Interaction between Thermal Convection and Mean Flow in a Rotating System

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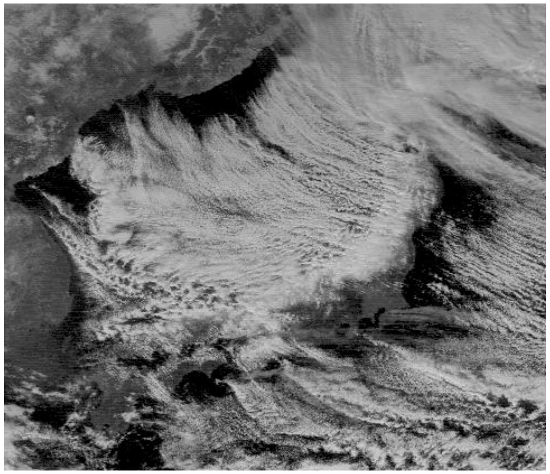
1. Introduction

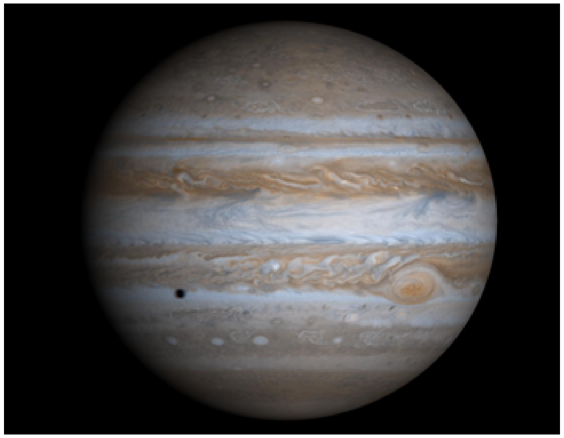
Thermal convection is an important motion in geophysical fluid.



Thermal convection in a shear flow (flow with velocity gradient)

- Vertical shear · · · Asai (1970), etc.

  e.g., snowbands over the Japan Sea in winter



- Horizontal shear

  · · · Davies-Jones (1971), Yoshikawa&Akitomo (2003), etc.

  e.g., zonal band structure of Jovian atmosphere

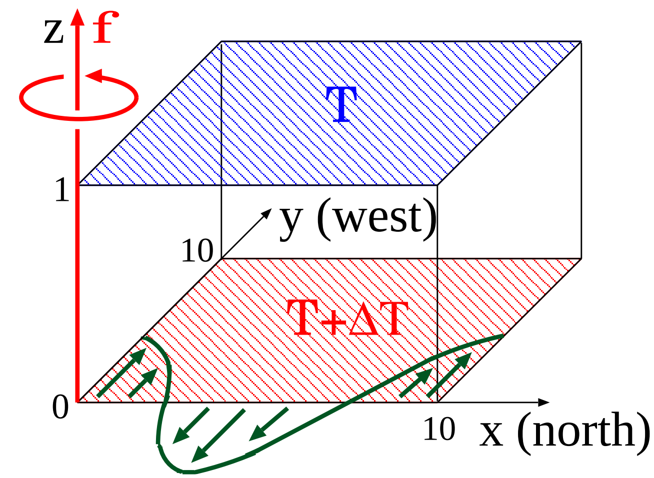
There are few studies on thermal convection in a horizontal shear flow.

In this study : Thermal convection in a sine-type horizontal shear flow in a rotating system

◎ In the case of vertically-directed rotating axis

"Wave Pattern Formation from Thermal Convection in a Horizontal Shear Flow"

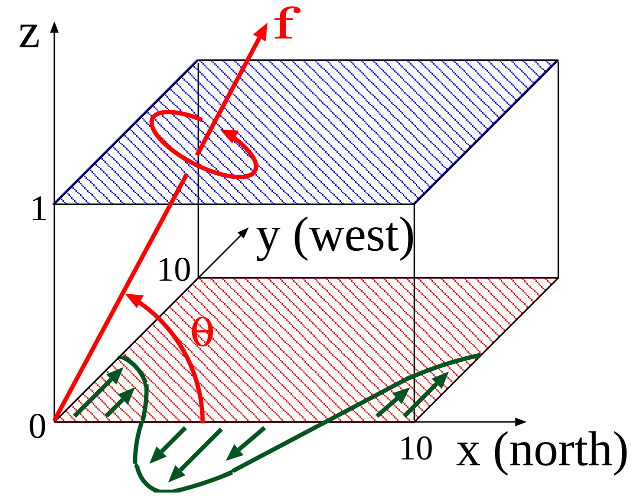
Previous study : Furukawa & Niino (2006)



◎ In the case of tilted rotating axis

"Interaction between a Sine-type Horizontal Shear Flow and Thermal Convections in a Rotating System with a Tilted Axis"

Previous study : Hathaway & Somerville (1987)



2. Model Equations and Computing Configuration

Basic equations

Boussinesq fluid in a system rotating at an angular velocity of $f/2$ around a rotating axis

Eq. of Motion : $$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p - \boldsymbol{f}\times(\boldsymbol{u}-\boldsymbol{u}_B) + Ra\,\boldsymbol{b} + \nabla^2(\boldsymbol{u}-\boldsymbol{u}_B)$$

Thermal Eq. : $$\frac{\partial b}{\partial t} + u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} + w\frac{\partial b}{\partial z} = w + \nabla^2 b$$

Continuity Eq. : $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

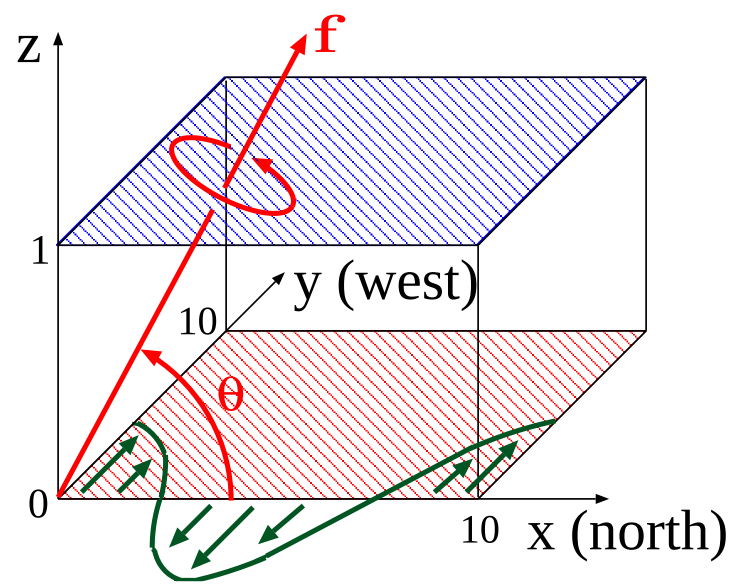
$$\boldsymbol{f} = \begin{pmatrix} f\cos\theta \\ 0 \\ f\sin\theta \end{pmatrix},\ \boldsymbol{b} = \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix},\ \boldsymbol{u}_B = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix}.$$

($0 \le \theta \le \pi/2$: **latitude**)

b : **buoyancy**, Ra : **Rayleigh number**,

∇^2 : **laplacian**, v_B : **basic flow.**

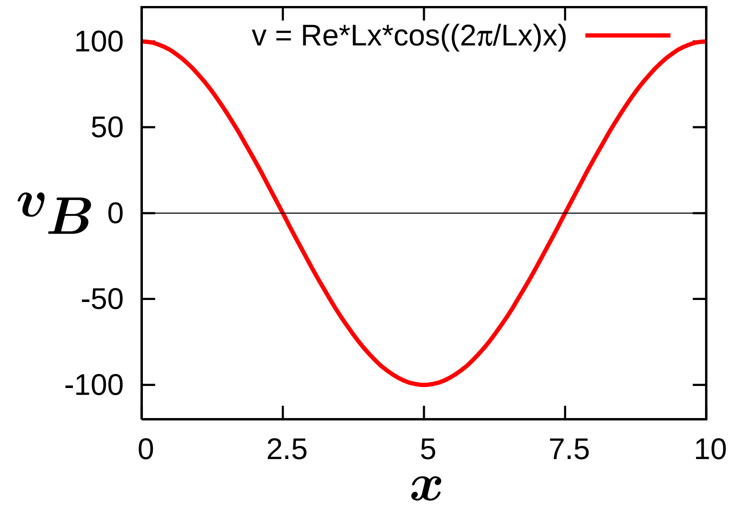
Prandtl Number $= 1$



Boundary condition

Periodic in x, y of period L_x, L_y .

Free-slip and fixed temp. at $z=0,1$.



Basic flow (horizontal shear flow)

$$v_B = Re\, L_x \cos\left(\frac{2\pi}{L_x}x\right)$$

Re **: Reynolds number**

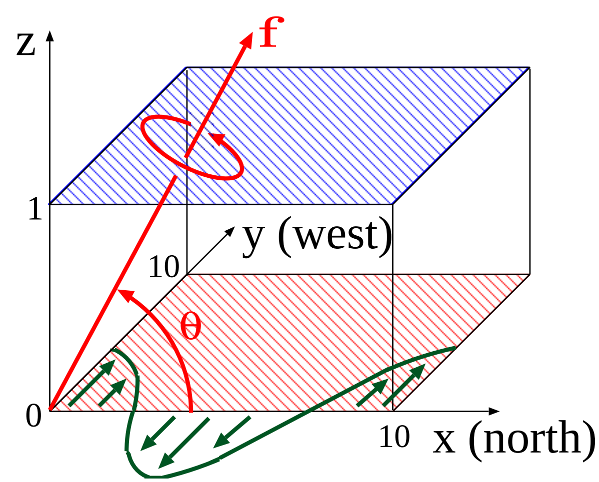
Numerical computation method

- **Space discretization : spectral method**

  x, y **direction : Fourier series expansion (truncation wavenumber 21)**

  z **direction**

  $$\begin{cases} \theta = \pi/2 : \sin, \cos \text{ series expansion } (10) \\ \theta \neq \pi/2 : \text{Legendre polynomial expansion } (20) \end{cases}$$

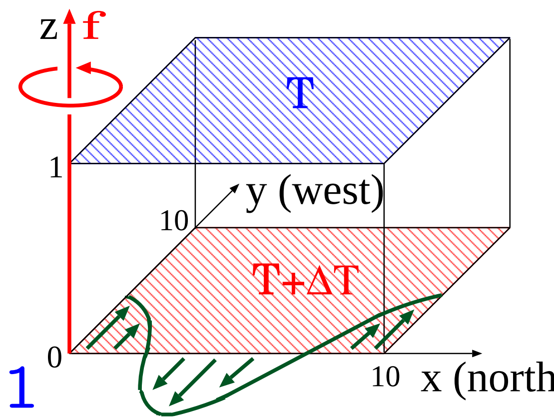


- **Time evolution : 4th order Runge-Kutta method**

Computational domain : $L_x = L_y = 10$

Attention : For convenience of modeling, positive x direction is northward, positive y direction is westward. Please no flames.

3. Wave Pattern Formation from Thermal Convection in a Horizontal Shear Flow $(\theta = \pi/2)$

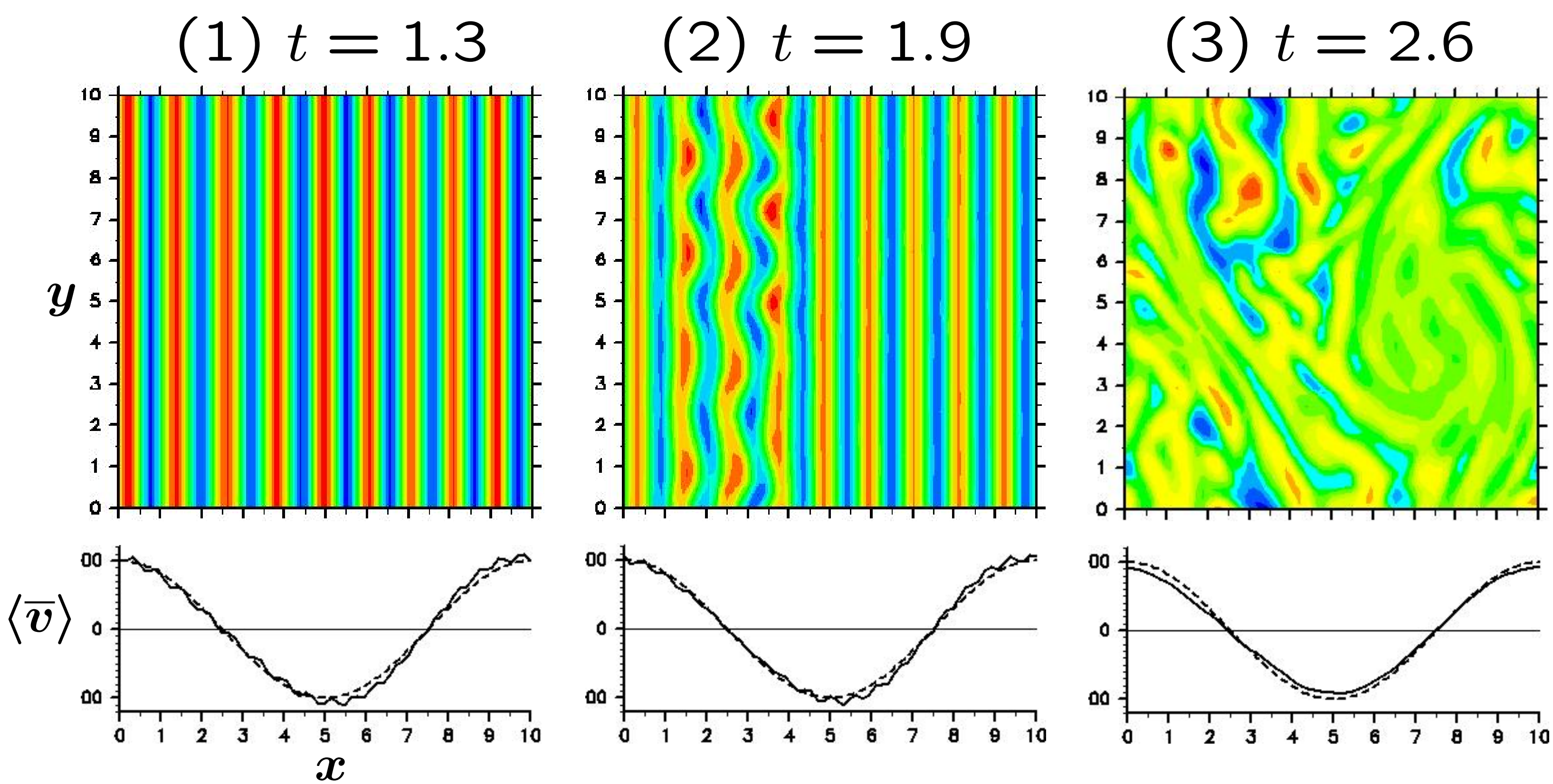


Nonlinear time evolution

(1) **Roll convection parallel to mean flow**

→(2) **Disturbance of zonal wavenumber** $4, 5$

→(3) **Barotropic eddy of zonal wavenumber** 1



Top : horizontal sectional view of temperature field $(z = 0.5)$

Bottom : mean flow profile (vertically and zonally averaged velocity v)

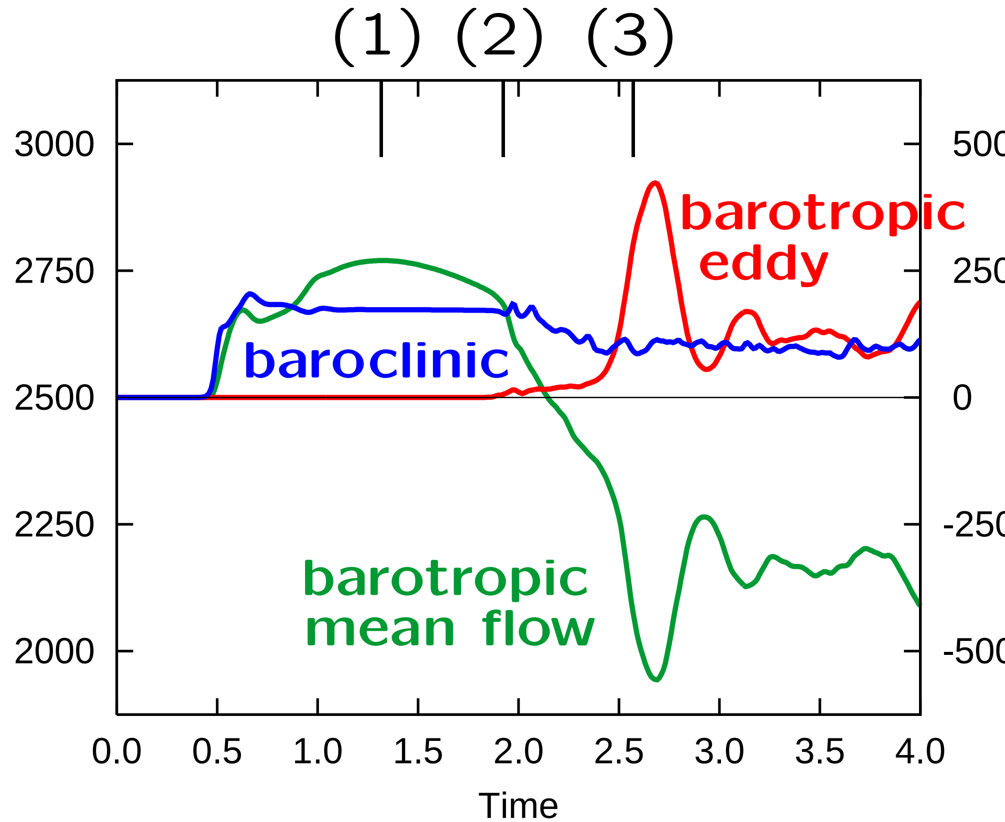
Analysis

It turns out that barotropic eddy of zonal wavenumber 1 is formed by barotropic instability.

However, initial sine-type shear flow is barotropically stable.

Roll convections distort barotropic field and makes horizontal shear flow unstable.

【Two-stage instability】



Time evolutions of energy for each component

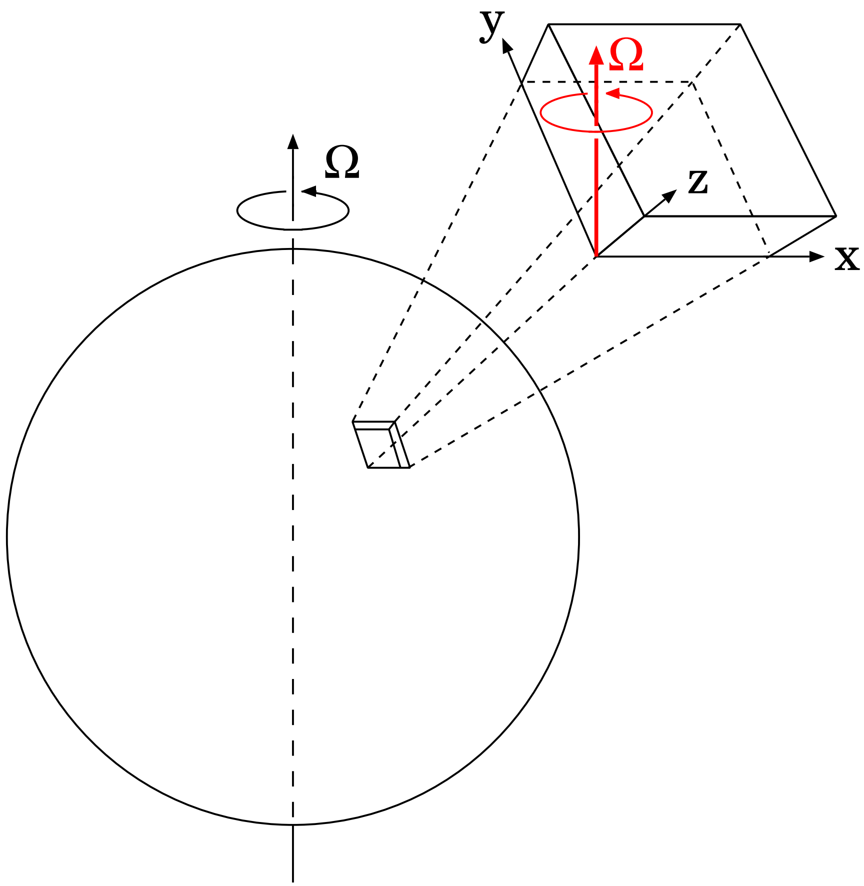
Naoaki SAITO, Keiichi ISHIOKA, 2008 :
Wave Pattern Formation from Thermal Convection in a Horizontal Shear Flow. *Nagare Multimedia* **.**
http://www.nagare.or.jp/mm/2008/saito/

4. Interaction between a Sine-type Horizontal Shear Flow and Thermal Convections in a Rotating System with a Tilted Axis

4-1. Review of Hathaway & Somerville (1987)

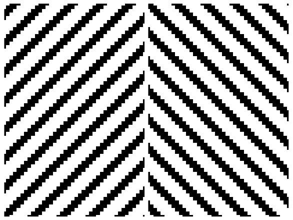
Nonlinear time evolution of thermal convection in sine-type horizontal shear flow at low latitudes in a rotating atmosphere.

- Basic equations and configuration are the same as my study, except for vertical rigid boundary condition. ($u = v = 0$ at $z = 0, 1$)



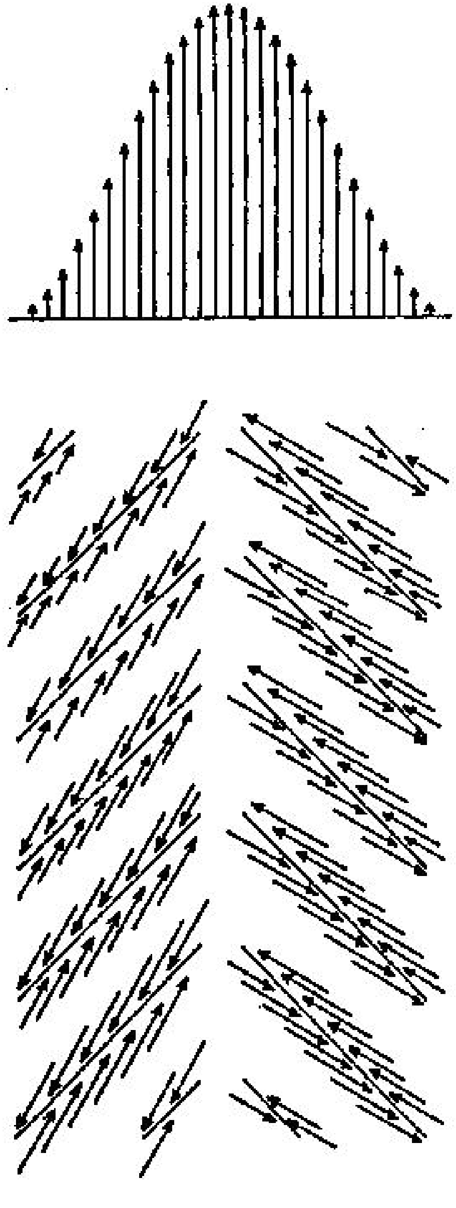
Result

Roll convection forms a herringbone pattern, and mean flow is accelerated.

Herringbone pattern : 

Interpretation

The Coriolis force turns the convective flow, and momentum transport accelerates mean flow.



Top : mean flow
Bottom : convective flow

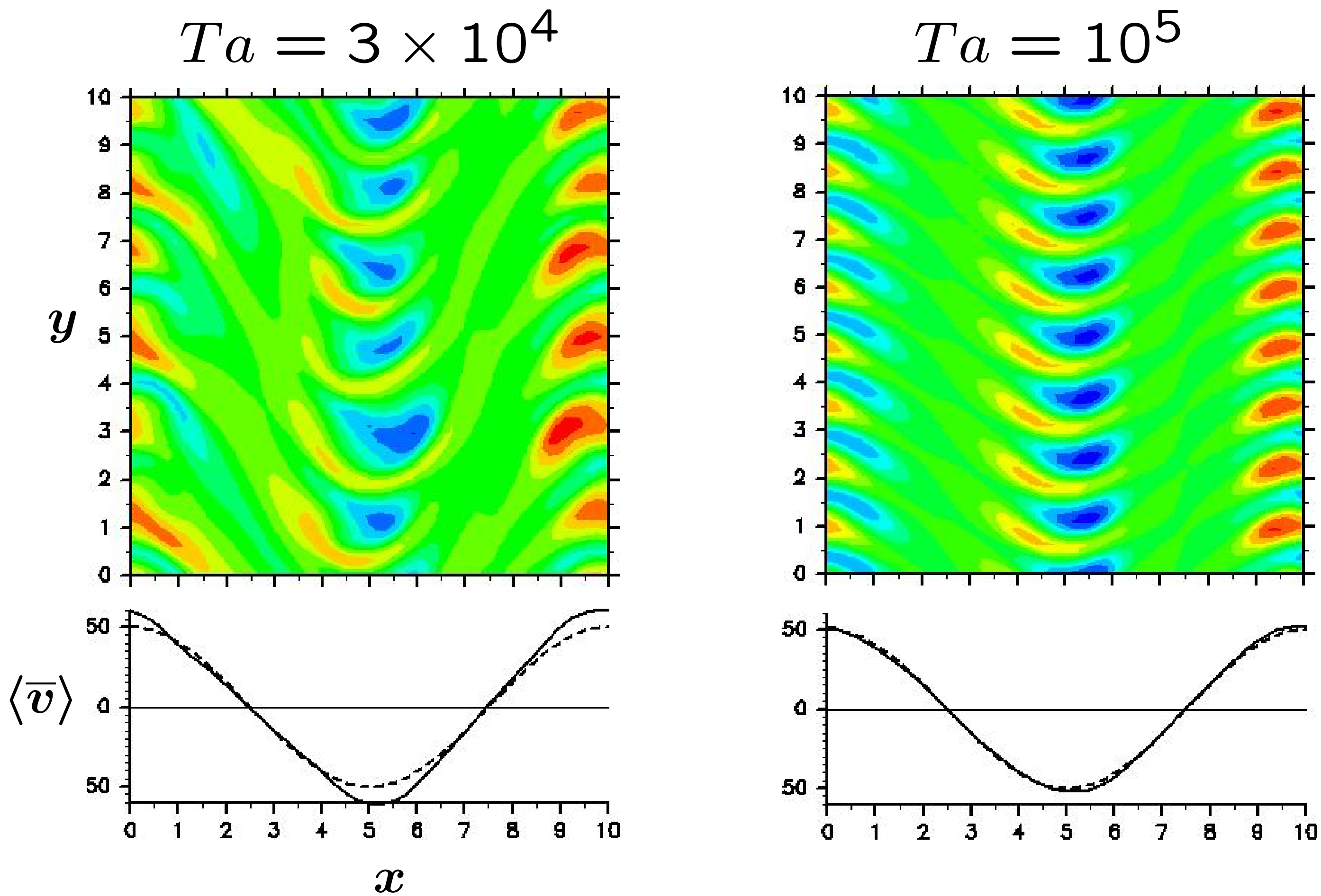
> The aim of my study is
> to explore the mechanism of
> acceleration of mean flow
> by more detailed analyses
> such as linear stability analyses.

4-2. Nonlinear Time Evolution

Config. $\theta = \pi/12, Re = 5, Ra = 10^4, Ta = f^2 = 0, 10^4, 3 \times 10^4, 10^5.$

Result $Ta = 0$ **and** 10^4 **: Herringbone pattern is not formed.**

$Ta = 3 \times 10^4$ **and** 10^5 **: Herringbone pattern is formed.**



Top : horizontal sectional view of temperature field $(z = 0.5)$

Bottom : mean flow profile (vertically and zonally averaged velocity v**)**

4-3. Linear Stability Analysis

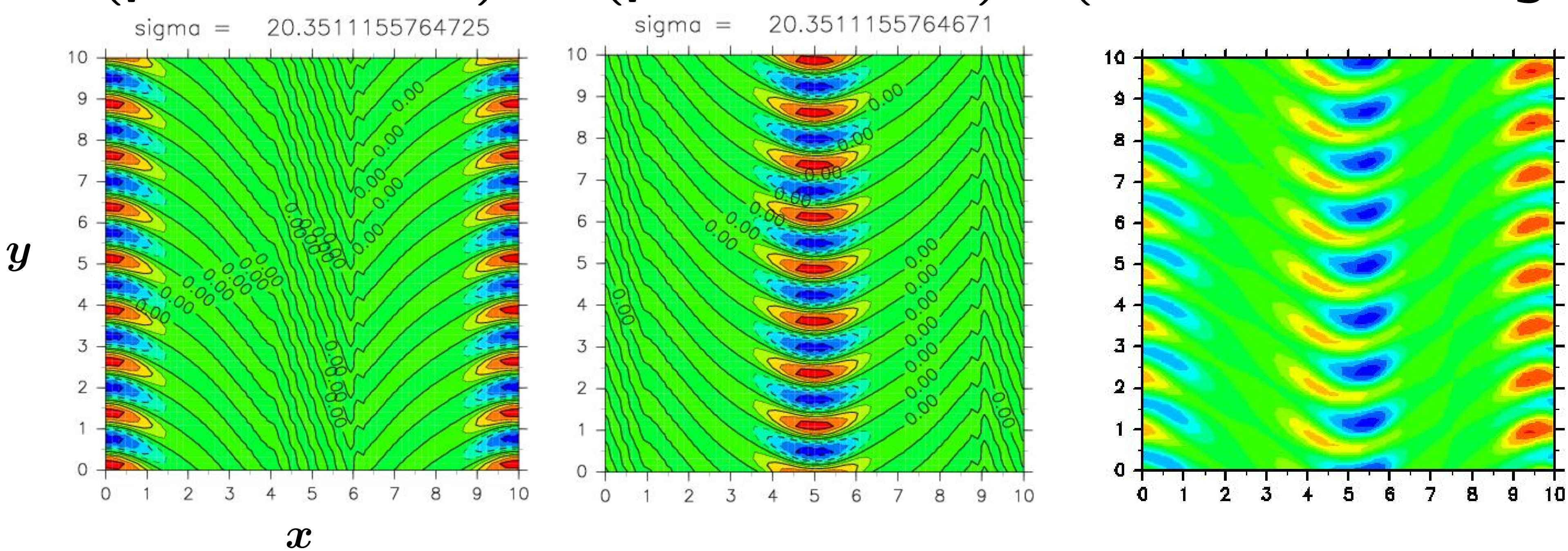
Analyze the initial field in the cases of $Ta = 3\times10^4$ and 10^5 .

Result

The wavenumber, the structure and the growth rate of the largest growing eigenmode are consistent with the time evolution.

- **In the case of $Ta = 10^5$ (Horizontal sectional view of temp. field)**

eigenmode (peak : $x = 0$) | **eigenmode (peak : $x = 5$)** | **time evolution (※ nonlinear stage)**



Herringbone pattern $\Leftarrow$ the structure of the eigenmode

Result 2

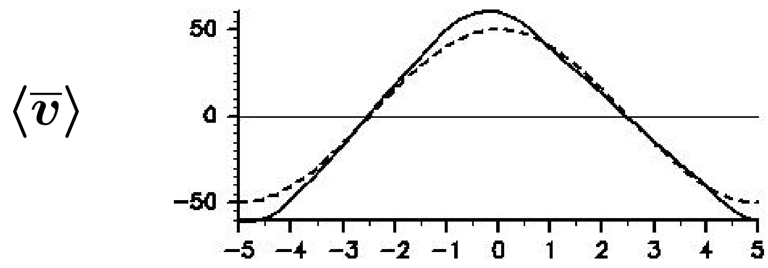
Growth rate of the deviation of mean flow velocity $\langle \overline{v} \rangle$ from initial velocity v_B is twice as large as the largest eigenvalue.

$\Rightarrow$ The acceleration of mean flow is due to the second-order effect of the eigenmode.

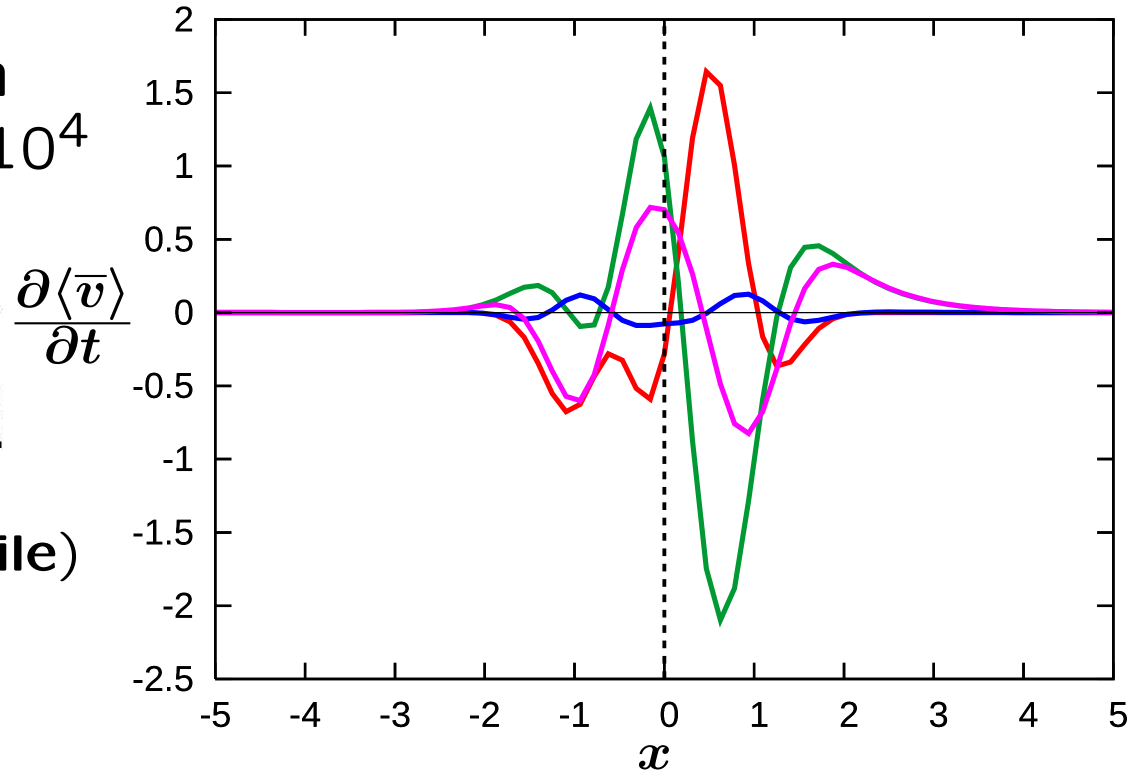
4-4. Analyses of second-order effects of the eigenmode

Acceleration by the eigenmode with a peak at $x=0$ in the case of $Ta=3\times10^4$

Time evolution in the case of $Ta=3\times10^4$



(dashed line : initial profile)



(Direct momentum transport, the Colioris force acting on the second-order vertical flow, viscosity, summation)

Contribution of the Colioris force acting on the second-order vertical flow is larger than that of direct momentum transport proposed by Hathaway & Somerville (1987).

4-5. Discussion on the mechanism of the acceleration

Further detailed analyses show that the following process is important.

1. Roll convection localizes around the fastest area of mean flow.
   Axis of roll tilts parallel to f.
   Rotation of roll is strong in the inertially unstable area.

↓

2. Heat transport by disturbances generates buoyant deviations. The upper deviation is larger.

↓

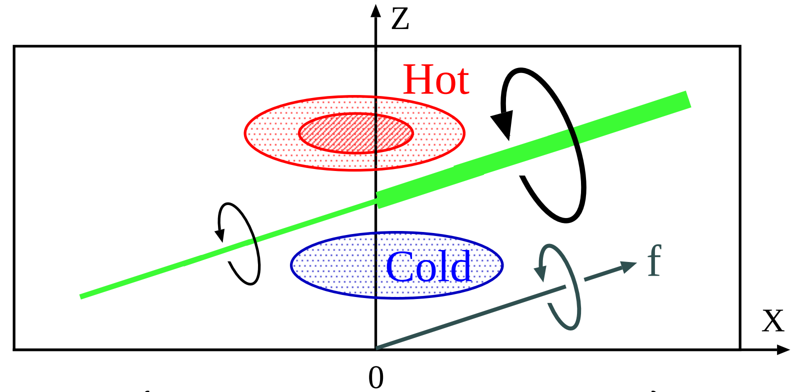
3. Second-order vertical flow occurs. Vertical mean near $x=0$: $\langle \overline{w} \rangle > 0$

↓

4. The Colioris force acting on $\langle \overline{w} \rangle$ accelerates the mean flow.

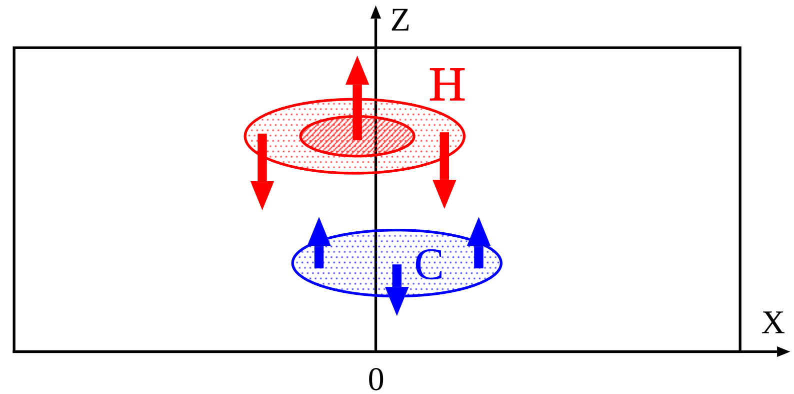
(Horizontal comp. of f is effective.)

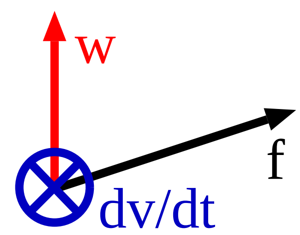
★ This mechanism also accounts for the asymmetric structure in nonlinear stage.

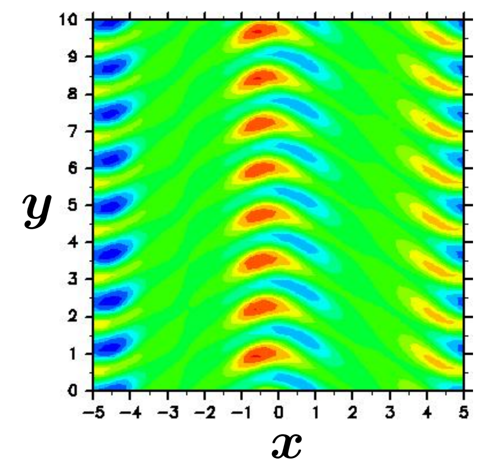


(green : axis of roll)

↓







(temperature field)

5. Conclusion

Thermal convection in a sine-type horizontal shear flow in a rotating system is studied numerically.

◎ In the case of vertically-directed rotating axis

- Barotropic eddy of zonal wavenumber 1 is formed by barotropic instability.
- Two-stage instability :

  Initial barotropically-stable field is destabilized by roll convection. ⇒ Barotropic instability occurs.

◎ In the case of tilted rotating axis

- $Ta = 3 \times 10^4$ and 10^5 : Herringbone pattern is formed.
  $Ta = 3 \times 10^4$: Mean flow is largely accelerated.

- Herringbone pattern : eigenmode of initial field

- Acceleration of mean flow :

direct momentum transport	$<$	the Colioris force acting on the second-order vertical flow

- vertical heat transport by disturbances
  $\Rightarrow$ buoyant deviations
  $\Rightarrow$ vertical flow + the Colioris force
  $\Rightarrow$ acceleration of mean flow

- This process may work as a new mechanism of the acceleration of zonal flows in rotating planets.