termination time of integration: $t=5$
- viscous coefficient: $\nu=1.5\times10^{-5}$
- maximum of the initial vorticity: $\omega_0=10$
- initial aspect ratio of the vortex: $\sqrt{10}$
- the other initial conditions: same as the Melander, *et al.*(1987) and the compact support case in Kimura and Herring(2001).

3.3 Determination of the core-filaments boundary



- Red solid lines are the boundaries between the vortex core and filaments.

3.4 Results

3.4.1 Contribution of the vortex core

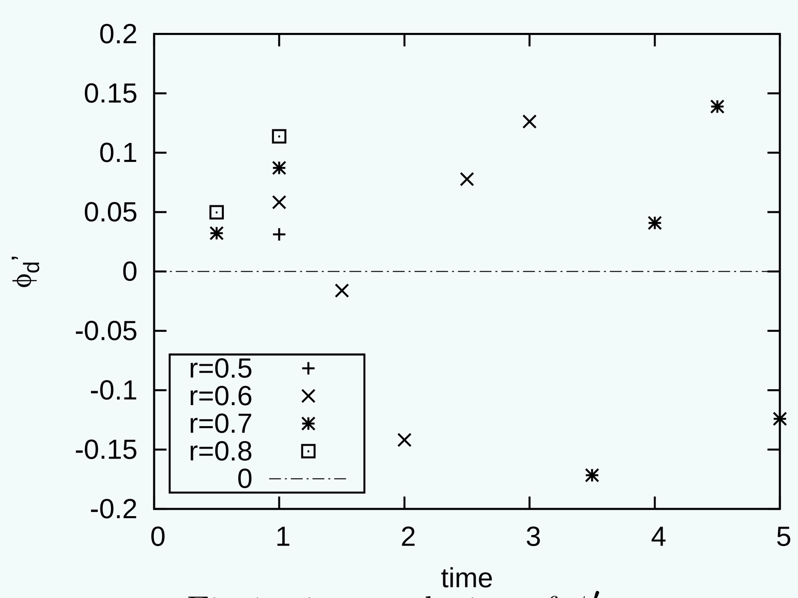


Fig.1: time evolution of ϕ'_d.

3.4.2 Contribution of the filaments

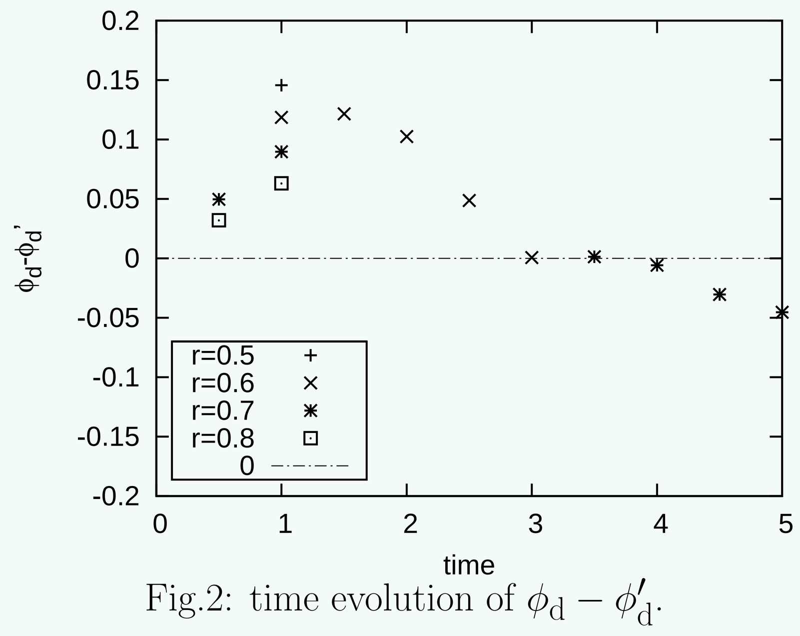


Fig.2: time evolution of $\phi_d-\phi'_d$.

Fig.1 and Fig.2 shows that

- amplitude of ϕ'_d is almost constant during the evolution.
- ϕ'_d is positive in the early time of evolution, and rapidly oscillates compared to $\phi_d-\phi'_d$.
- Contribution of the filaments largely depends on the definition of the filament in $0\le t\le 1.0$.
  - We choose $r\approx 0.7$ as the boundary between the core and the filaments. In the early time evolution, the division between the vortex core and the filaments is obscure from the morphology of the vorticity field. However in the late time evolution, $r\approx0.7$ seems to be a reasonable estimate to the boundary of the core and the filaments.
- The contribution of the filaments is larger than that of the core at $1.5\le t\le 2.0$.
- The contribution of the core is larger than that of the filaments at $2.5\le t\le 5.0$.

4 Conclusion

The contribution of the filaments to the axisymmetrization process of the elliptic vortex has been investigated quantitatively.

- We calculated the difference angle ϕ_d between orientations of elliptic vorticity and streamline contours which is a diagnostic tool for understanding the axisymmetrization of the elliptic vortex proposed by Melander, *et al.*(1987).
- We calculate the contribution of the vortex core and the filaments to ϕ_d, the difference angle between the principal axes of the ellipses with a vorticity contour and a streamline, respectively. from the filaments and the core, respectively.
  - The filaments largely contributes to ϕ_d in early stage.
  - In the early time of evolution, the contributions of the filaments to ϕ_d is comparable to that of the vortex core.
  - In the later time, the contribution of the filaments seems to be less important.

We confirm numerically that the filaments play an important role for the axisymmetrization of the elliptic vortex.

References

- Melander, M. V., McWilliams, J. C., Zabusky, N. J., 1987: Axisymmetrization and vorticity-gradient intensification of an isolated two-dimensional vortex through filamentation, *J. Fluid Mech.*, **178**, 137 – 159.
- Kimura, Y., Herring, J. R., 2001: Gradient enhancement and filament ejection for a non-uniform elliptic vortex in two-dimensional turbulence, *J. Fluid Mech.*, **439**, 43 – 56.
- Fitzgibbon, A., Pilu, M., Fisher, R. B., 1999: Direct Least Square Fitting of Ellipses, *Pattern Analysis and Machine Intelligence*, **21**, 476 – 480.