

# *EVOLUTIONS OF SMALL BODIES IN OUR SOLAR SYSTEM*

*Dynamics and collisional processes*

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# *Plan*

## ⊕ Chapter I:

A few concepts on dynamics and transport mechanisms in the Solar System; application to the origin of Near-Earth Objects (NEOs)

## ⊕ Chapter II:

On the strength of rocks and implication on the tidal and collisional disruption of small bodies

# Preliminaries: orbital elements

$a$  = semi major axis

$e$  = eccentricity

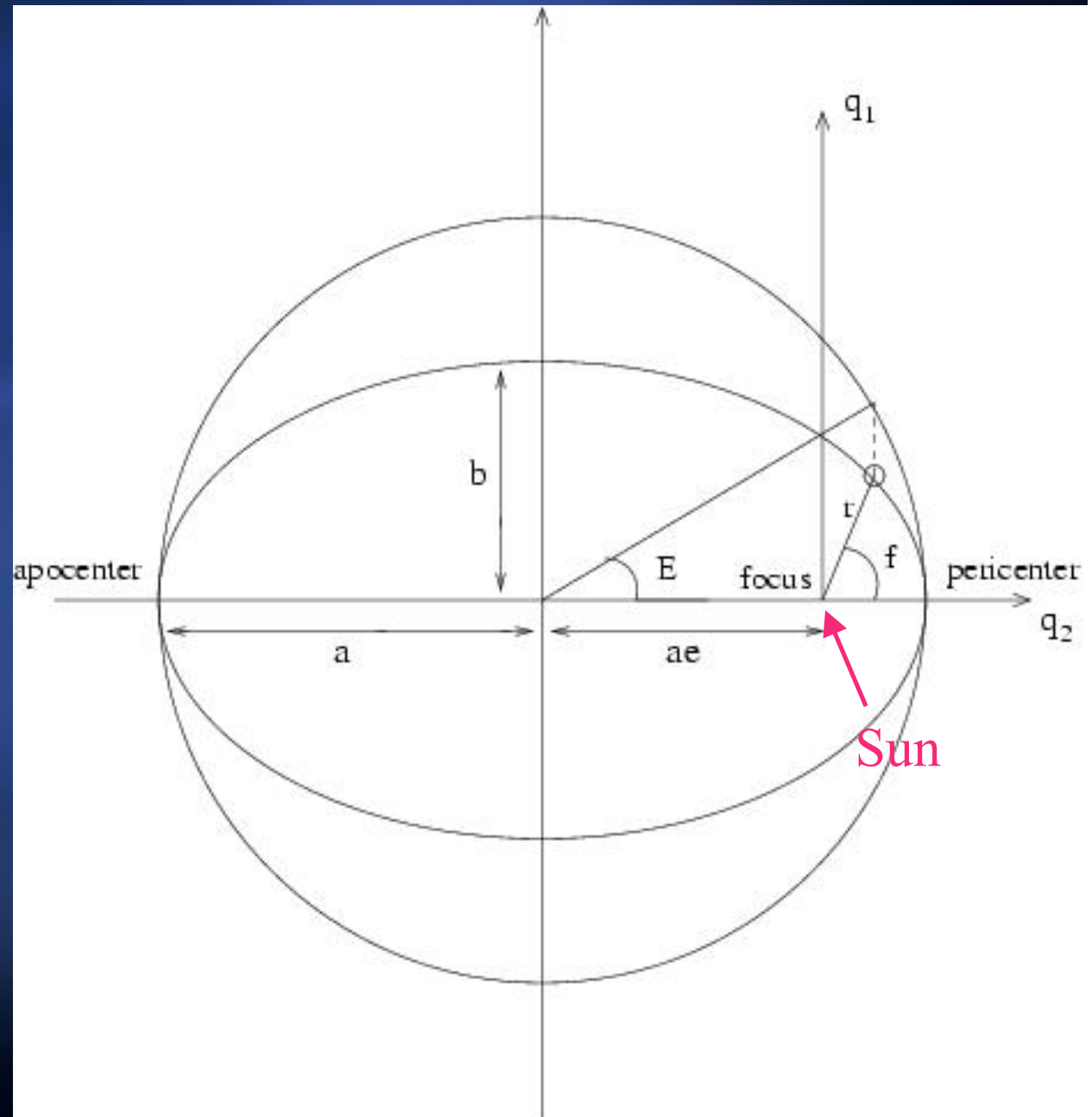
$f$  = true anomaly

$E$  = eccentric anomaly

Mean anomaly:

$M = E - e \sin E = n t$

with  $n = (GM_*)^{1/2} / a^{3/2}$   
(orbital frequency)



# Preliminaries: orbital elements

$i$  = inclination

$\Omega$  = longitude of node

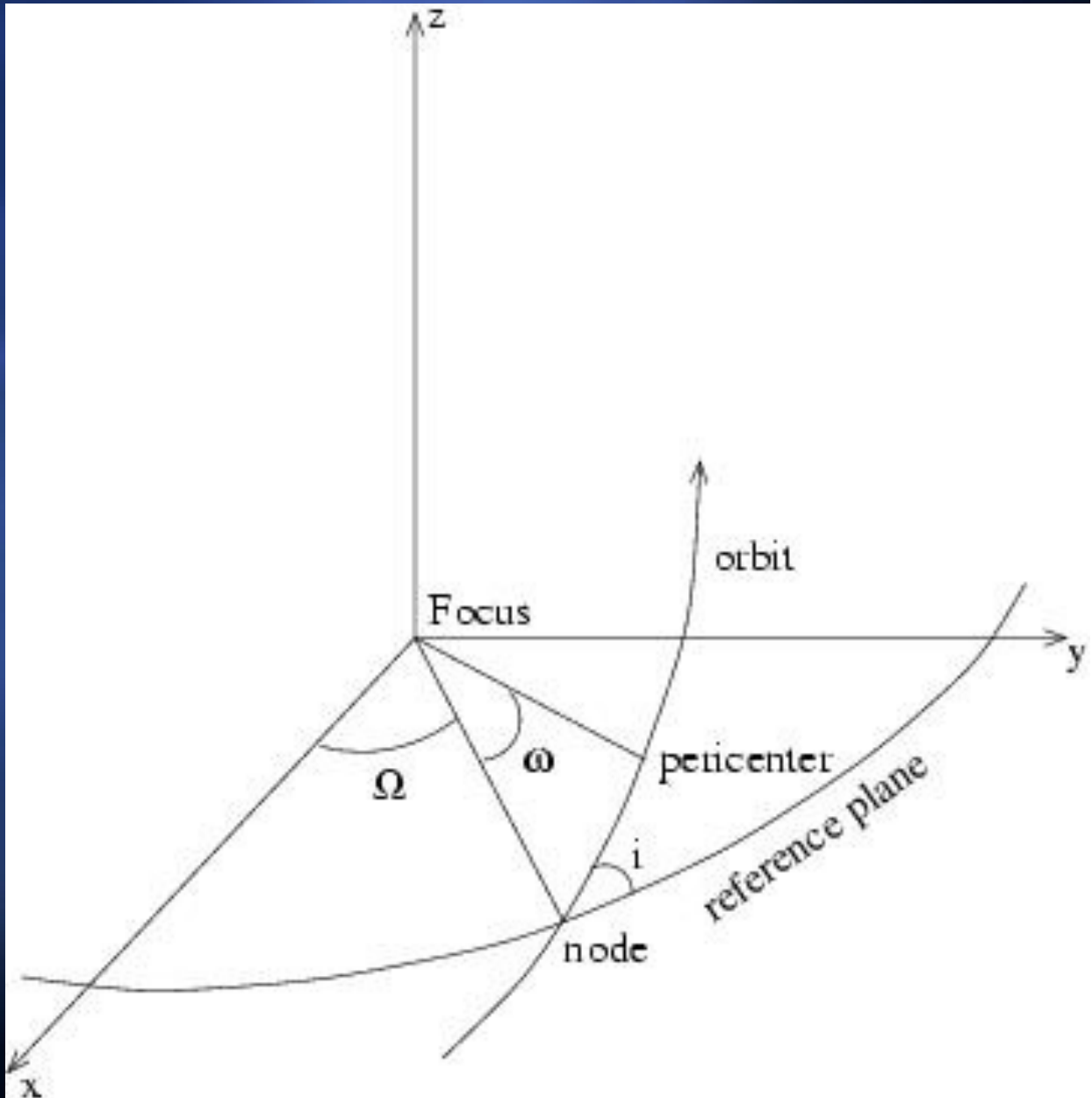
$\omega$  = argument of pericenter

Longitude of pericenter:

$$\varpi = \omega + \Omega$$

Mean longitude:

$$\lambda = M + \varpi$$

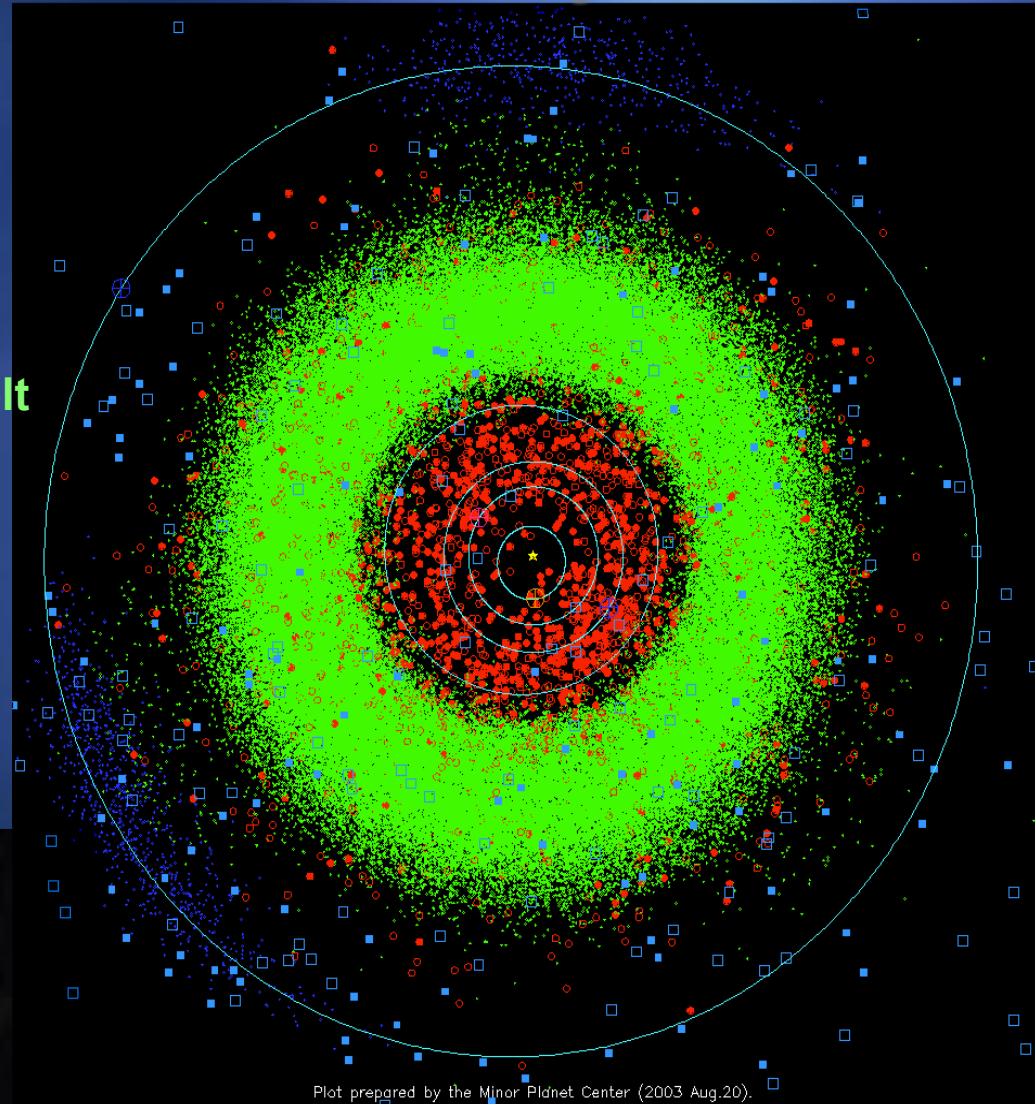


# The small body populations in the Inner Solar System

Green: Asteroid Main Belt

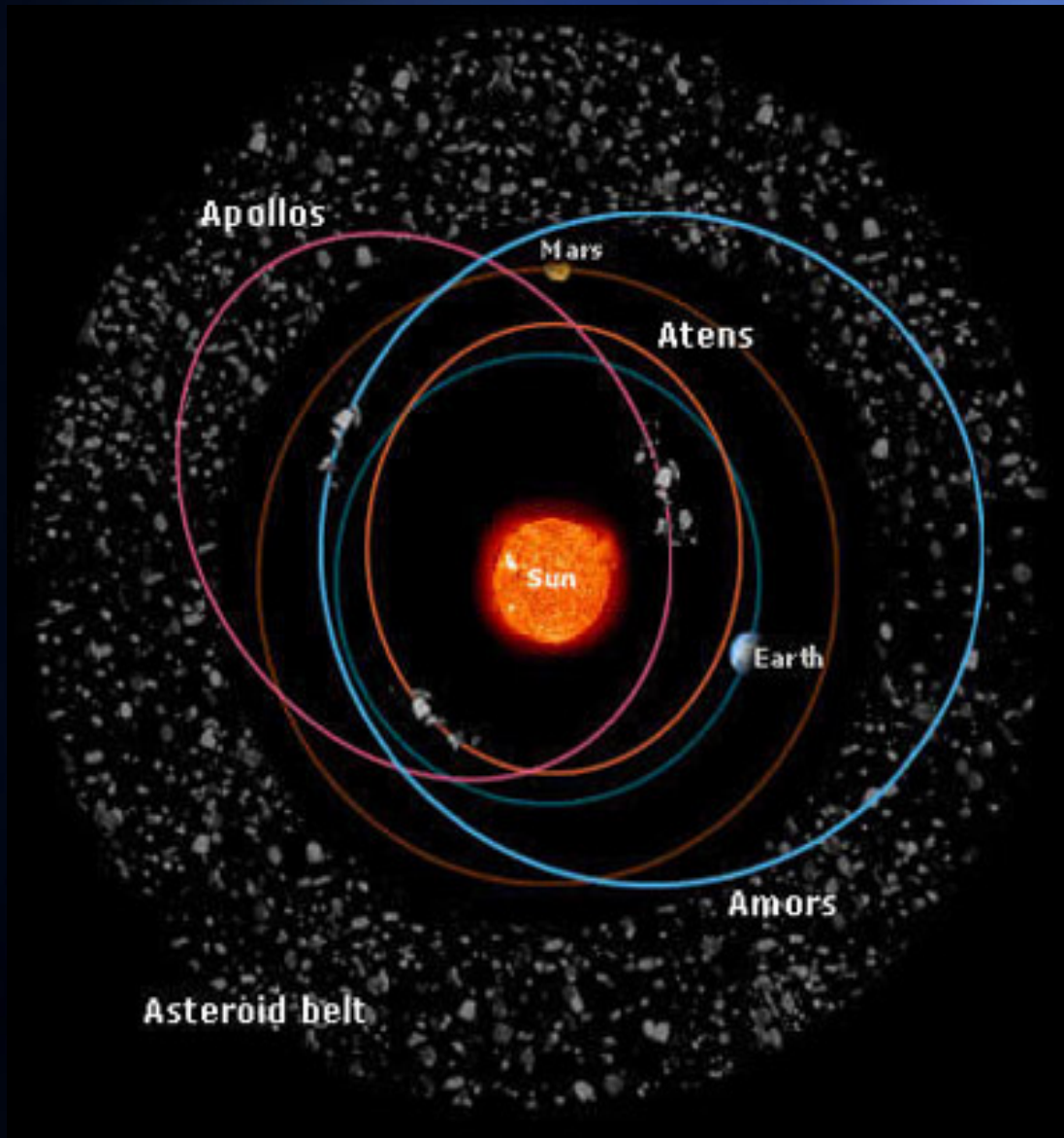
Blue squares: Comets

Red: objects with  
perihelion distance  
 $q < 1.3$  AU



Plot prepared by the Minor Planet Center (2003 Aug. 20).

# The NEO population



•Amors:  $a > 1$  AU  
 $1.017 < q < 1.3$  AU

•Apollos:  $a > 1$  AU  
 $q < 1.017$  AU

•Atens:  $a < 1$  AU  
 $Q > 0.987$  AU

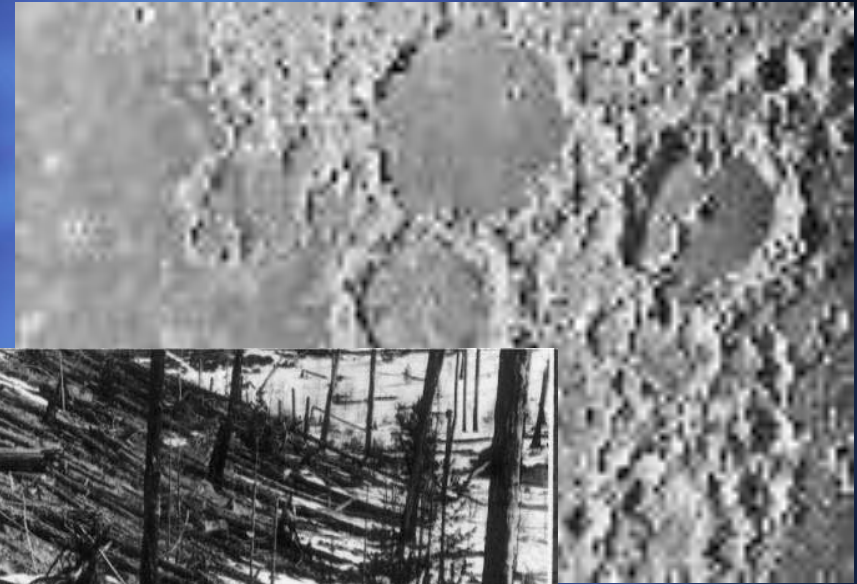
•IEOs:  $a < 1$  AU  
 $Q < 0.987$

1000 Objects  
with  $D > 1$  km,  
 $\approx 500$  discovered

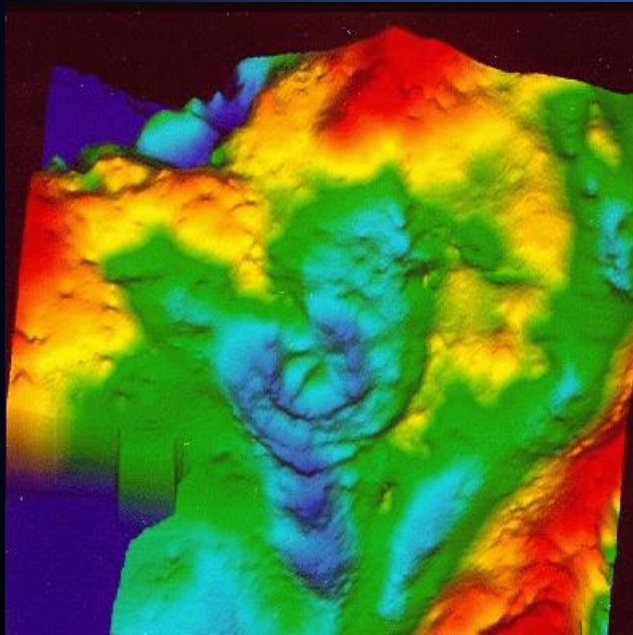
# The NEO threat!

**Impacts are real facts!**

Moon



Venus



Earth:  
Tunguska, 1908

**The least likely natural disaster BUT the only that may be predicted and avoided!**

# *Main transport mechanisms in the Solar System*

## ⊕ Fast mechanisms:

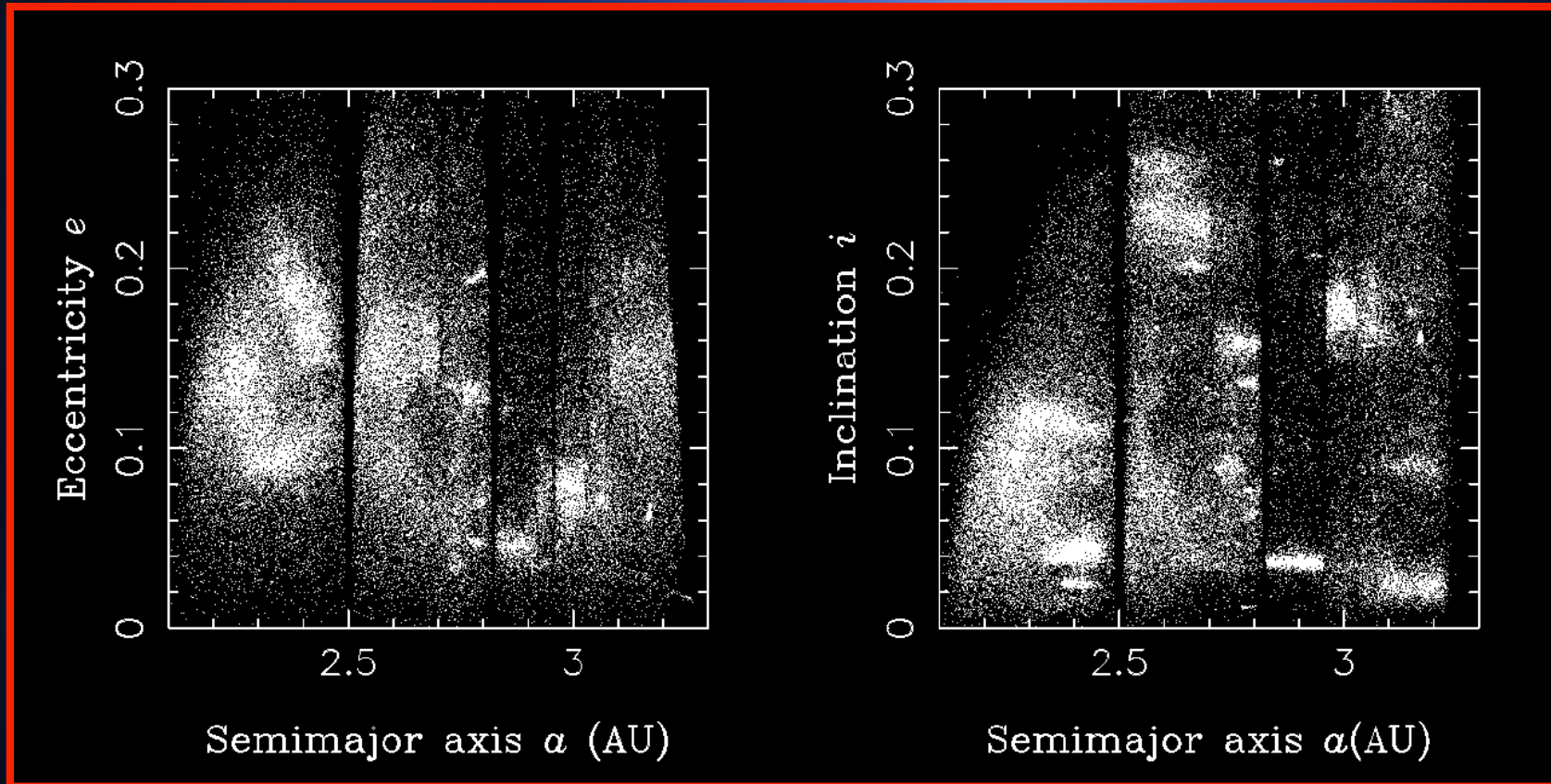
- Mean motion resonances with planets
- First-order secular resonances with planets

## ⊕ Slow diffusions (not described in this lecture):

- Non-gravitational effects (Yarkovsky)
- High-order and three-body resonances

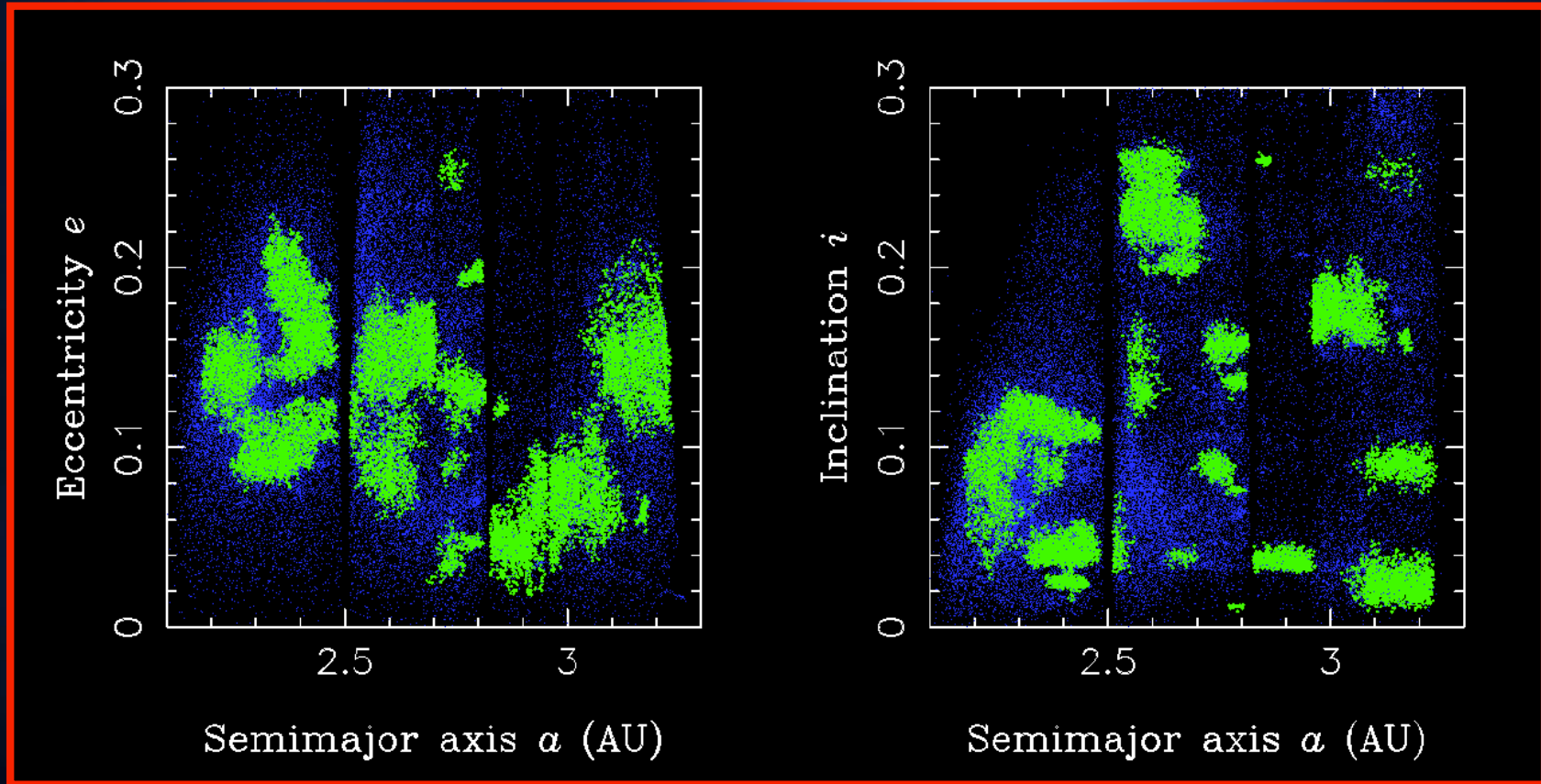


# The Kirkwood gaps in the asteroid Main Belt

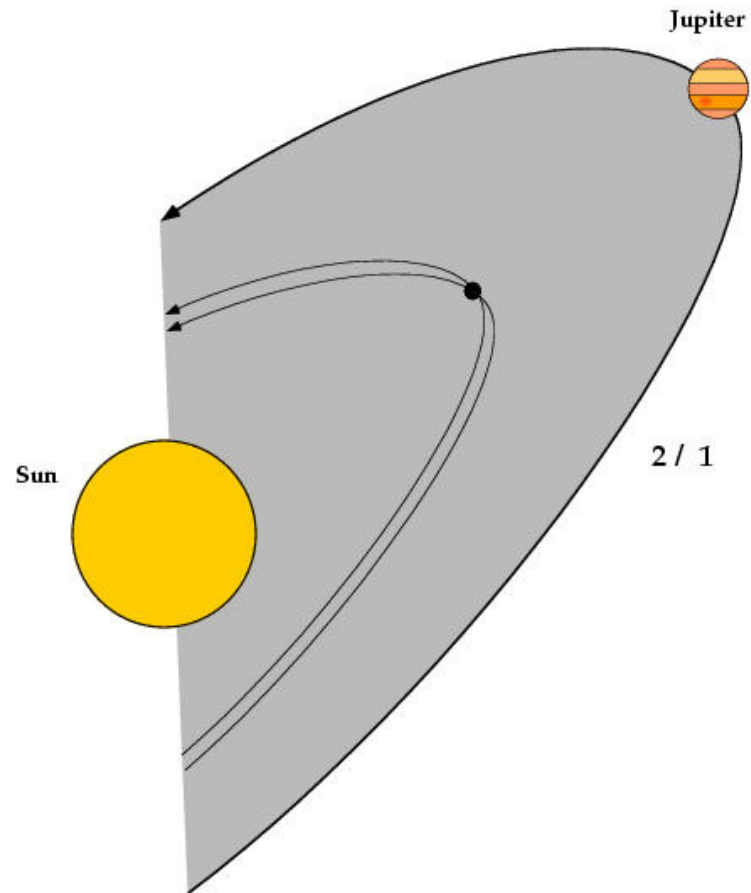


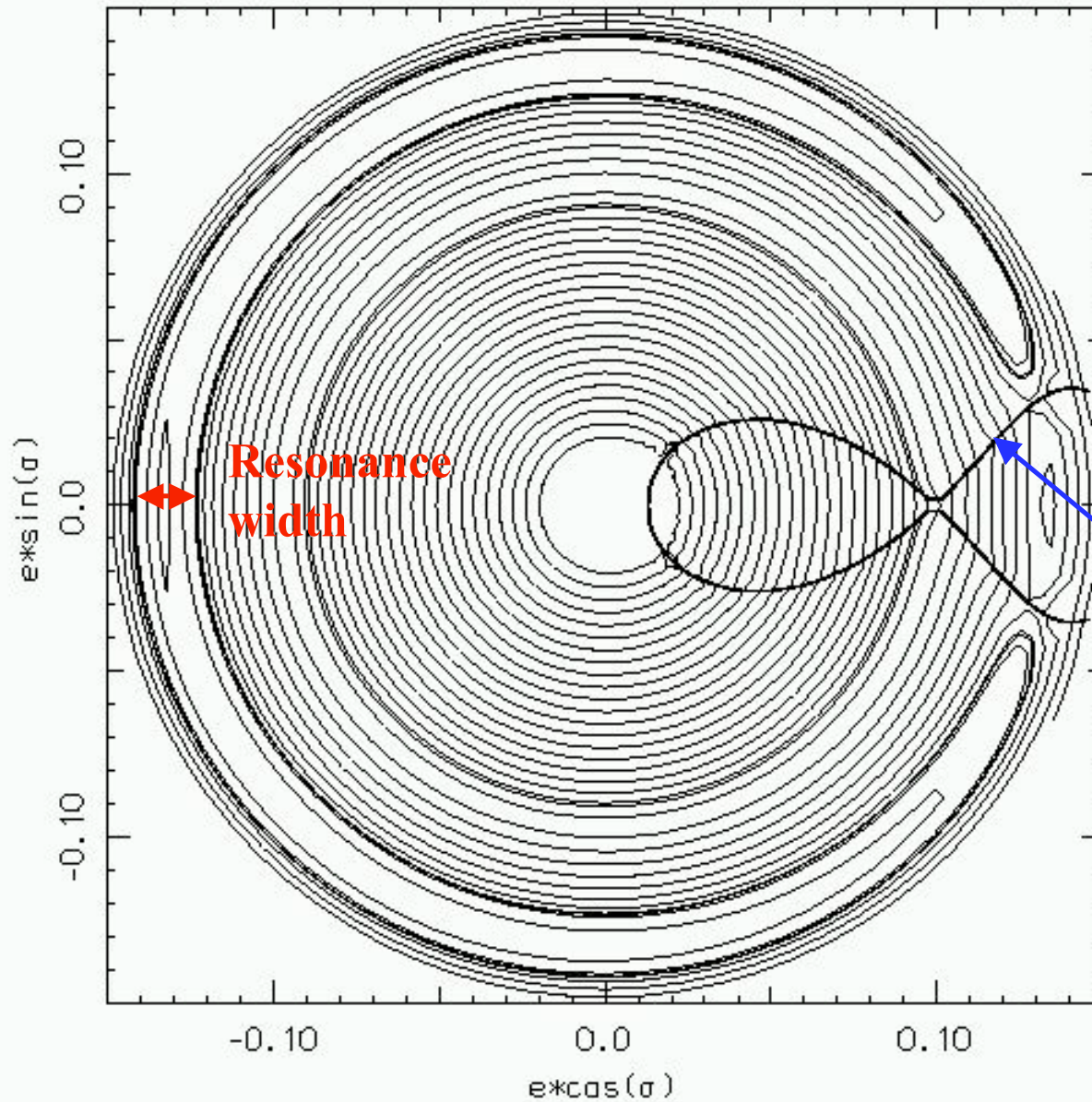
**Collisions produce asteroid families!**

**This will be addressed in Chapter II**



## Mean Motion Resonances

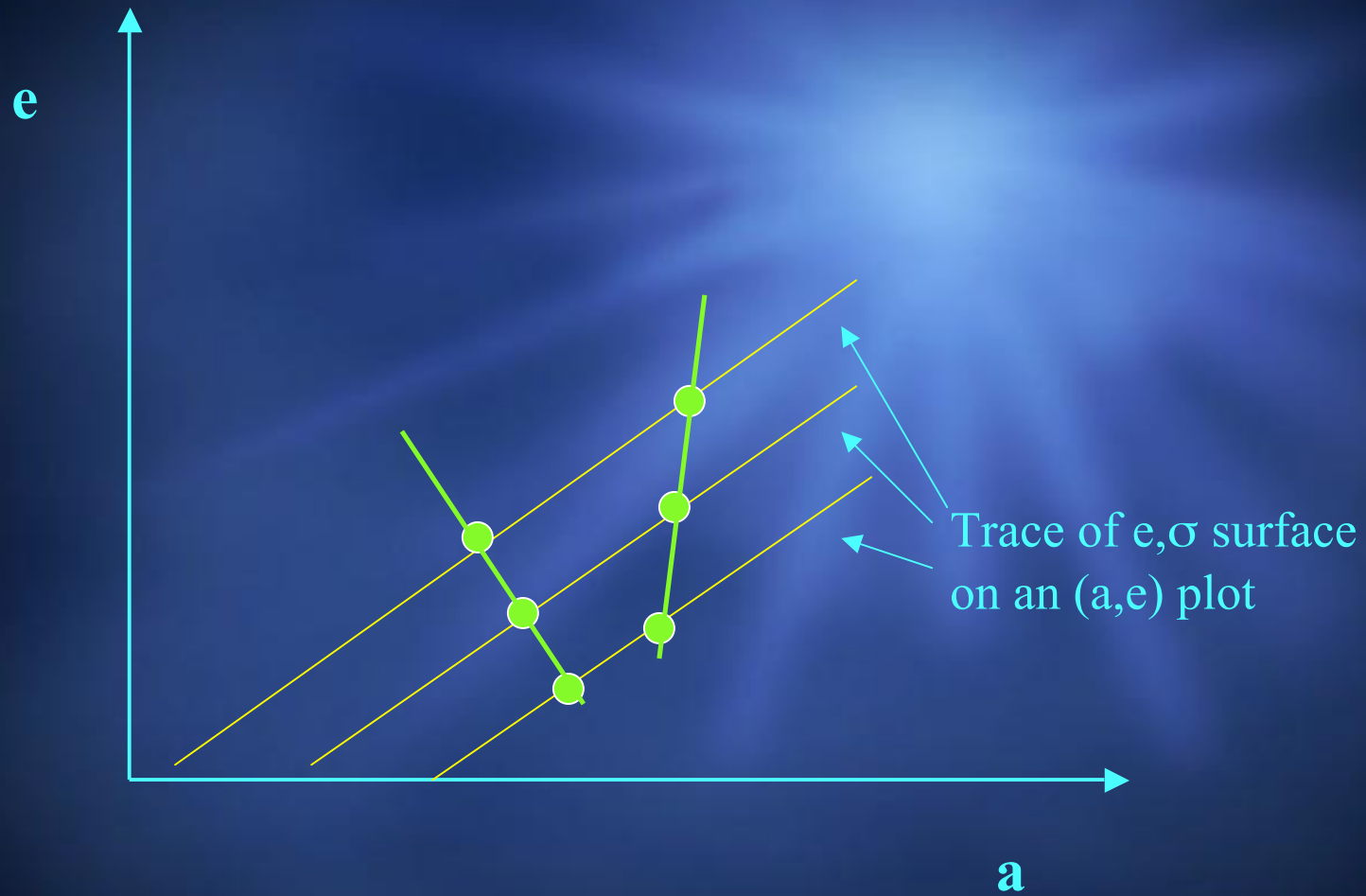




MM Resonance  
 (e,σ) surface plot

$$\sigma_{i/j} = i\lambda_p - j\lambda - (i-j)\varpi$$

Planet collision line

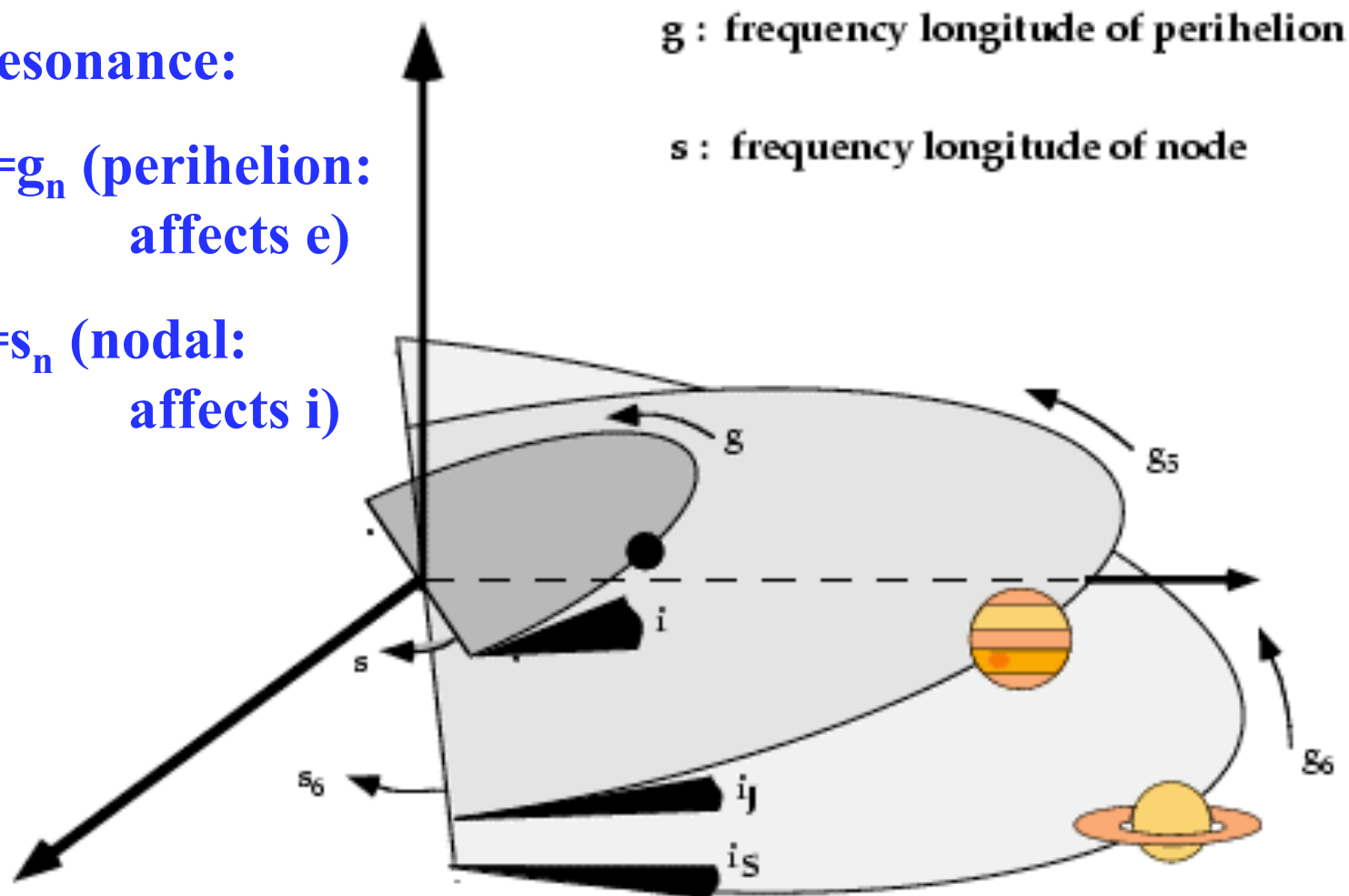


# SECULAR RESONANCES

**Resonance:**

$g=g_n$  (perihelion:  
affects  $e$ )

$s=s_n$  (nodal:  
affects  $i$ )



# Main principle

$$H(\dot{\mathbf{r}}, \dot{\mathbf{r}}_j; \mathbf{r}, \mathbf{r}_j) = \frac{1}{2} \|\dot{\mathbf{r}}\|^2 - \frac{1}{\|\mathbf{r}\|} - \sum_{j=1}^{N_p} m_j \left[ \frac{1}{\|\Delta_j\|} - \frac{\mathbf{r}_j \cdot \mathbf{r}}{\|\mathbf{r}_j\|^3} \right]$$

At first order in planetary mass ( $j = \text{planet index}$ ), the hamiltonian of a massless body expresses as:

$$H = -\frac{1}{2L^2} - \sum_{j=2}^{N_p} m_j P_j(L, G, H, L_j, G_j, H_j; l, g, h, l_j, g_j, h_j),$$

Keplerian part

Planetary perturbations

$$L = \sqrt{a}$$

$$l = M$$

$$G = \sqrt{a(1 - e^2)}$$

$$g = \omega$$

Delaunay variables

$$H = \sqrt{a(1 - e^2)} \cos i \quad h = \Omega$$

# *Assumption: the small body is not in a mean motion resonance*

- ⊕ The Hamiltonian (at 1st order in planet masses) can be averaged over all mean anomalies  $l$  and  $l_j$  (fast angles)

$$\bar{H} = -\frac{1}{2L^2} - \sum_{j=2}^{N_P} m_j \bar{P}_j(-, G, H, -, G_j, H_j; -, g, h, -, g_j, h_j)$$

$L = \text{cste}$ , so we omit the first term and expand the perturbation w.r.t. planetary eccentricities and inclinations:

$$\bar{H} = - \sum_{j=2}^{N_P} m_j \left[ K_0^j + (e_j, i_j) K_1^j + (e_j, i_j)^2 K_2^j + \dots \right]$$

$(e_j, i_j)^r$  are terms prop. to  $e_j^a i_j^b$ , with  $a+b = r$  and  $a, b \geq 0$



# Isolate the first term $K_0$

⊕ It can be shown that:

$$K_0 = \sum_{\substack{\geq 0 \\ p, q \in \mathbb{N}}} c_{0, -v, v, 0, p, q, 0, 0} e^{i^{|2v|+2p} i^{|2v|+2q}} \cos(2v(\underbrace{\varpi - \Omega}_{\omega}))$$

Thus,  $K_0 = f(\varpi - \Omega) = f(g)$

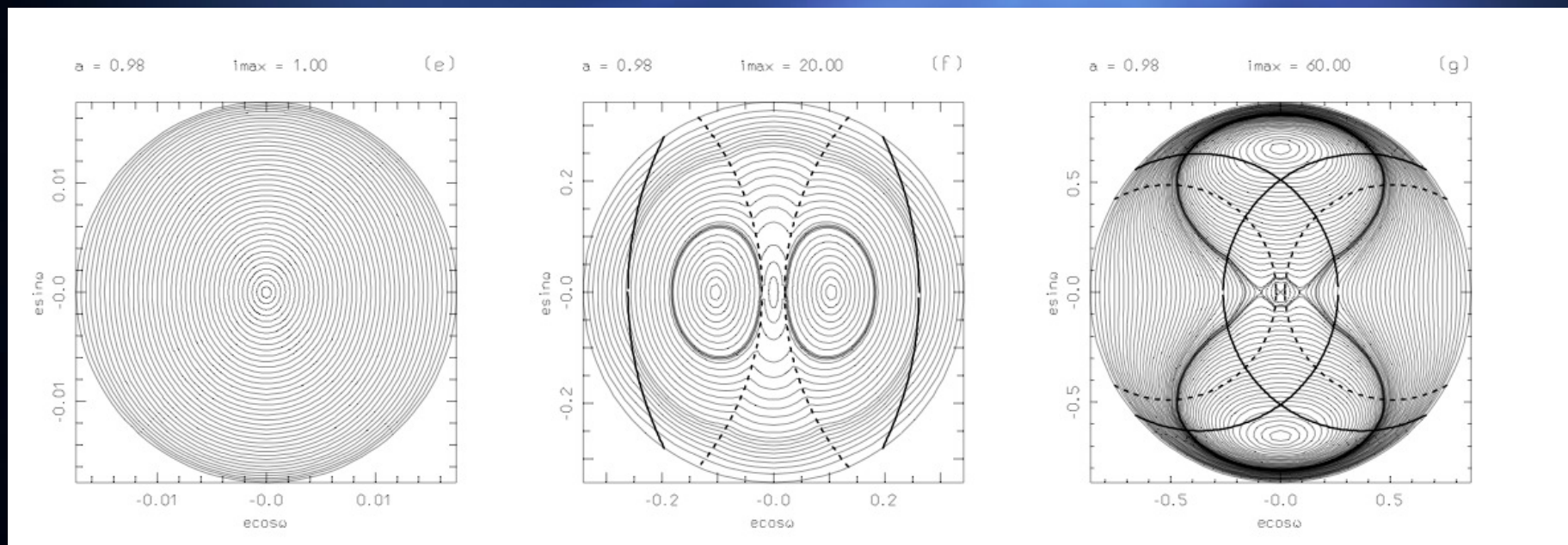
$K_0 = 1$  degree of freedom integrable hamiltonian in the variables  $G, g$  as it depends only on the angle  $g (= \omega)$ .

It is parametrized by the constants actions  $L$  and  $H$ .

Its highly non-linear dynamics can be studied in details (Kozai 1962) by drawing level curves in the  $(e, \omega)$  plane on a surface  $H = \text{constant}$ .

# Dynamics of $K_0$ at $a=0.98$ AU on 3 different surfaces of $H=cst$ , each characterized by a value of $i_{max}$ ( $1^\circ$ , $20^\circ$ , $60^\circ$ )

⊕ Polar diagram ( $e, \omega$ )



From Michel & Thomas (1996, AA 307)

# Location of secular resonances

- ⊕ The free frequencies of  $\varpi$  and  $\Omega$  of the asteroid's orbit in the (a,e,i) space are obtained by integrating wrt time:

$$\dot{G} = - \left( \frac{\partial K_0}{\partial g} \right),$$

Proper frequencies = average values over a complete cycle of the free oscillations with period T (from  $g=0$  to  $g=g(T)=2\pi$ )

$$\dot{g} = \left( \frac{\partial K_0}{\partial G} \right),$$

**Secular resonance:** (a,e,i) for which:

$$\dot{h} = \left( \frac{\partial K_0}{\partial H} \right)$$

$$\langle \dot{\varpi} \rangle = g_{planet}$$

$$\text{Ex: } \nu_6 \rightarrow g_6 \approx 28.22 \text{ ''/yr}$$

or

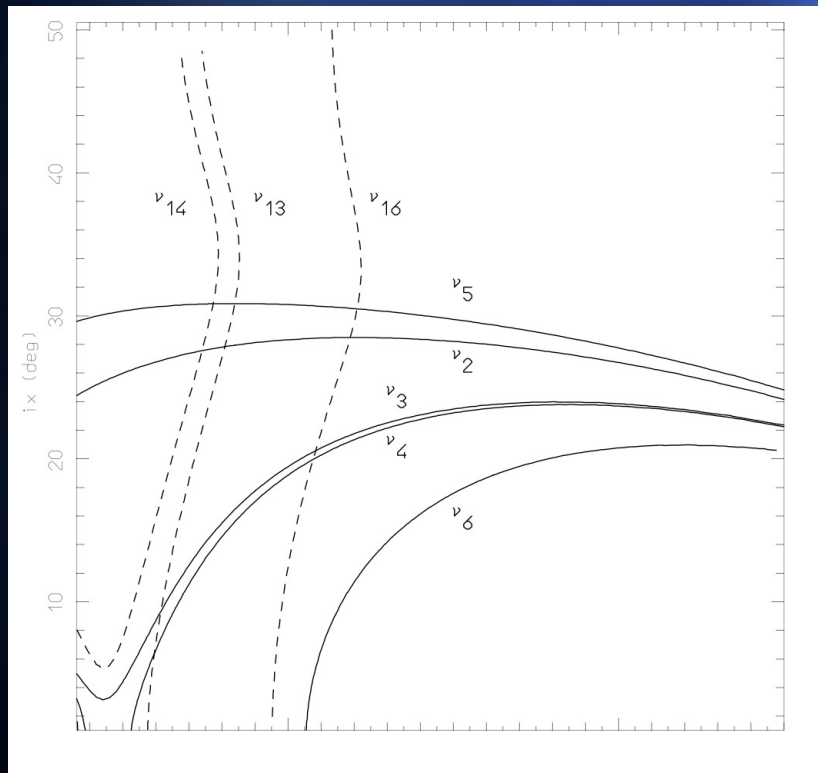
$$\langle \dot{\Omega} \rangle = S_{planet}$$

# Some secular resonance locations (left: main belt, right: NEO region)

From Michel & Froeschlé (1997, *Icarus* 128)

50°

$i^\circ$



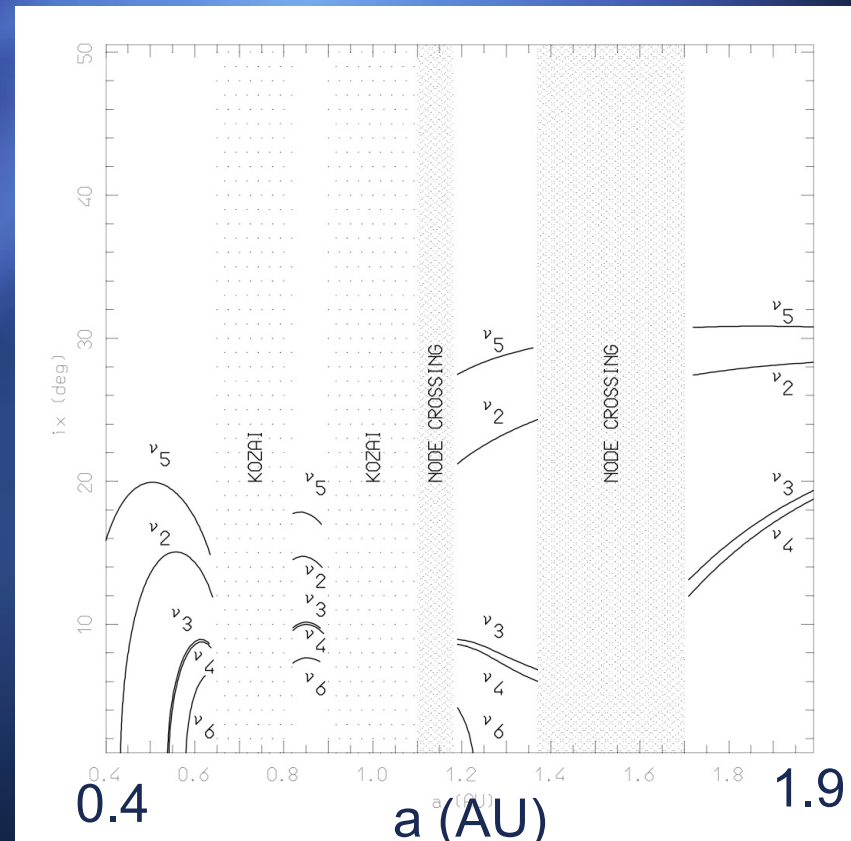
1.5

$a$  (AU)

3.5

COE Planetary  
School 12/4/2006

$e=0.1$



0.4

$a$  (AU)

1.9

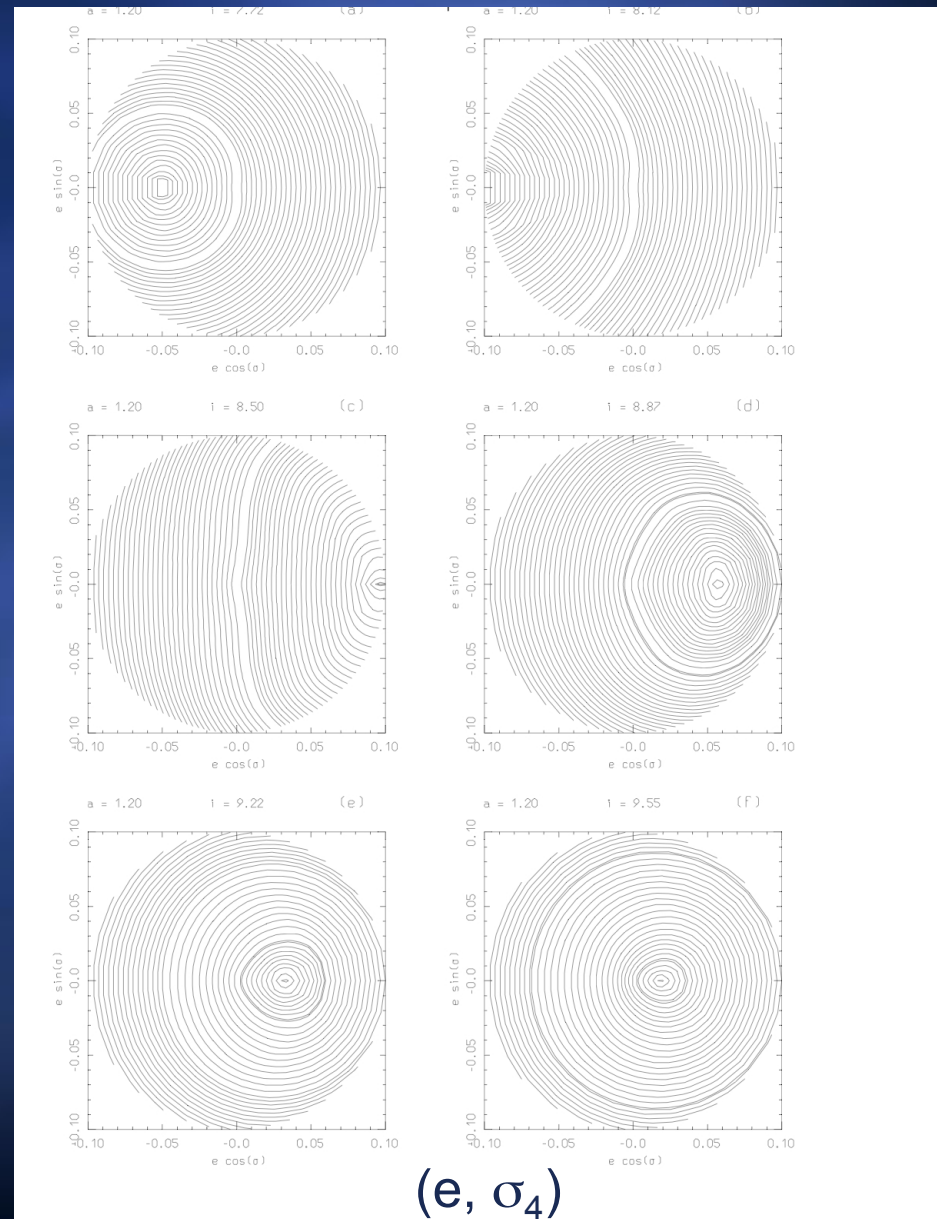
20

# Effect of secular resonances

Needs to consider  
the first-order term  
in  $e_j$  and  $i_j$  of  
the Hamiltonian  
( $K_1$ )

0.1

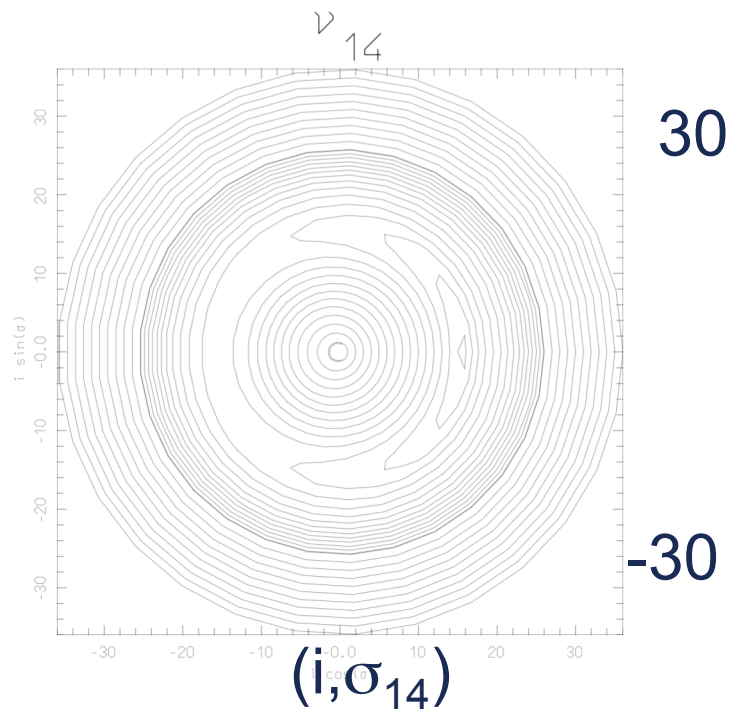
*Ex: effect on  
eccentricity of  $v_4$   
at  $a=1.2$  AU*



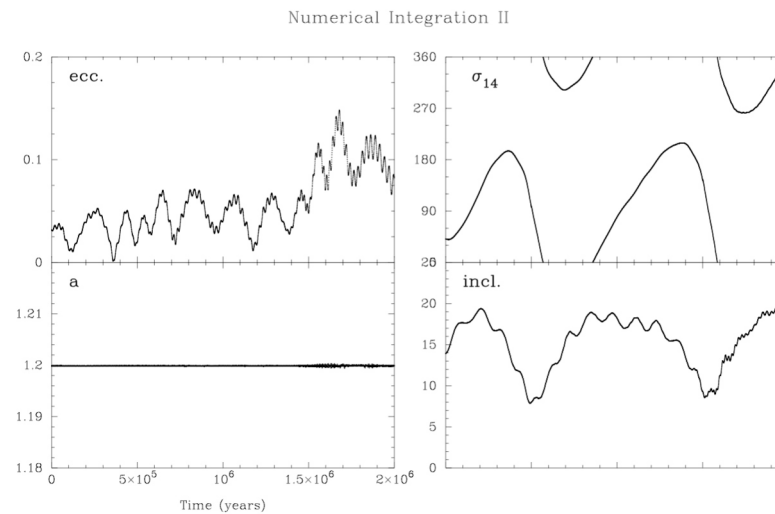
# Effect of secular resonances (II)

Semi-analytical theory

$a = 1.2 \text{ AU}$



Numerical integration



# *Effect of resonance overlapping*

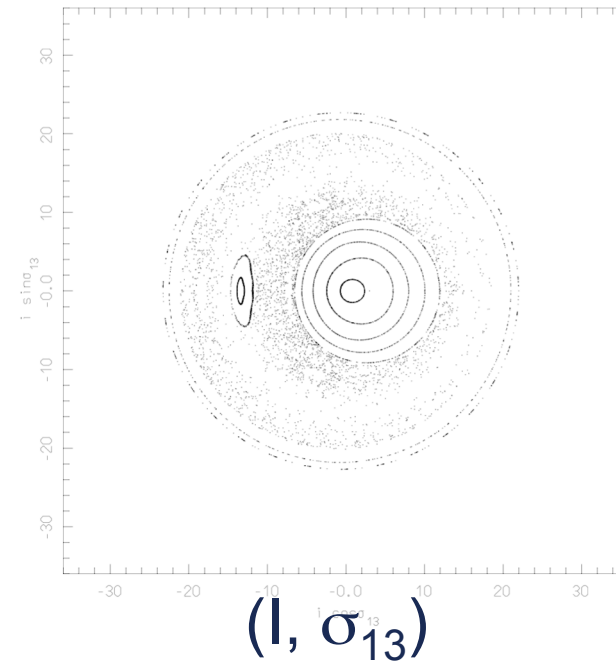
Overlapping of  
 $\nu_{13}$  and  $\nu_{14}$

Surface of section  
at  $\sigma_{14} = \pi$

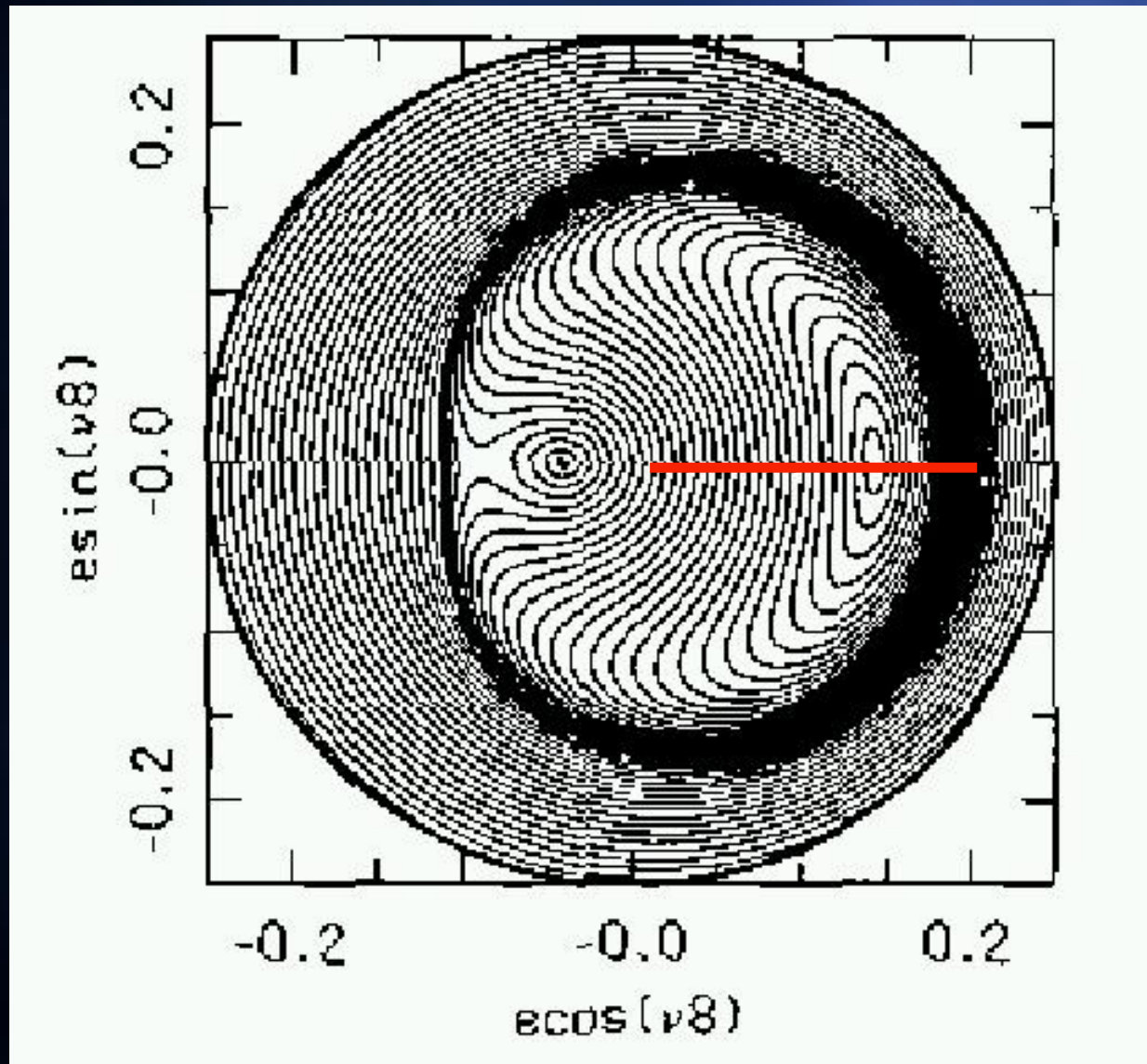
30

-30

$a = 1.2 \text{ AU}$



## Example: Dynamics of the $g=g_8$ resonance at 41 AU



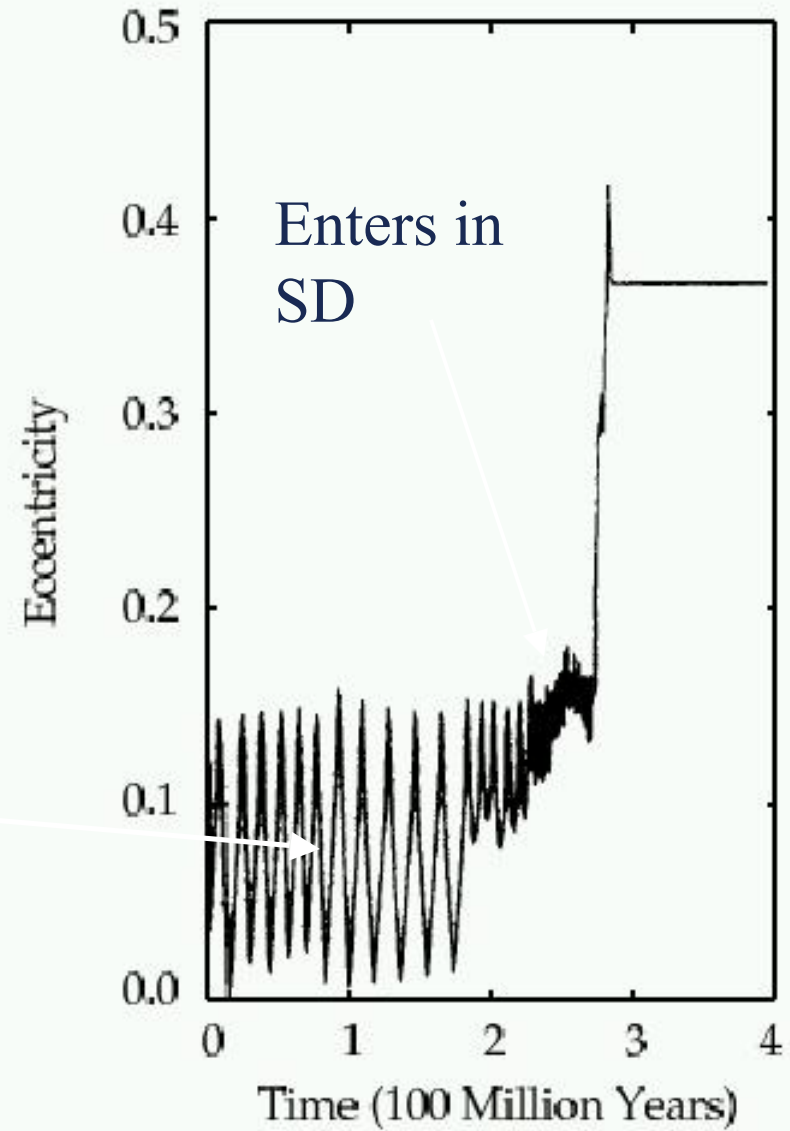
$$\nu_8$$

$$\sigma_8 = \varpi - \varpi_N$$



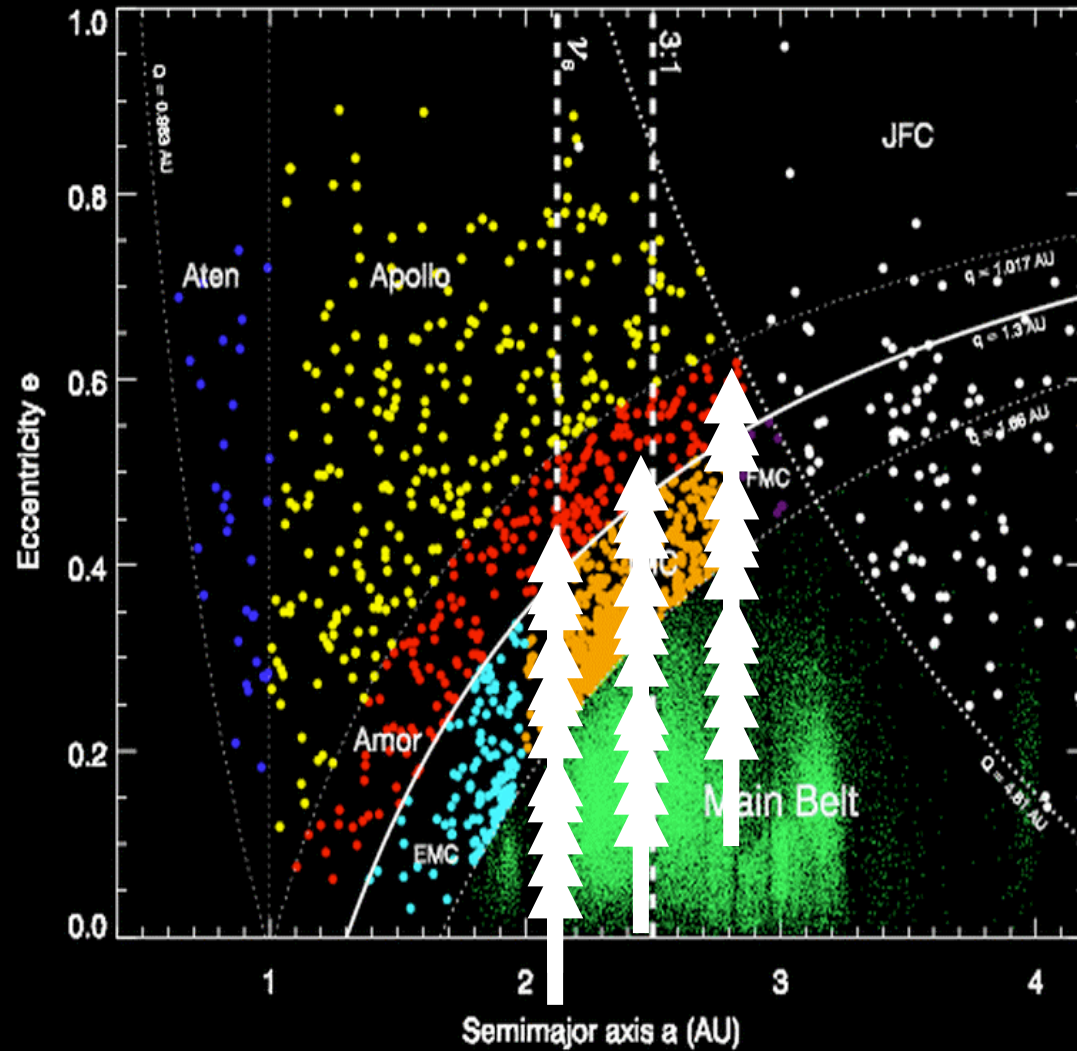
**Simulation of the evolution  
of a body in the  $g=g_8$   
resonance by Holman and  
Wisdom, 1993**

Secular resonance driven  
slow oscillations



# Origin of NEOs

Asteroids from different regions of the Main Belt (MB) are injected into resonances which transport them on Earth-crossing orbits



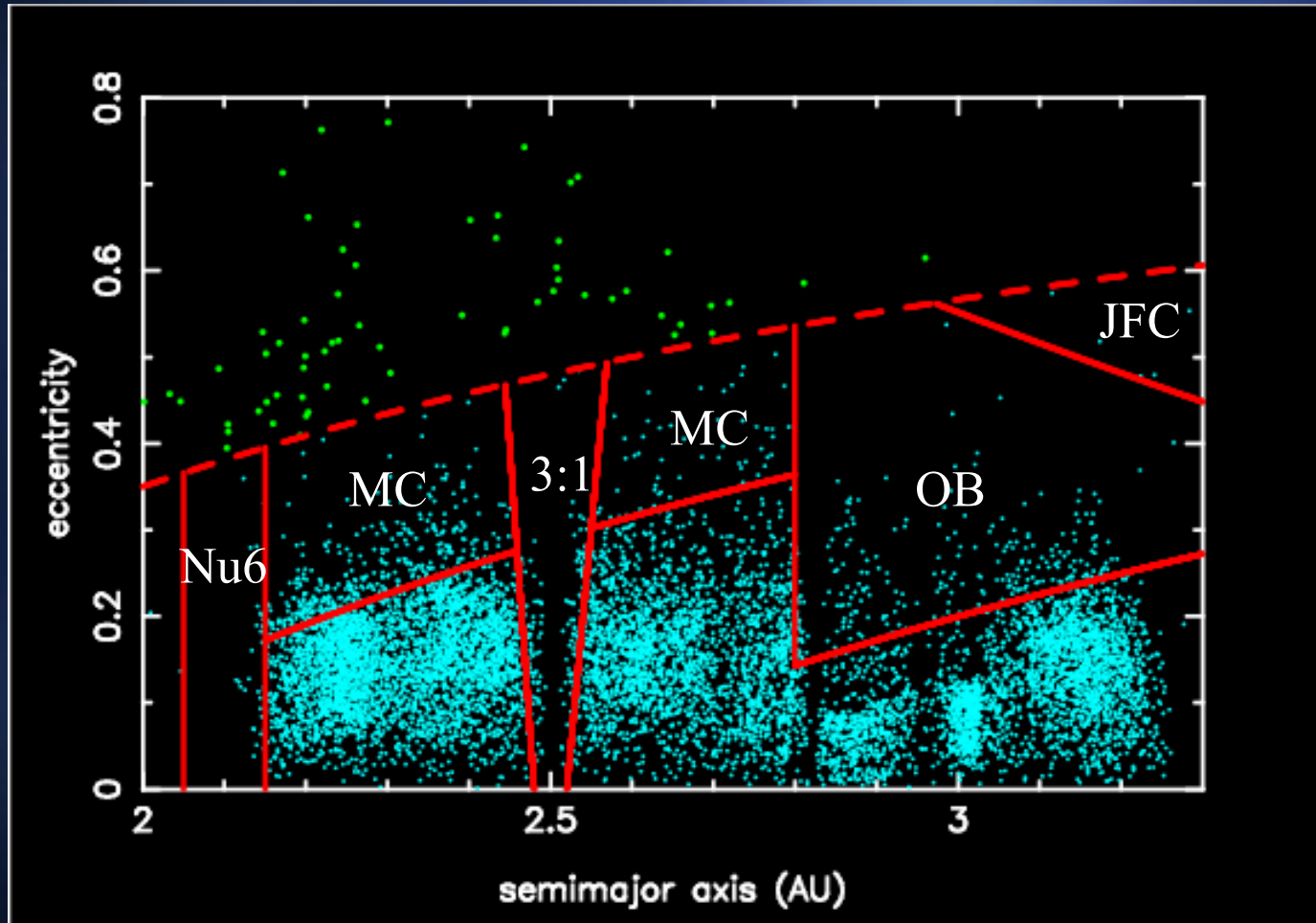
Hamiltonian describing the evolution of a massless body:

$$H(\dot{\mathbf{r}}, \dot{\mathbf{r}}_j; \mathbf{r}, \mathbf{r}_j) = \frac{1}{2} \|\dot{\mathbf{r}}\|^2 - \frac{1}{\|\mathbf{r}\|} - \sum_{j=1}^{N_p} m_j \left[ \frac{1}{\|\Delta_j\|} - \frac{\mathbf{r}_j \cdot \mathbf{r}}{\|\mathbf{r}_j\|^3} \right]$$

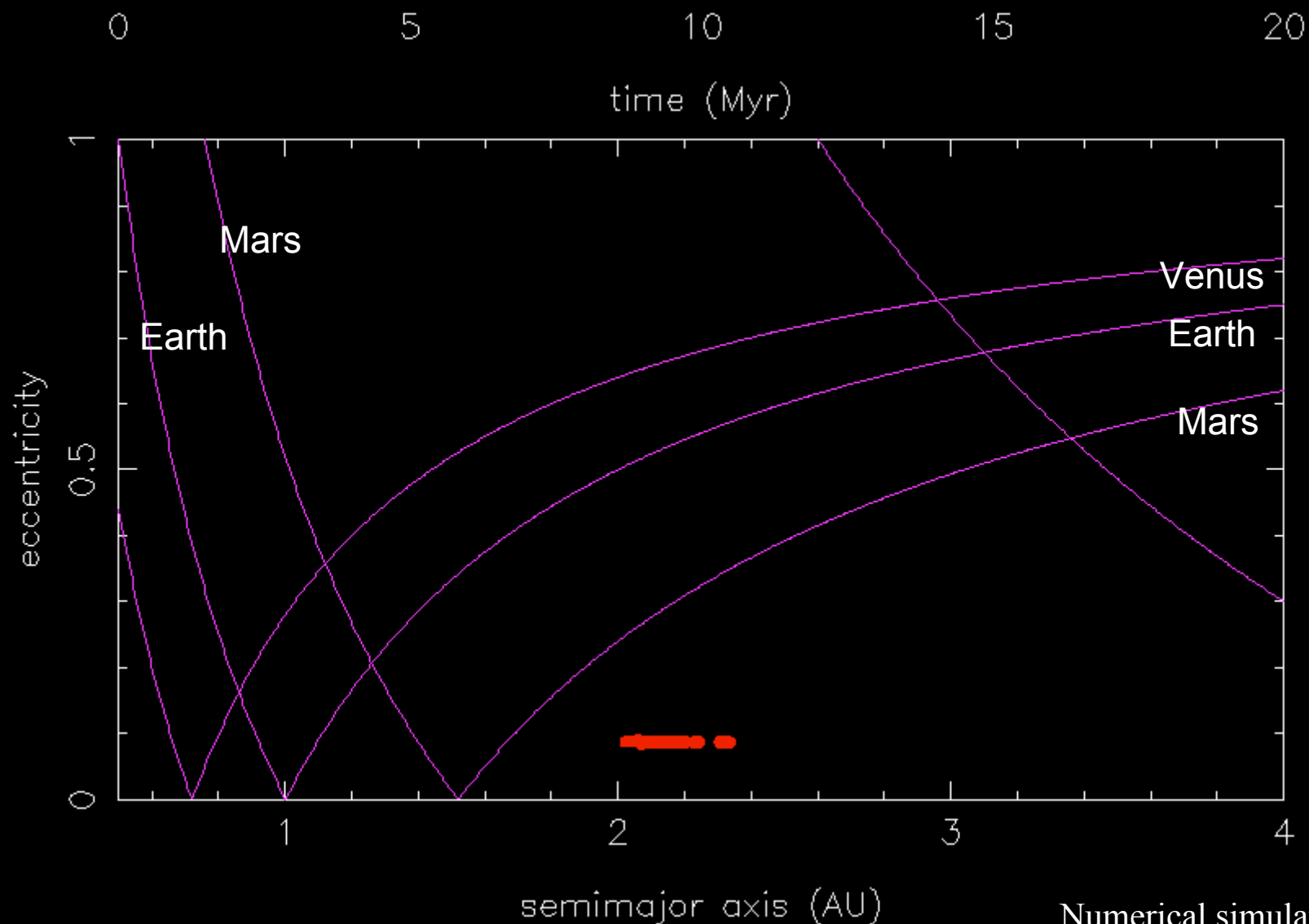
(heliocentric frame)

$$\Delta_j = \mathbf{r}_j - \mathbf{r} \quad G = M_{\text{sol}} = 1$$

# SPECIFIC SOURCES OF NEOs:

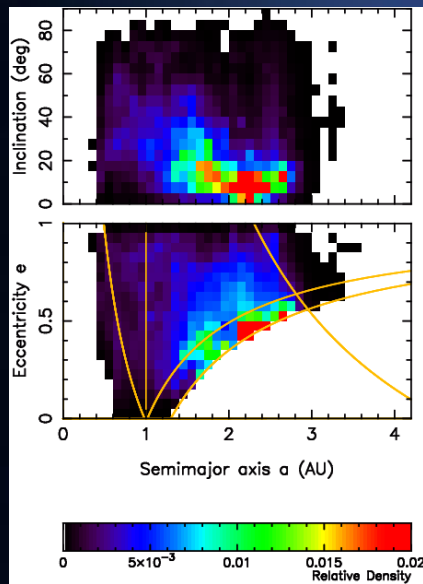


**Fast resonances:** Main Belt Asteroids become rapidly NEOs by dynamical transport from a source region (in a few million years)

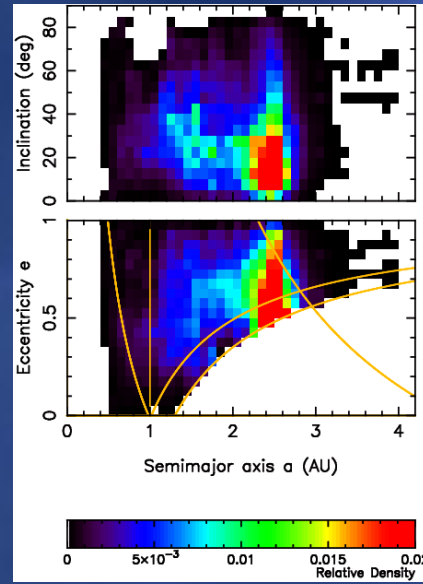


Numerical simulations:  
Several 1000 particles

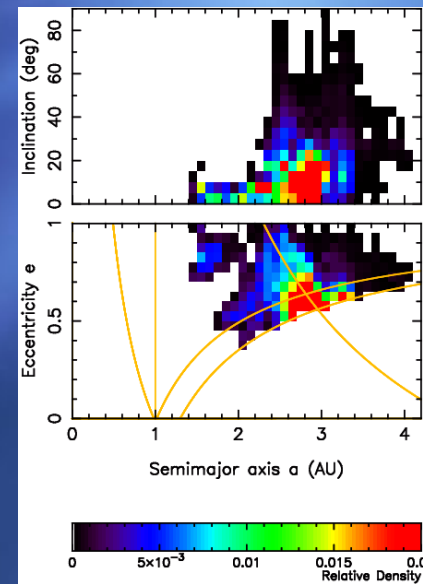
# Combine the sources of NEOs so that applying observational biases on the total distribution reproduces the observed distribution



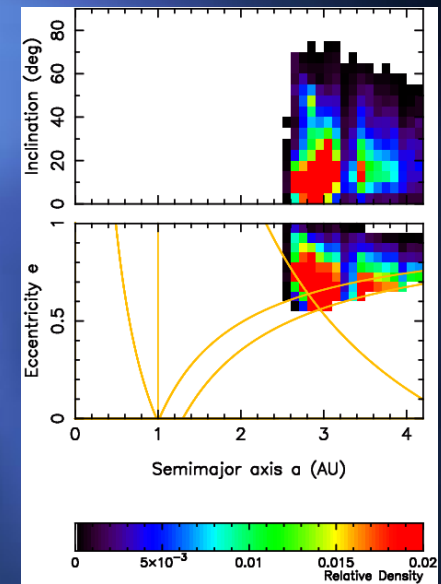
Mars Crossers



3:1



External Belt



Comets (Jup.)

Combine NEO Sources  
 $R(a, e, i)$

nu6

IMC

3:1

Outer MB

JFCs

# Comparison between the *biased* model of NEOs and real data

Continue Until "Best-Fit" Found

(5)

Compare with Spacewatch NEO Data  
 $n(a,e,i,H) = \text{"Known NEOs"}$ ?

(4)

"Observed" NEO Distribution  
 $n(a,e,i,H)$

(3)

Observational Biases  
 $B(a,e,i,H)$

Debiased NEO Orbits  
 $\text{Model}(a,e,i,H)$

(2)

Combine NEO Sources  
 $R(a,e,i)$

Abs. Mag. Distribution  
 $N(H)$

(1)

nu6

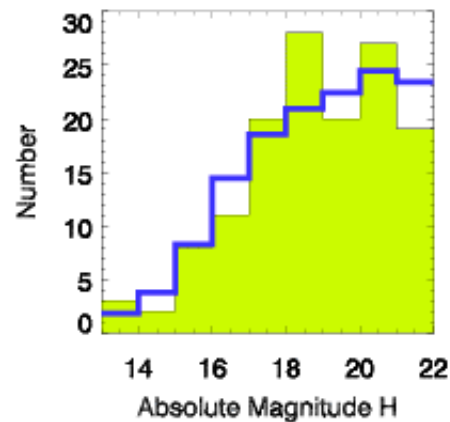
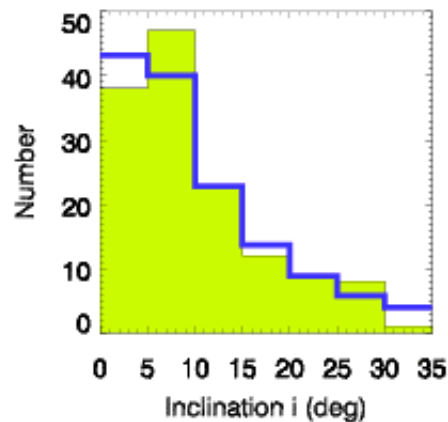
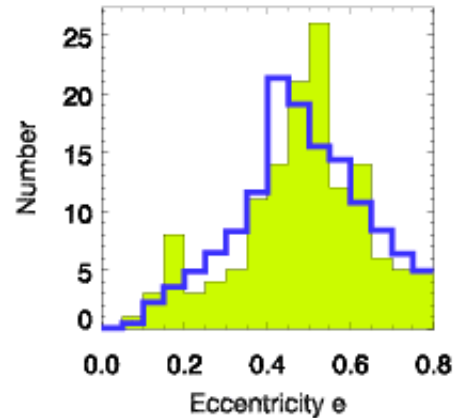
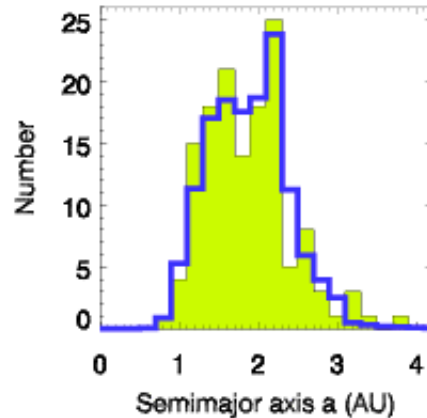
IMC

3:1

Outer MB

JFCs

# Comparison Between Discovered NEOs and Best-Fit Model



## Weighting factors

$v_6$	$0.36 \pm 0.09$
IMC	$0.29 \pm 0.03$
3:1	$0.22 \pm 0.09$
Outer MB	$0.06 \pm 0.01$
JFC	$0.07 \pm 0.05$

Model fit to 138  
Spacewatch NEOs  
with  $H < 22$

# Our model of real orbital and absolute magnitude distributions of *Near Earth Objects*

~1000 NEOs with  $H < 18$  and  $a < 7.4$  AU

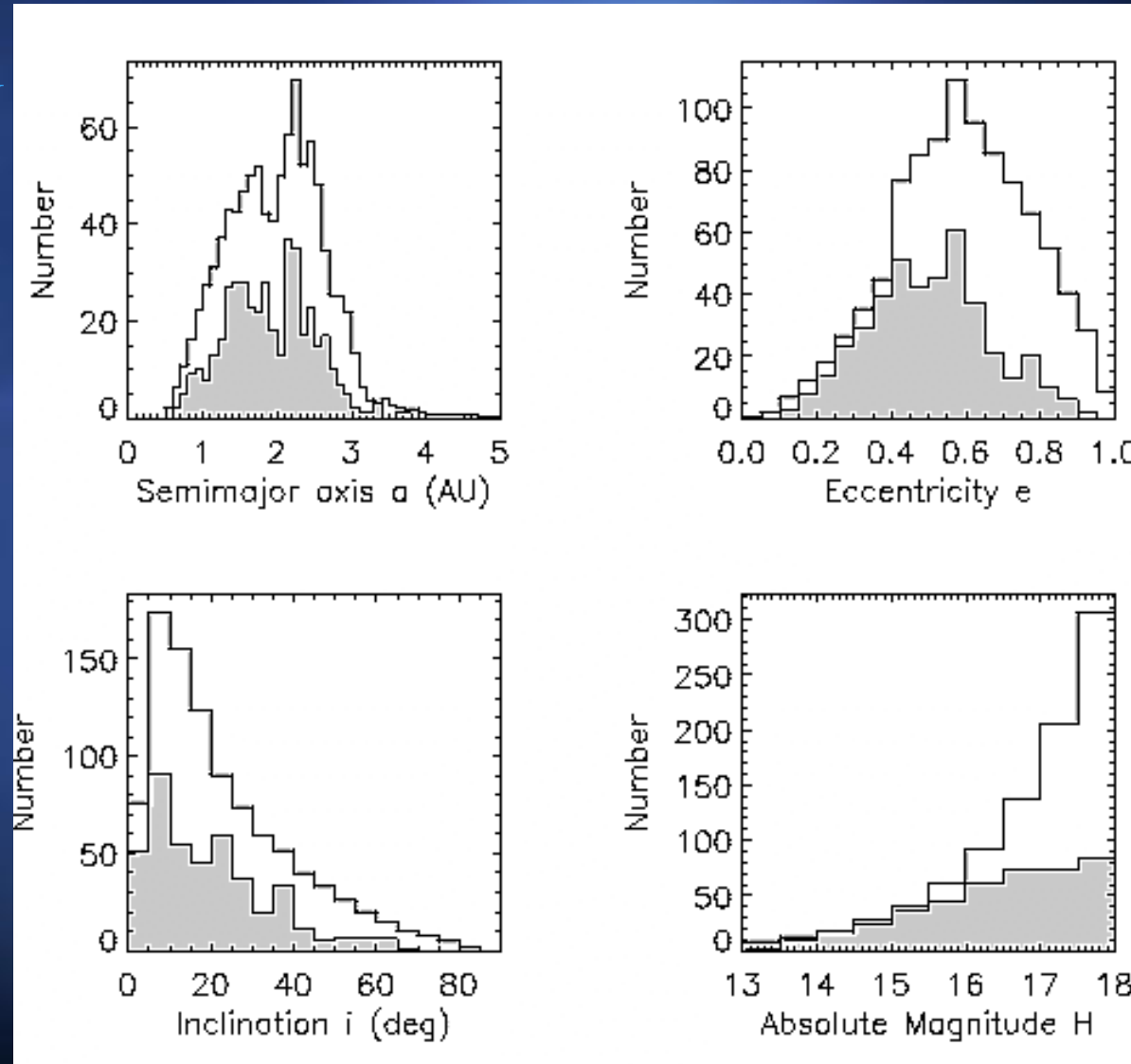
32% Amors

61% Apollos

6% Atens

94% of asteroidal origin

6% dormant comets (Jupiter family)

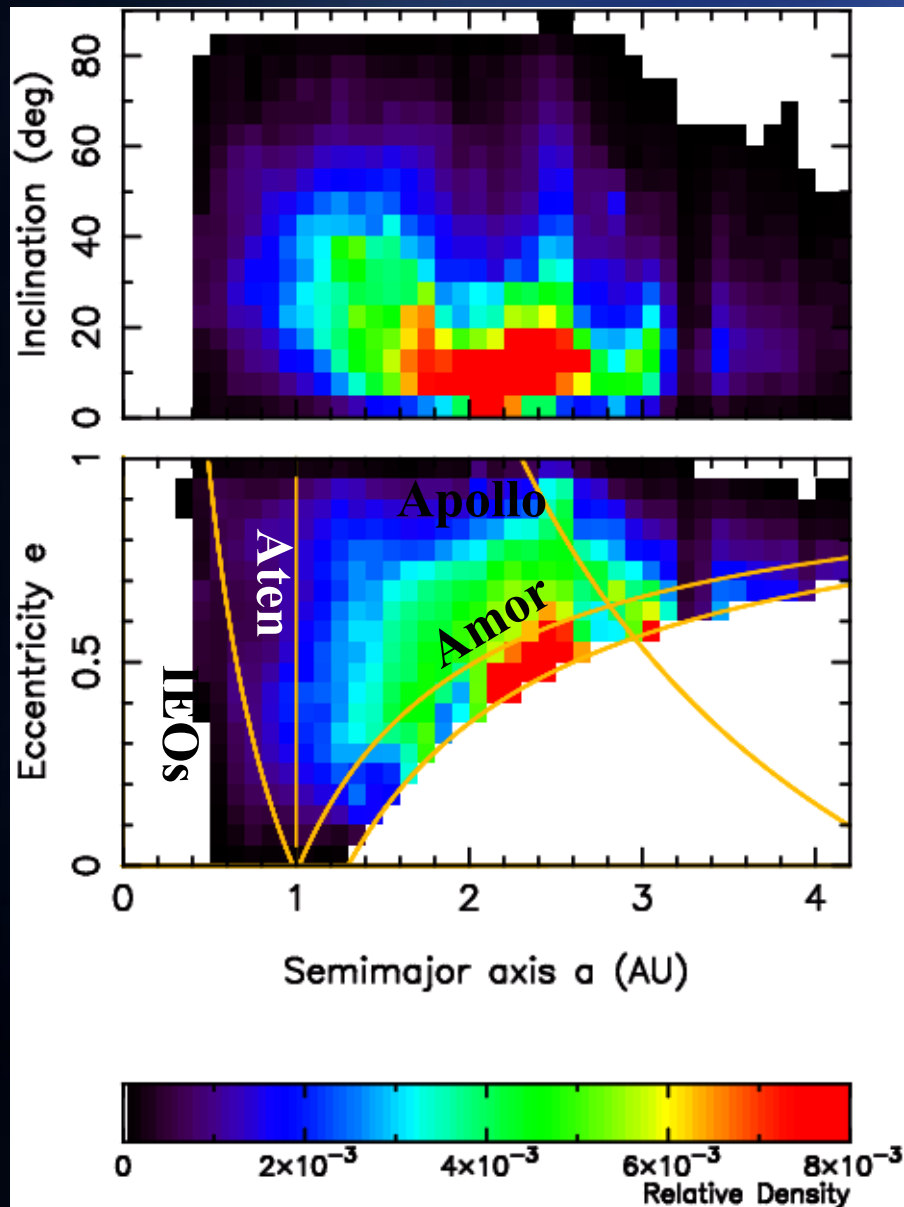


(Bottke et al., 2000, 2002)

White = model; Gray = observations



# Debiased NEO Orbital Distribution



- The NEO population having  $H < 22$  and  $a < 7.4$  AU consists of:
  - 32% Amors.
  - 61% Apollos.
  - 6% Atens.
- 2% are IEOs (Inside Earth's Orbit).

**Estimate of 1 impact with energy > 1,000MT per 64,000 years**

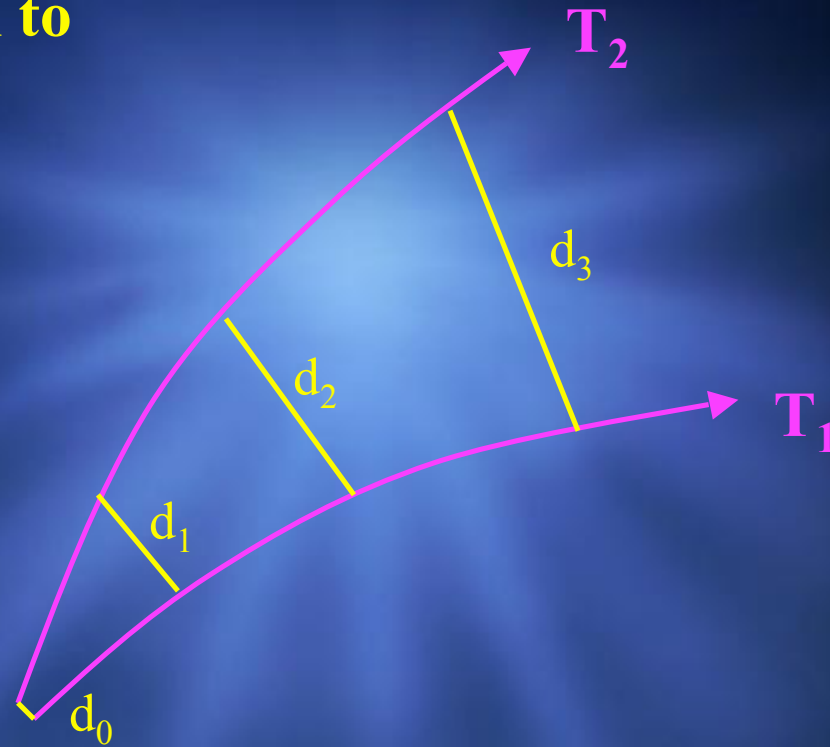
**Known NEOs carry only 18% of this total collision probability  
( $H < 20.5$ )**

Morbidelli et al. 2002

<b>Impact Energy</b>	<b>Mean Frequency (years)</b>	<b>Mean projectile's size</b>	<b>Completeness</b>
<b>1,000 MT</b>	<b>63,000</b>	<b>277 m (<math>H=20.5</math>)</b>	<b>16%</b>
<b>10,000 MT</b>	<b>241,000</b>	<b>597 m (<math>H=18.9</math>)</b>	<b>35%</b>
<b>100,000 MT</b>	<b>935,000</b>	<b>1,287 m (<math>H=17.5</math>)</b>	<b>50%</b>
<b>1,000,000MT</b>	<b>3,850,000</b>	<b>2,774 m (<math>H=15.6</math>)</b>	<b>70%</b>

**The Lyapunov exponent: a tool to  
characterize the chaotic  
nature of an evolution**

$$L = \lim_{t \rightarrow \infty} \text{Log}(d_t)/t$$



NEOs have positive Lyapunov exponent indicating chaotic evolutions

⇒ impossible to make long term predictions of individual trajectories



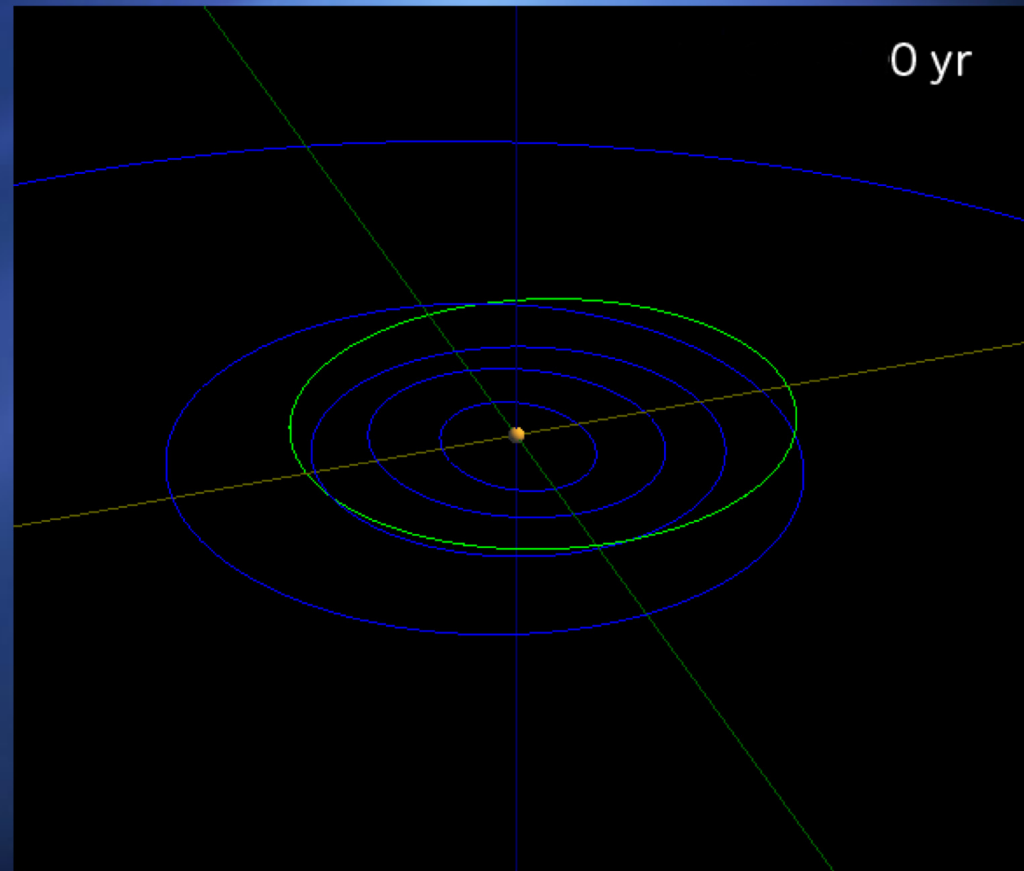
# *NEOs have chaotic evolutions*

## *Example of Itokawa*

Computation of the evolutions  
of **100 initially very close orbits**

**Expected timescale for a  
Collision of Itokawa with  
the Earth: 1 Myr**

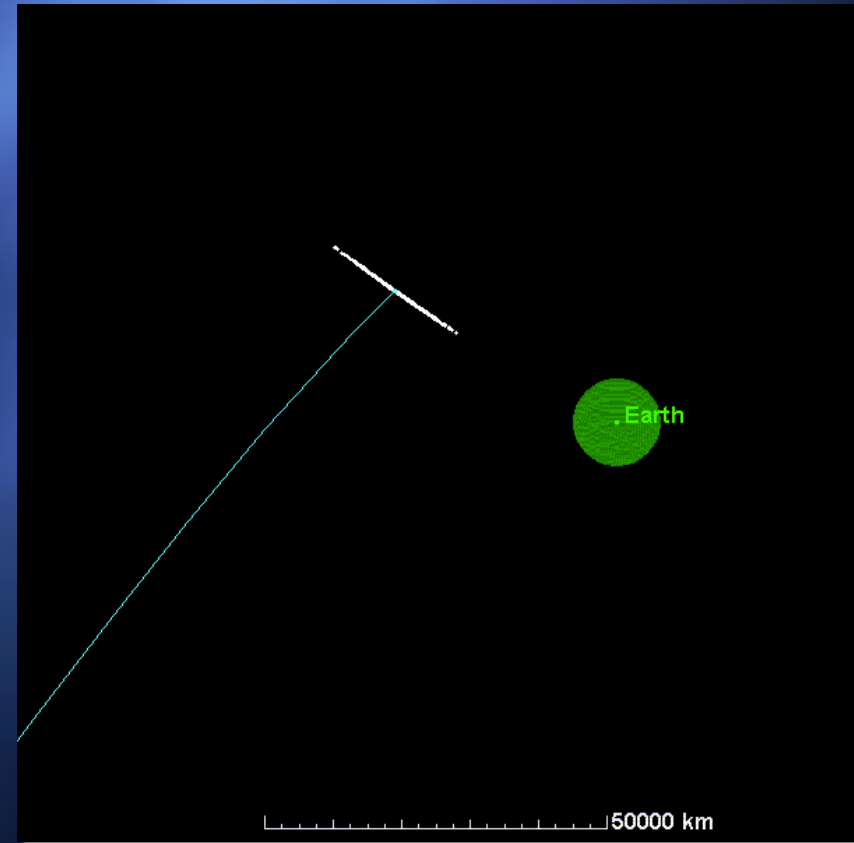
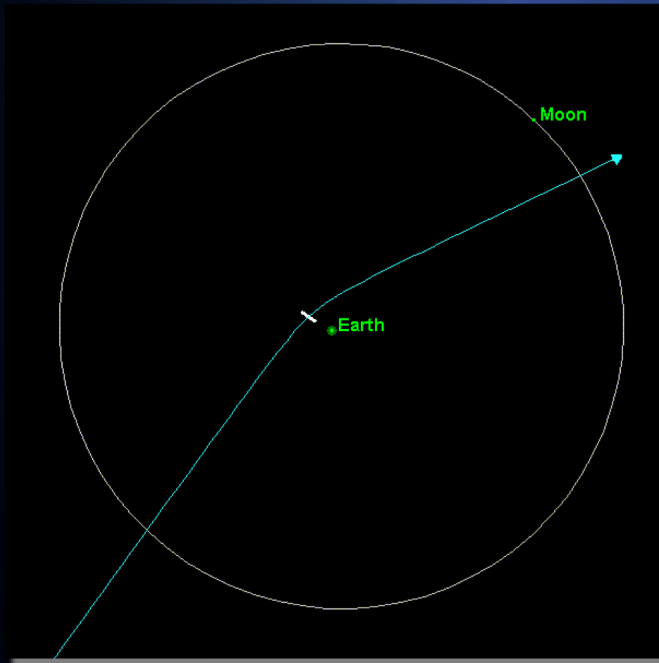
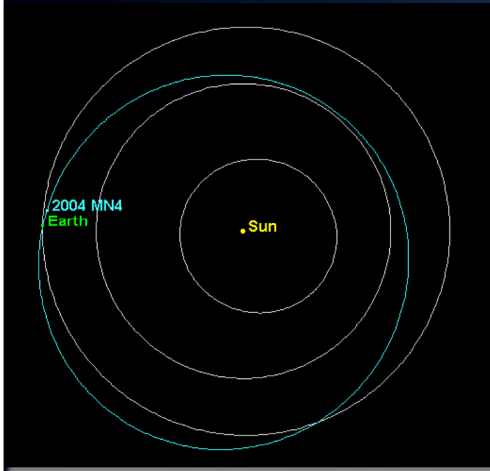
P. Michel & M. Yoshikawa, 2005,  
Icarus 179, 291-296.



# On a shorter term: the threatening object Apophis (size: 300 m)

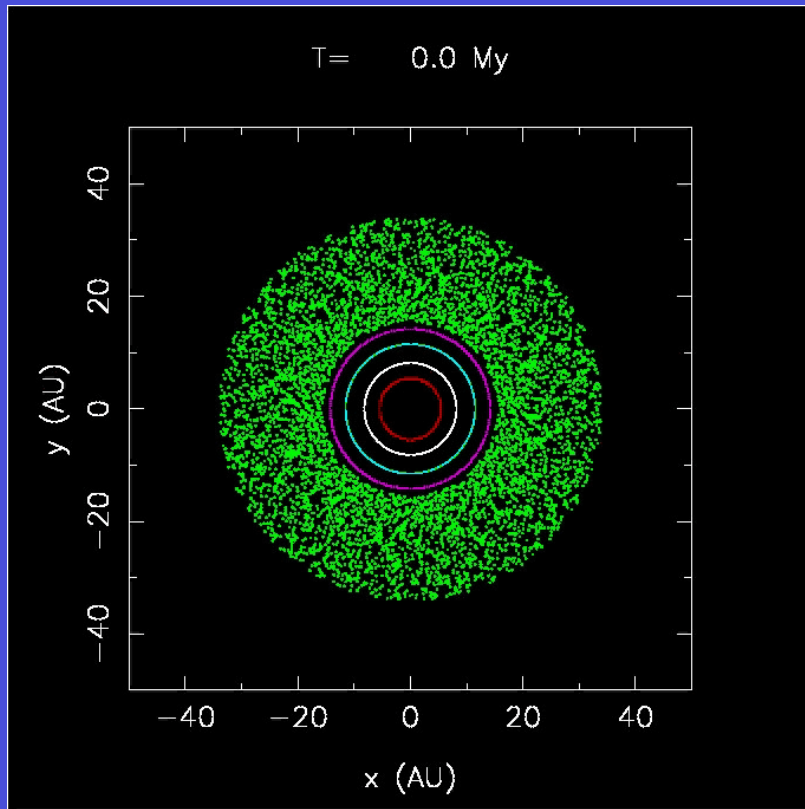
Trajectory uncertainty: 600 m  
within which a solution leads to  
a collision in 2036

In 2029: approach within 32,000 km!!

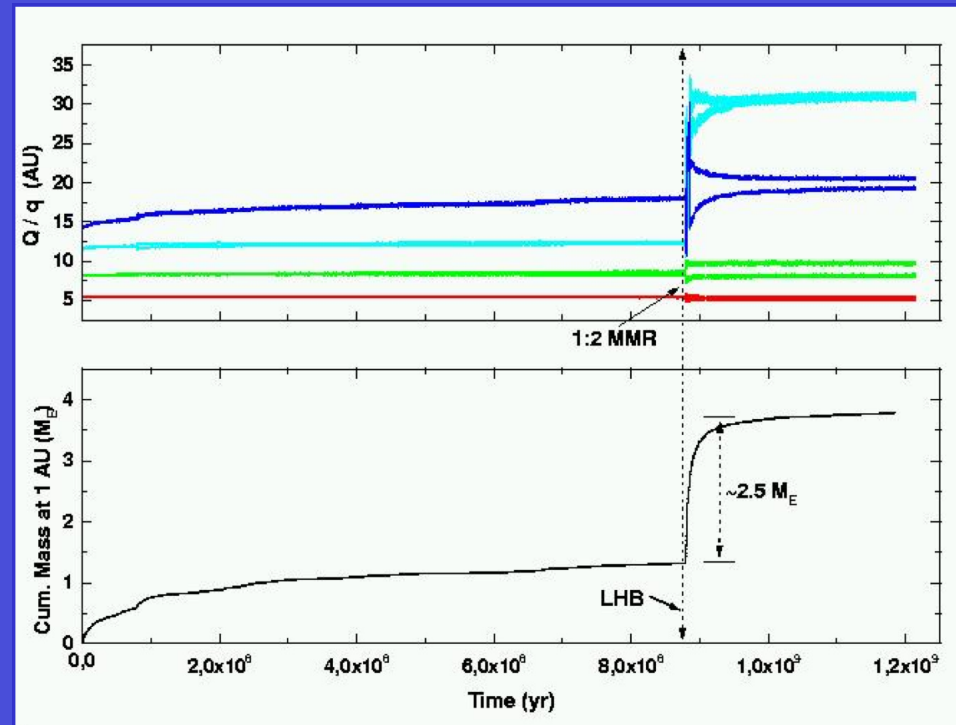


# Origin of the Late Heavy Bombardment (3.9 Byr ago)

Lunar craters



3 articles published in Nature  
(Vol. 435, 2005)



External Solar System (**in red:** Jupiter)  
**In green:** disk of planetesimals

**1st scenario which simultaneously explains:** giant planet eccentricities, origin of Trojans, LHB, and structure of the Kuiper Belt !

# *Conclusion I*

- ⊕ Mean motion and secular resonances = efficient transport mechanisms by increasing eccentricities or inclinations
- ⊕ Most NEOs come from the main belt through resonance channels
- ⊕ LHB can be explained by passage of Jupiter and Saturn in the  $\frac{1}{2}$  MM resonance

## Nice Observatory



## Cannes



sand and water:  
2 cohesionless materials!



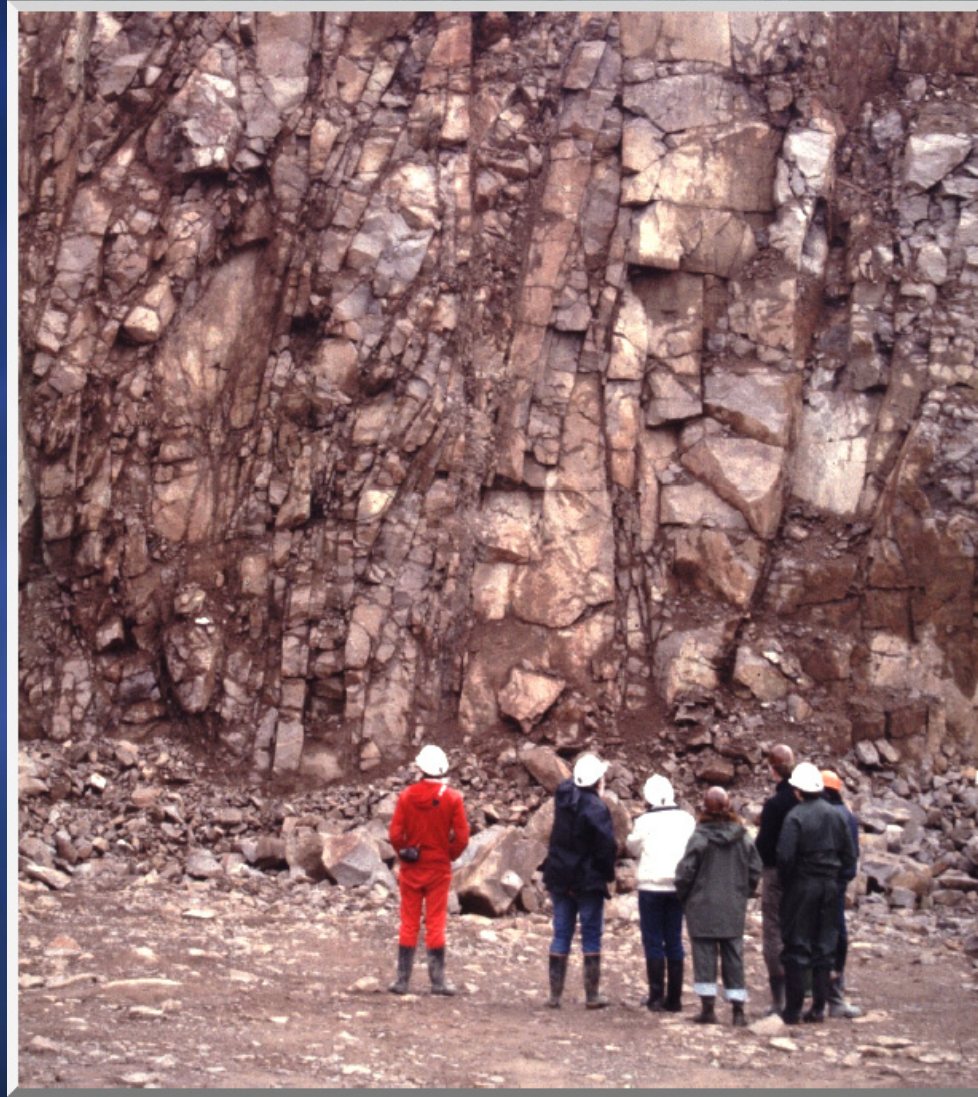
The Alps  
in the snow (another material)!



*Both **dynamical AND physical** properties must be characterized*

- ⊕ To determine the global (collisional and dynamical) evolution of small body populations
- ⊕ To determine the origin of observed properties (e.g. existence of binaries)
- ⊕ To define efficient mitigation strategies

*Rocks:  
A Modeling  
Challenge*

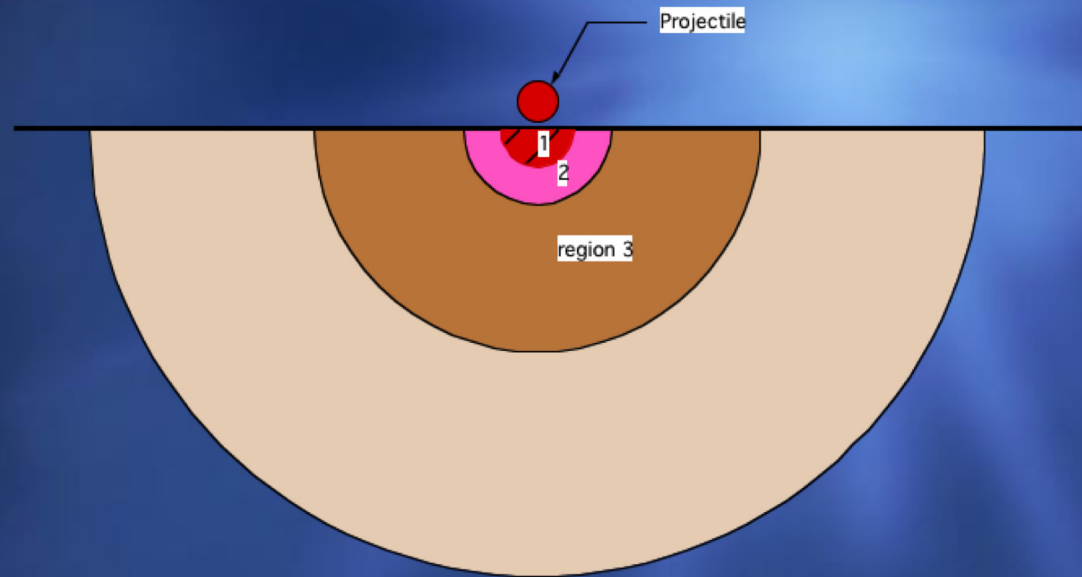


*The modeling of material behavior is the biggest shortcoming in code calculations, and the primary reason for bad results..*

# *What I won't talk about, but are important:*

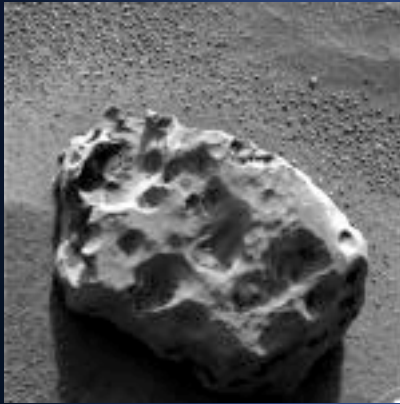
Eulerian v. Lagrangian codes  
Handling Mixtures in Eulerian codes  
Boundaries in Eulerian codes  
Grid distortion in Lagrangian  
Equations of States of rock materials

# Understanding the process: Regions of Impact Process

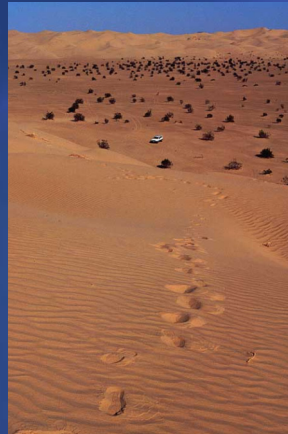


1.  $r \sim 0 \rightarrow a$ : Coupling of the energy and momentum of the impactor into the asteroid
2.  $r \sim a \rightarrow 2a$ : Transition into point source solution, shock breakaway.
3.  $r \sim 2a \rightarrow +\infty$ : Shock decays with distance, strength (& gravity) become important

# Strength v. Strength v. Strength



Rock Strength



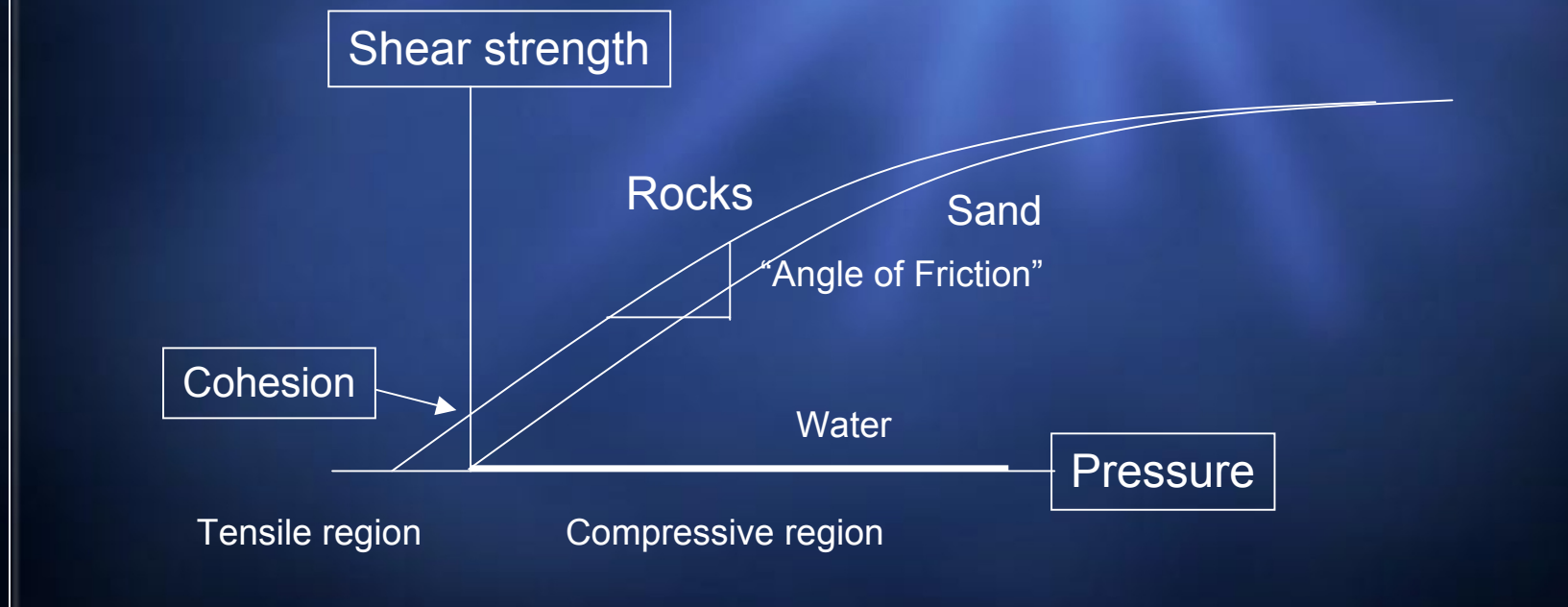
Sand Strength



Water Strength

# Strength:

⊕ The Mohr-Coulomb (or Drucker-Prager) model:

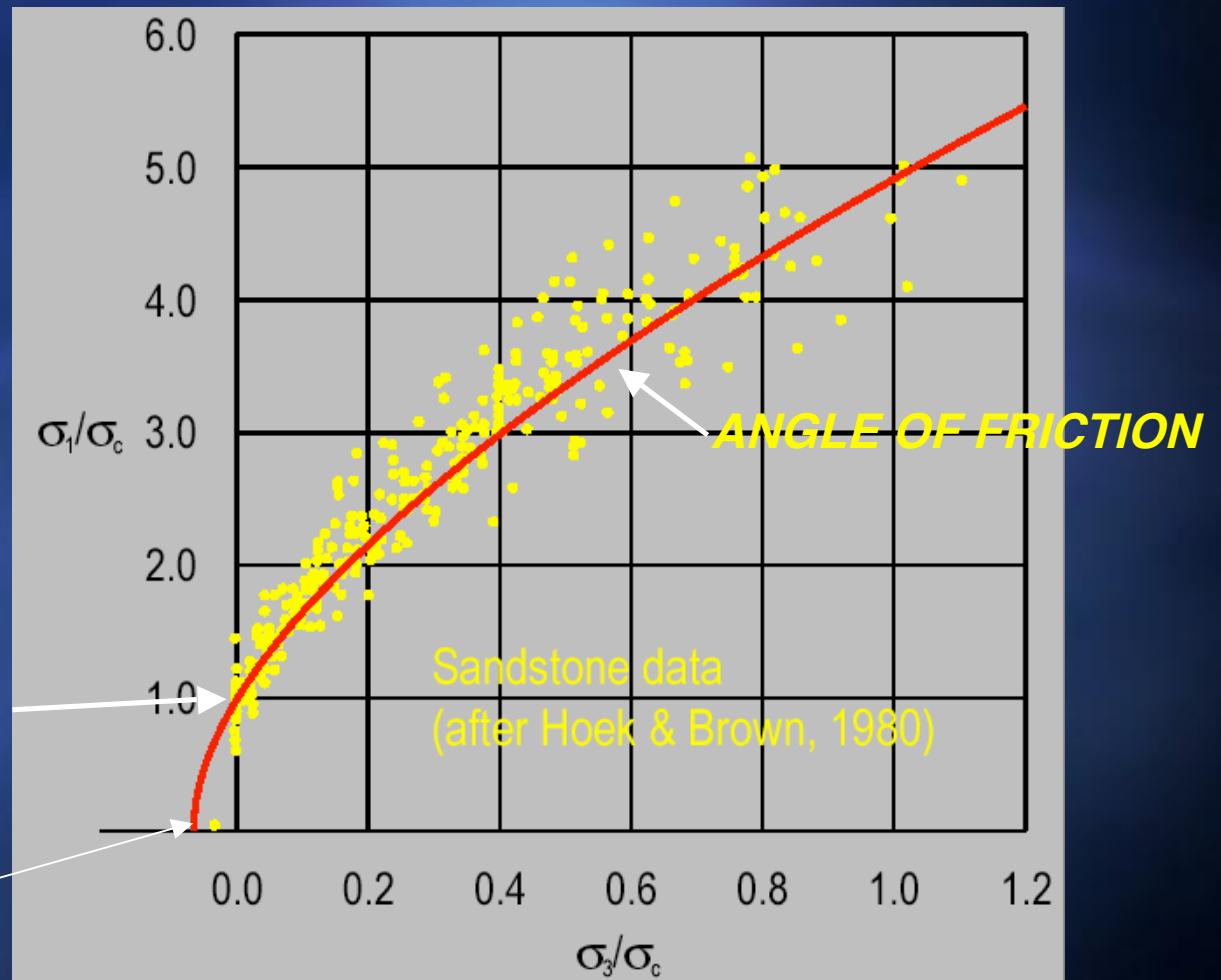


## Some real data

*Yield  
depends  
on  
pressure*

*Cohesion*

Tensile  
strength

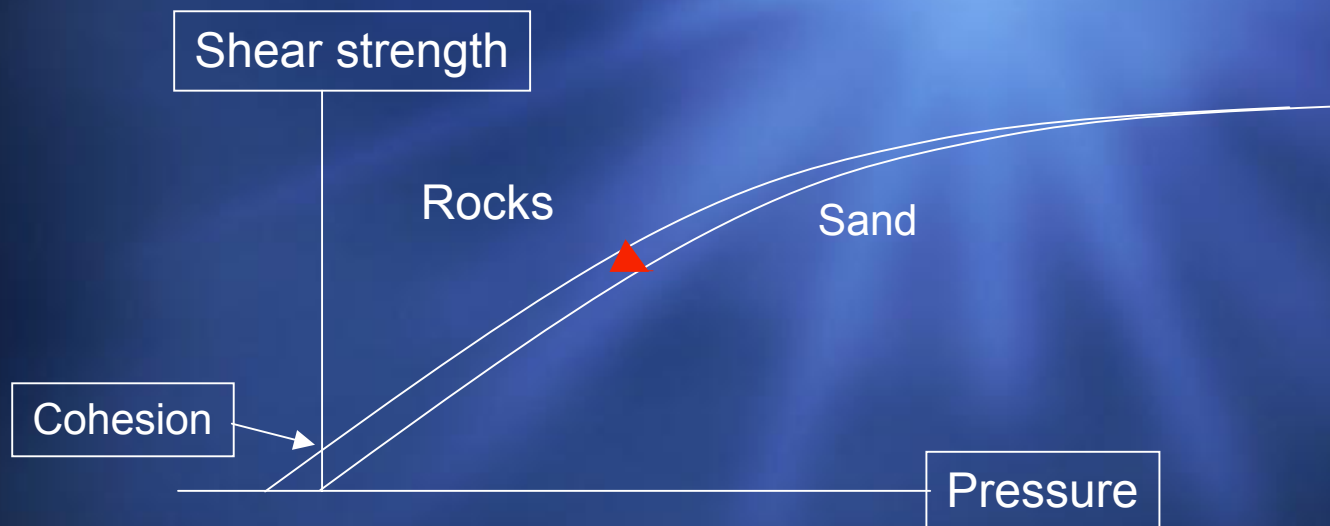




# *Strength*

- ⊕ A rock has each of:
  - ⊕ Tensile strength
  - ⊕ Shear strength (cohesion) ~same as tensile
  - ⊕ Compressive strength ~5-7\* tensile
  
- ⊕ But at large pressure, the cohesion can be ignored...

*And we have the model for large cohesion- less bodies or rubble piles:*



The 'strength' is due to the pressure, which is a result of self gravity holding the body together (*but it has no tensile strength*)

# *First application: Roche limit of cohesionless bodies*

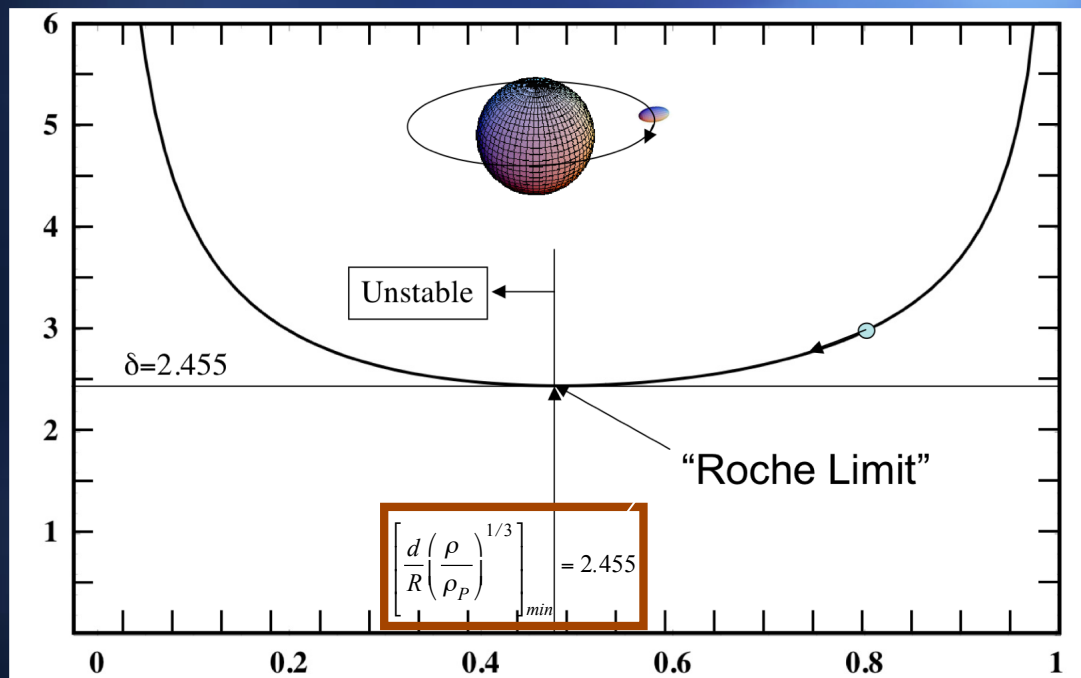
⊕ THE ROCHE LIMIT IS A WELL KNOWN FEATURE FOR SMALL Orbiting (or passing) BODIES.

- But:

- It assumes a **fluid** body

- It requires an **almost prolate** shape with aspect ratios  $\sim 2.1:1$

# So here is the **fluid** tidal disruption problem



Semi-major Axes:  $a, b, c$

Aspect ratios:

$$\alpha = \frac{c}{a}$$

$$\beta = \frac{b}{a}$$

Aspect ratio  $\alpha$

# *An Example (Phobos):*



Does this look fluid to You??

Is it anywhere near the required shape for a fluid body??  
(No:  $a=0.7$ , not 0.49)

**A fluid model is not mandatory for any solid body, even when dominated by gravity (see further)**

So:

“Satellites can orbit within their Roche limit because they have non-zero strength”

But, what is “strength”?

Here, we do not mean cohesion

**BUT** shear strength under pressure

# *The Problem Solved:*

- ⊕ Determine the tidal disruption limits for a geological material such as rock or sand.
  - ⊕ Rubble Piles (Ignore cohesion)
  - ⊕ Then what are the limit tidal disruption distances?

Ref: Holsapple and Michel, 2006, Icarus 183, 331.

## *Step 1: Determine the stress state*

- ⊕ Include spin, gravity, and tidal forces
  
- ⊕ But there are different ways to do this:
  - ⊕ Elastic Theory: Can determine state for “first yield”  
(but that depends on residual stresses, which cannot be known)
  
  - ⊕ Plastic Limit Theory: Can determine states for “final failure” irrespective of past history.



# The stresses at 'final failure' in an ellipsoidal body

$$\sigma_x = -\rho k_x a^2 \left[ 1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 \right]$$

$$\sigma_y = -\rho k_y b^2 \left[ 1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 \right]$$

$$\sigma_z = -\rho k_z c^2 \left[ 1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 \right]$$

The magnitudes are determined by  $k_x$ ,  $k_y$  and  $k_z$  which have specific components from each of gravity, spin and tidal forces. For the long x-axis is pointed toward the primary center:

$$k_x = \left( -2\pi\rho G A_x + \omega^2 + 2\frac{GM}{d^3} \right) x,$$

$$k_y = \left( -2\pi\rho G A_y + \omega^2 - \frac{GM}{d^3} \right) y,$$

$$k_z = \left( -2\pi\rho G A_z - \frac{GM}{d^3} \right) z$$

$A_x=A_y=A_z=2/3$  for a sphere  
and are expressed in terms of elliptic  
integrals for an ellipsoid

$\omega$  = spin magnitude (about z)

$M$  = primary's mass,  $\rho$  = body's density

$d$  = distance (primary's and body's center)

## Step 2: Solve the failure criterion for the tidal disruption limit distance

⊕ The failure criterion is the Druker-Prager one (zero-cohesion):

$$\frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = s^2 [\sigma_1 + \sigma_2 + \sigma_3]^2$$

Define:  $\Omega = \frac{\omega}{\sqrt{\pi\rho G}}$

$$s = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)}$$

and solve for the dimensionless distance:

$$\delta = \left( \frac{\rho}{\rho_p} \right)^{1/3} \frac{d}{R} = F[\alpha, \beta, p, \phi, \Omega]$$

$$p = m/M$$

$\phi$  = angle of friction

$\rho_p$ ,  $R$ : primary density and radius

$\alpha$ ,  $\beta$ : aspect ratios

This corresponds to solve for “LIMIT” State where no further plastic re-adjustments are possible.

We have done that for many combinations of distance, spin, shape, and secondary size..

as a function of the angle of friction...

(so the fluid case with zero angle of friction is a special case)

# Example: prolate bodies, spin-locked

$$\delta = d/R$$

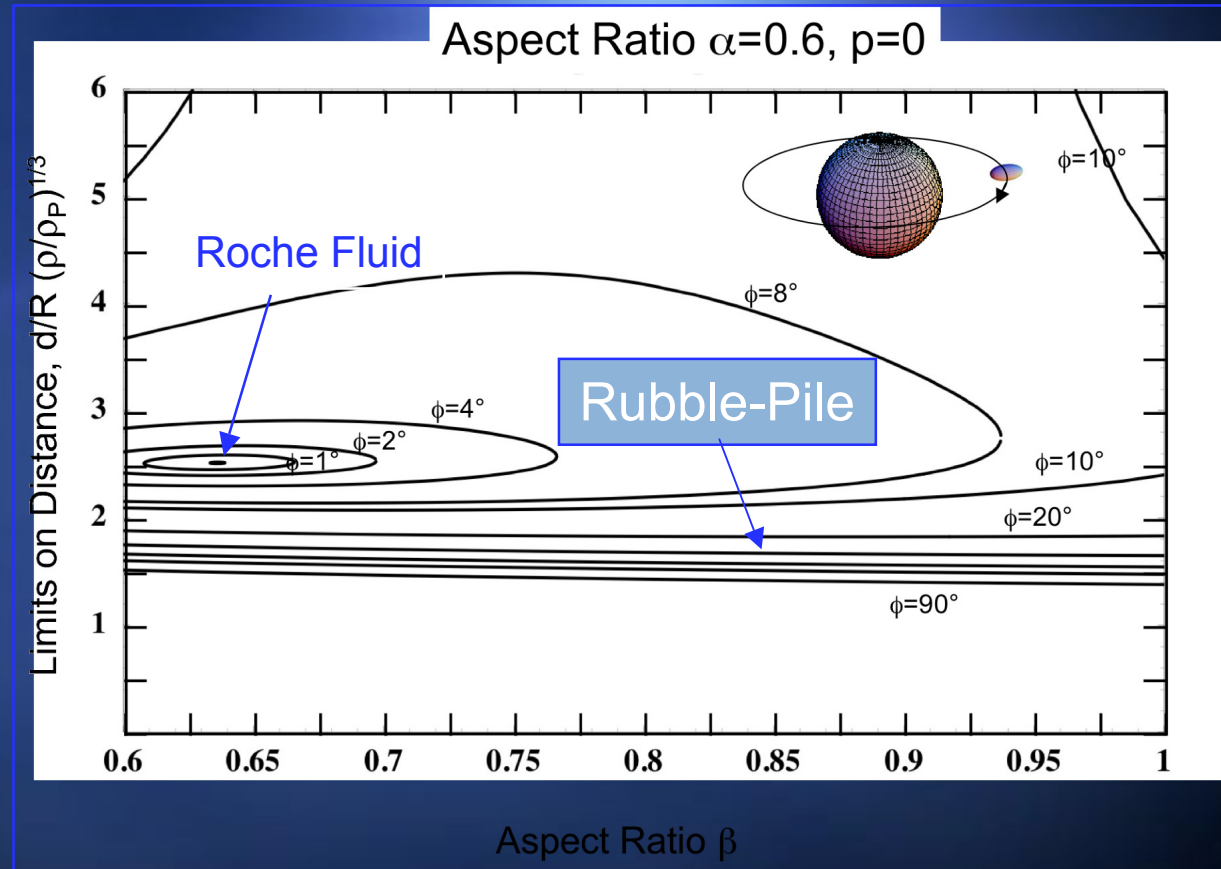
(for  $\rho = \rho_p$ )

Semi-major Axes:  $a, b, c$

Aspect ratios:

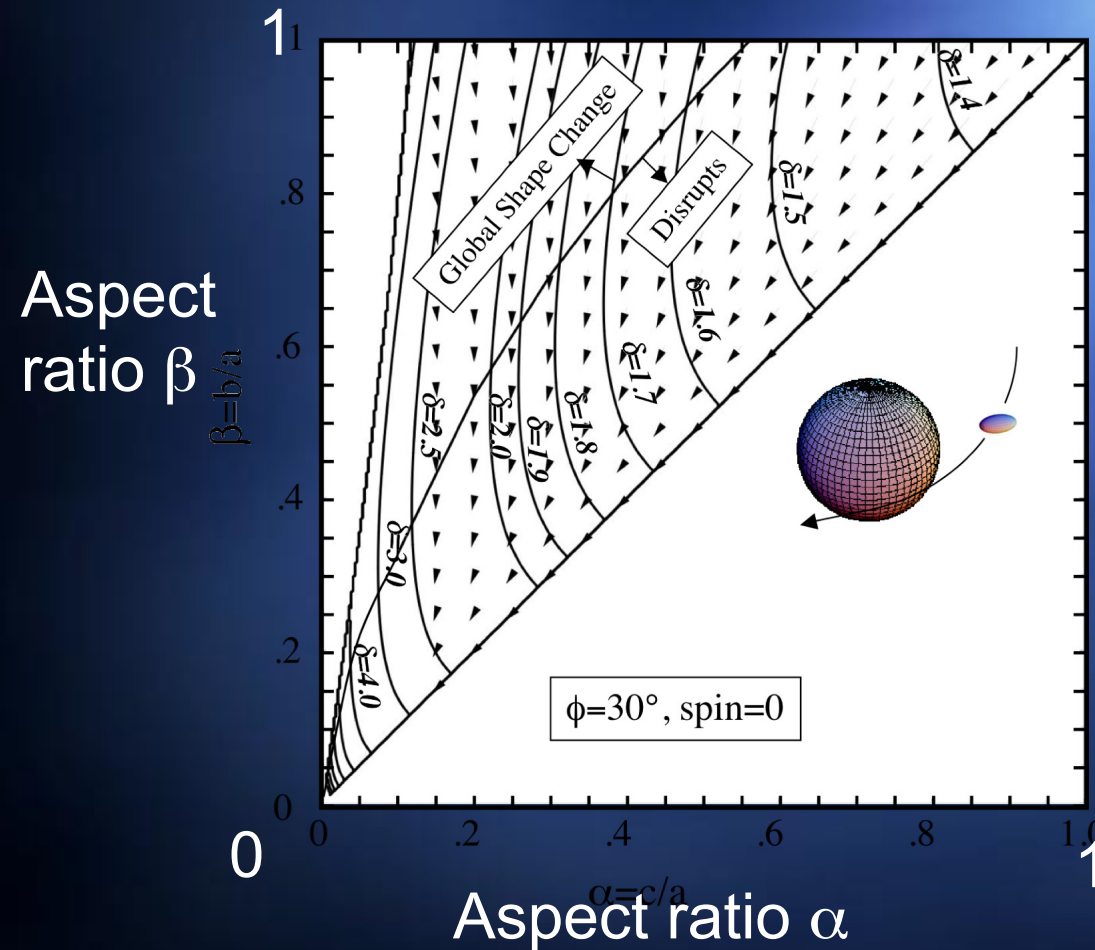
$$\alpha = \frac{c}{a}, \text{ (equal 0.6 here)}$$

$$\beta = \frac{b}{a}$$



Aspect ratio  $\beta$

*And finally, what if it does have 'final failure'?*



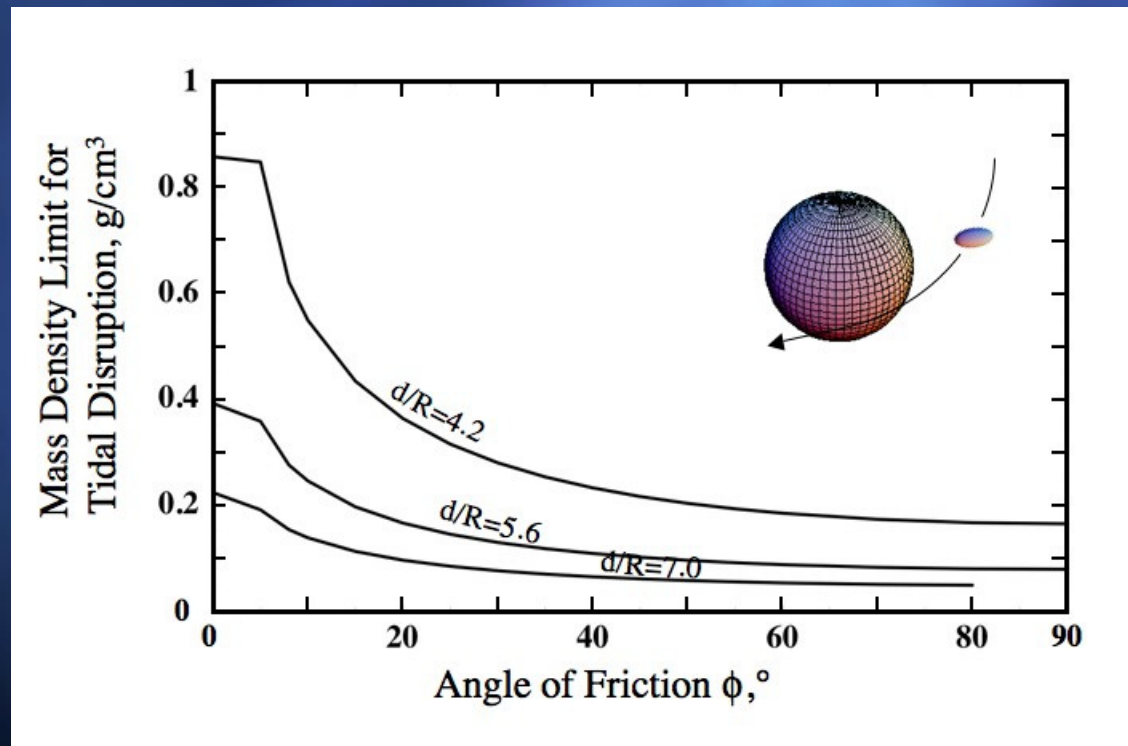
Prolate passing body,  
Long axis 'down',  
No spin,  
30° friction angle

# *Application:* **99942 Apophis (2004 MN4)**

- ⊕ In 2029: approach within  $5.6 \pm 1.4$  Earth's radii from Earth's center.
- ⊕ Ellipsoid with aspect ratios  $a=0.57$ ,  $b=0.71$  (Scheeres et al. 2005), rotation period=30 h.
- ⊕ We can determine the bulk density for tidal disruption or reshaping vs. the angle of friction  $f$

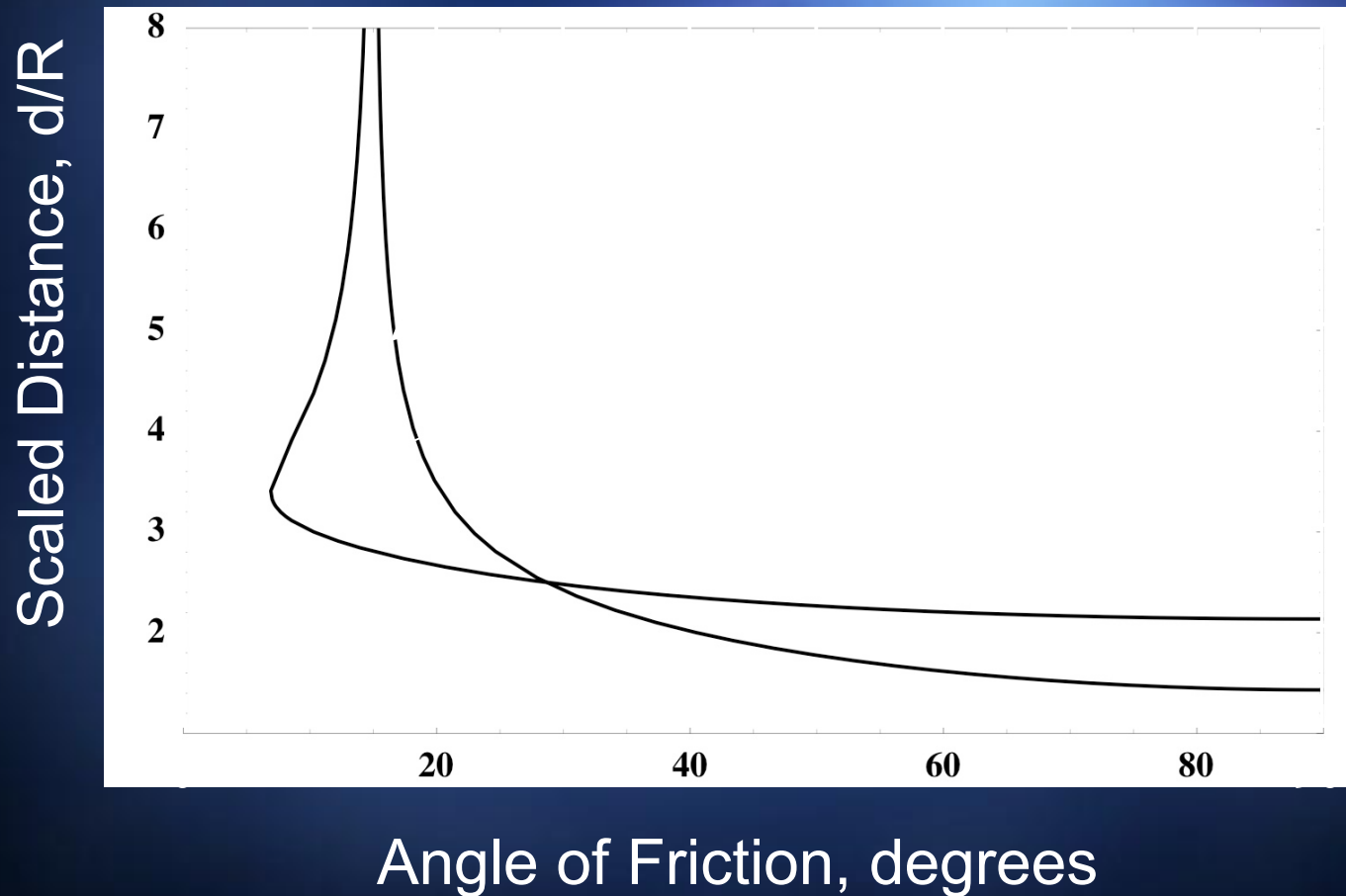
# Application: 99942 Apophis (2004 MN4)

Minimum bulk density of Apophis for survival without tidal breakup during the passage by the Earth at  $d/R=5.6$ ,  $4.2$ , and  $7.0$ , for various angles of friction (assumes the worst-case orientation of the longest axis pointed down)



# Application: (25143) Itokawa

## Minimal distance to Earth for tidal effects





# *FUTURE STEP: ADD COHESION*



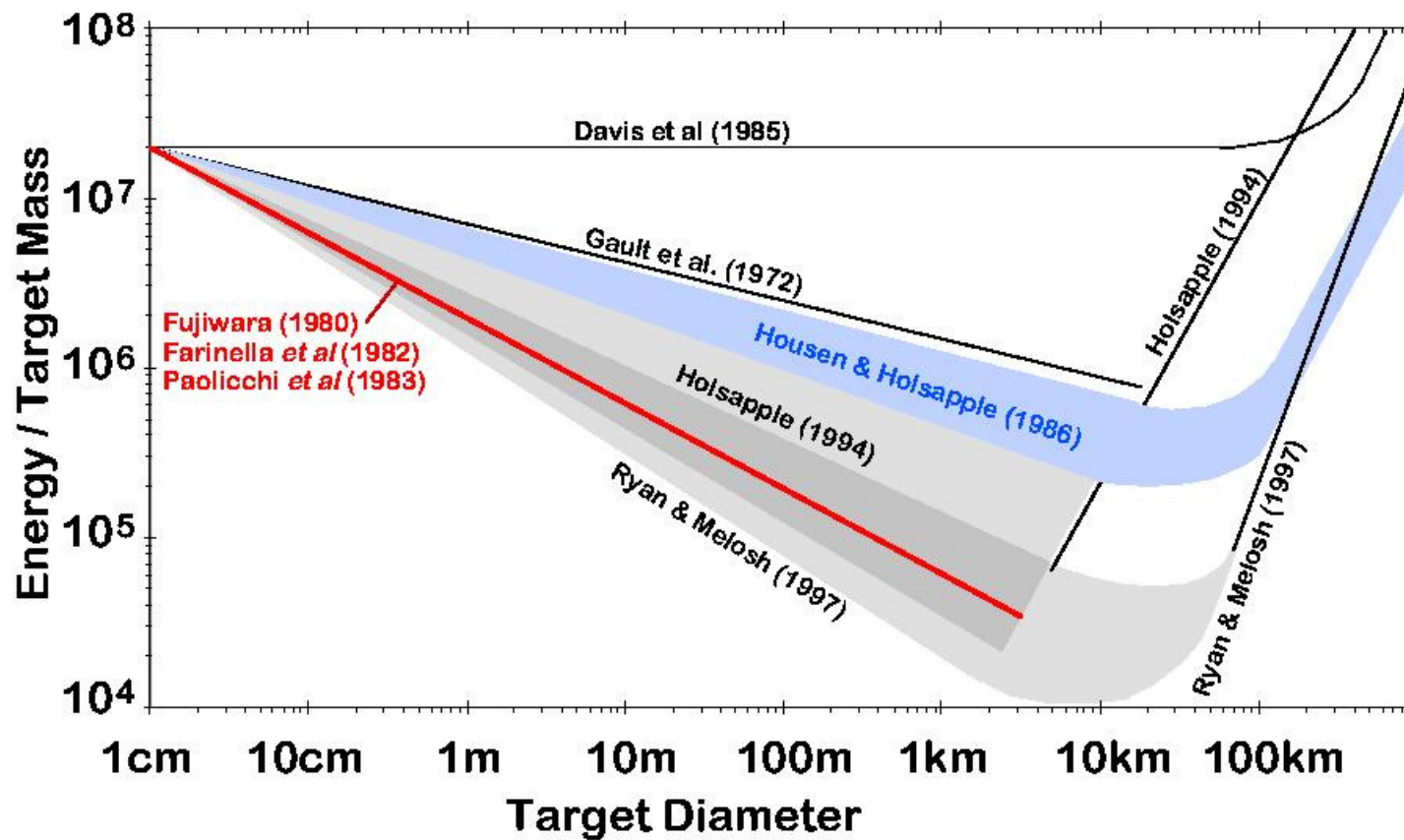
« Ostriches trying to stick their heads in the sand »

# *Simulating the collisional disruption of a small body: what do we need to know?*

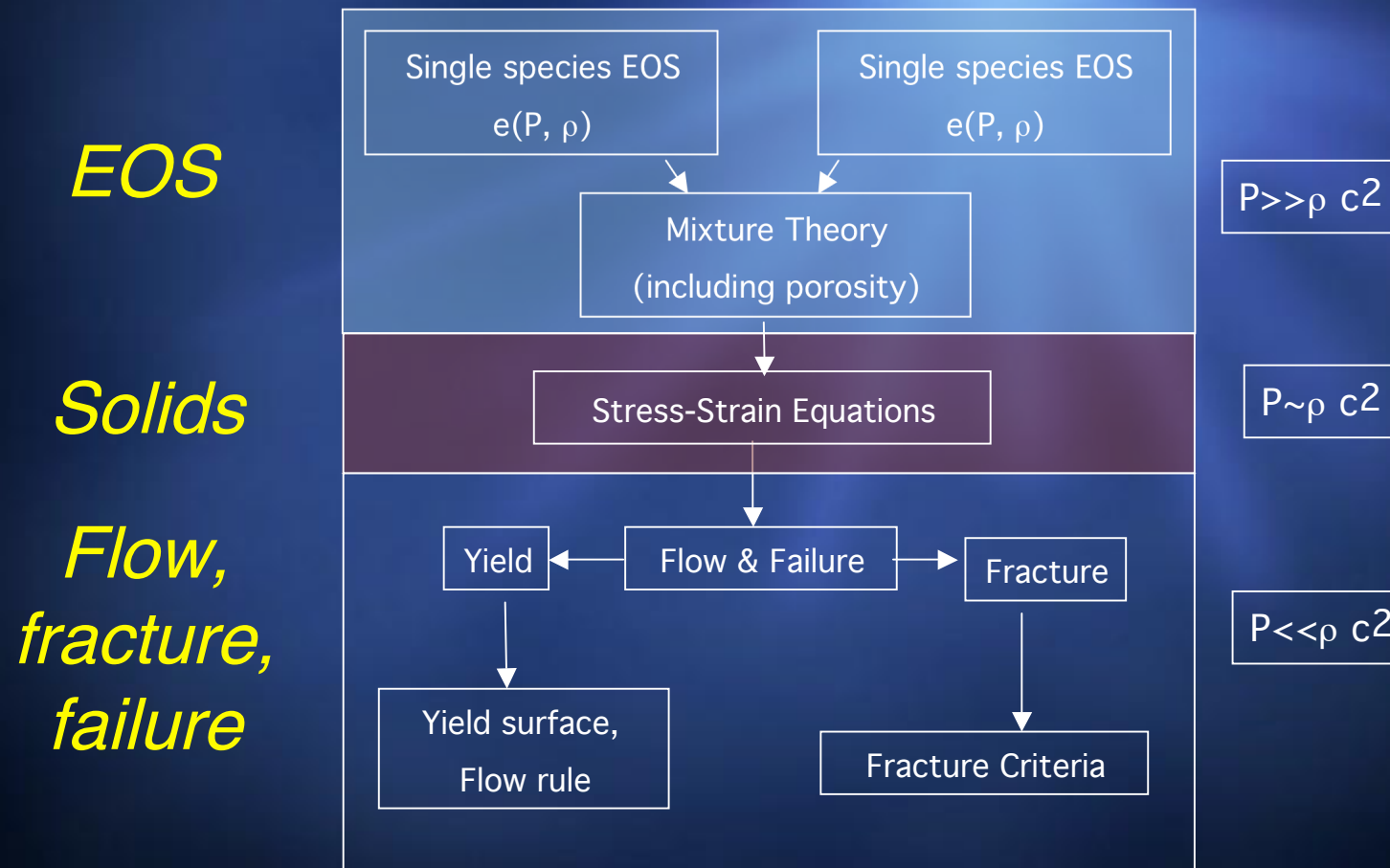
- Balance Laws (easy: continuum mechanics: balance of mass, momentum, energy)
- Material behavior (very hard: 100 Mbar down to partial bars!)
- Robust computer codes

# Comparison of scaling models

5<sup>th</sup> Catastrophic Disruption Workshop, Mt. Hood, June 30 - July 1, 1998



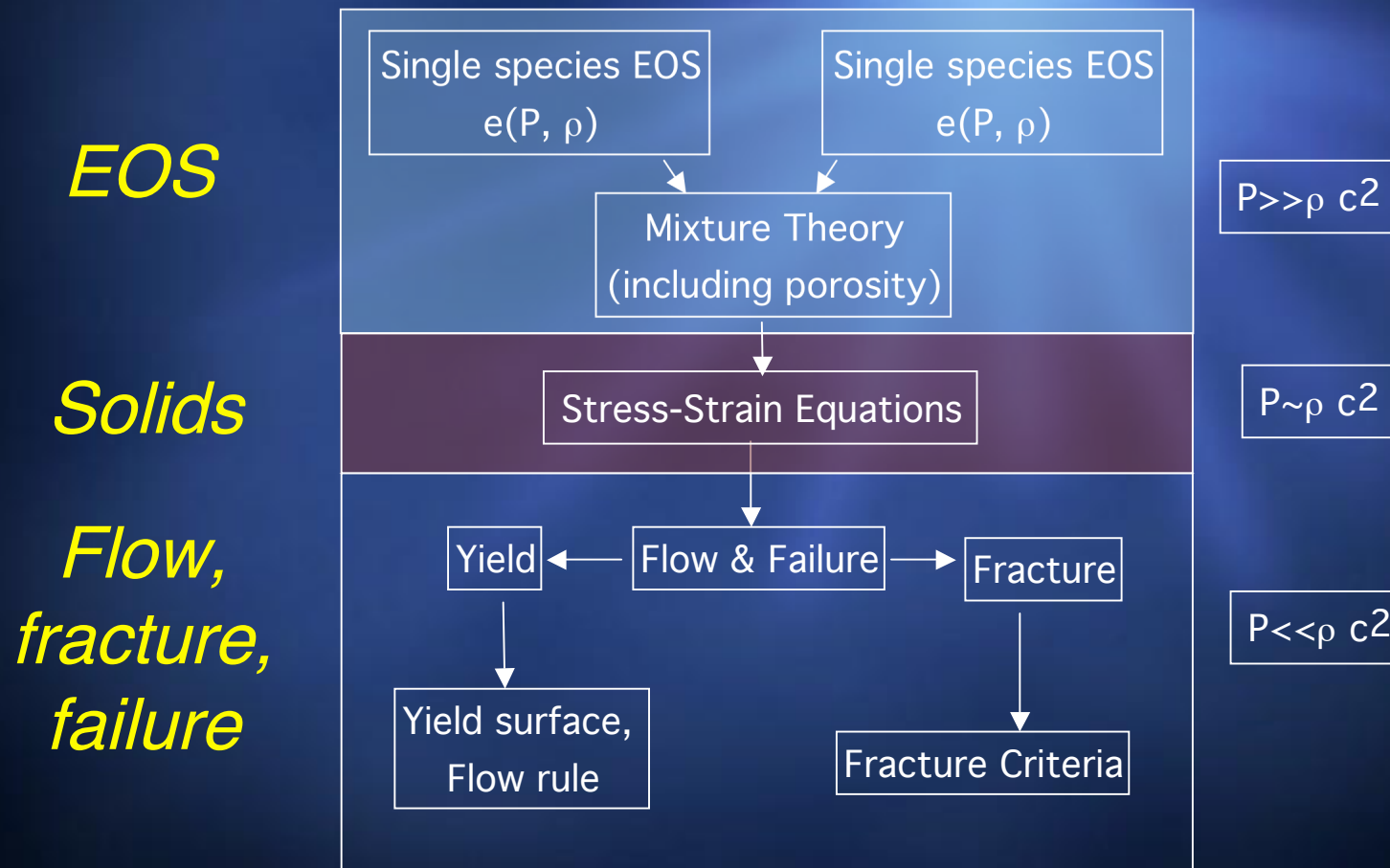
# Material Behavior: Three regimes



# *Stress-Strain behavior*

- When  $P \approx \rho c^2$  the material no longer behaves as a fluid.
- Then we need a constitutive equation for the stress-strain behavior
- Almost always, in wave codes that is simply an isotropic linear elastic relation (which is undoubtedly extremely crude).

# Which brings us to the strength parts..



# The “F” words: Flow, Fracture and Failure

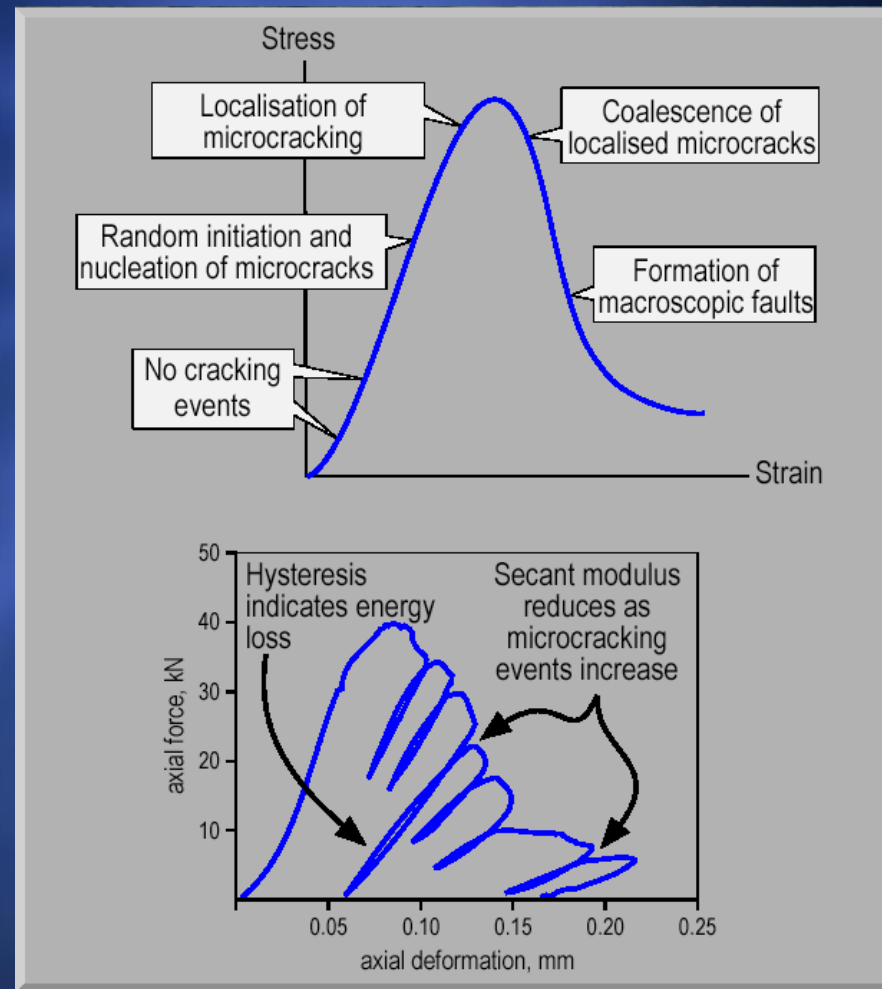
⊕ Models for these fall into three groups:

- *“Degraded Stiffness”, no explicit flow or fracture.*
- *“Flow” including plasticity and damage, used to model microscopic voids and cracks leading to an inability to resist stress.*
- *“Fracture”, involving actual macroscopic cracks and voids which are tracked, leading to an inability to resist stress.*

*In a continuum theory, the first two can be included directly, the latter is difficult, unless some statistical approach is used to smear them out.*



*Damage and degradation leading to ultimate failure occur at some limiting strain*



# *Flow and Fracture: Yielding and Cracking*

Initial Yield= $F(\text{stresses})$  or  $G(\text{strains})$

- Isotropic  $\Rightarrow \sigma_1, \sigma_2, \sigma_3$   
(Or three stress invariants)
- Commonly only 2, e.g.  
 $J_2 = F(P)$   
Or max shear =  $f(\text{pressure})$

# The Grady-Kipp Model

*Special nature*

- It is a Tensile Brittle Fracture Mechanism
  - For fragmentation in mining
- One-Dimensional Model
- Synthesized for constant strain rate histories only

- Governed by Crack Distributions (Weibull) and growth
- Implies rate and size-dependent strength

*But Attractive Physics*

# *There exists an initial distribution of incipient flaws in the target*

⊕ Weibull distribution:

$$N(\varepsilon) = k \varepsilon^m$$

where:

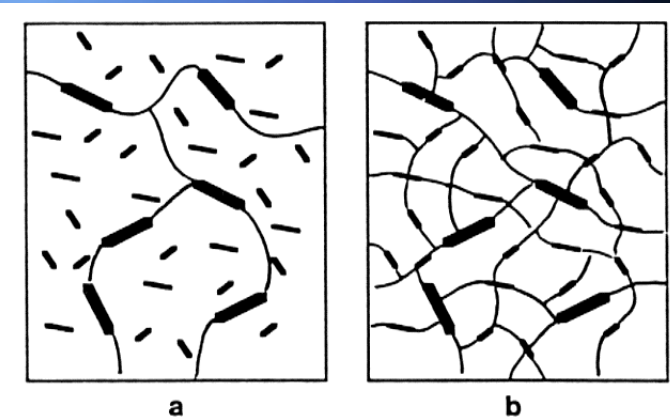
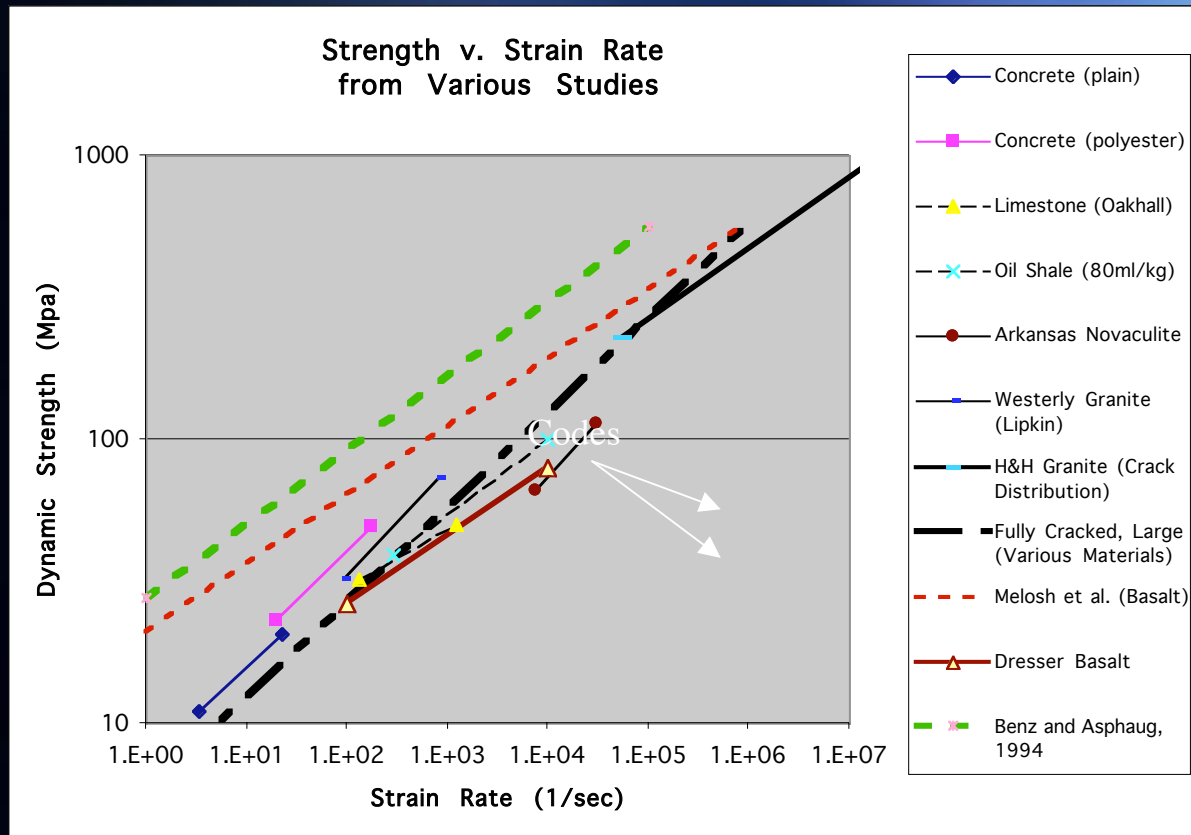
N = density number of flaws activating at or below the strain  $\varepsilon$

k, m: Weibull parameters (large m= more homogeneous material)

$$\varepsilon_{\min} = (1/kV)^{-m}$$

**Larger targets (volume V) activate largest crack at lower strain  
⇒ Larger targets are weaker**

# Tensile fracture depends strongly on strain rate



Low  
strain rate

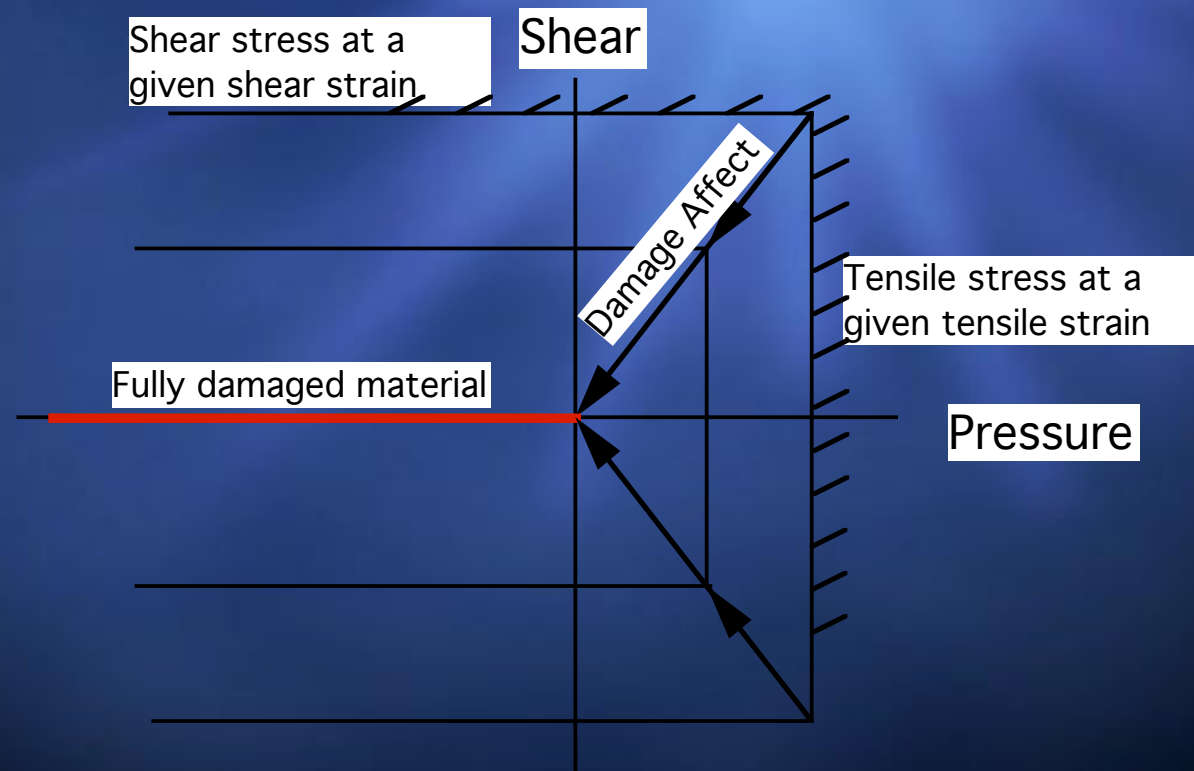
High  
strain rate

(From Asphaug)

# *A Grady Kipp Implementation in 3D*

- Damage is isotropic, so that when a crack is formed in one directions, all directions lose stiffness
- As damage accumulates, the stiffness in both tension and in shear decrease, eventually to zero.
- Therefore, material failed by the outgoing shock behaves as water.
- *Calibrated to disruption test, by adjusting the strength (Weibull) parameters*

# The Grady-Kipp Approach



# Fragmentation phase: principles

Equation of state  
 $P=f(E,\rho)$

Model of brittle  
Failure

Stress tensor

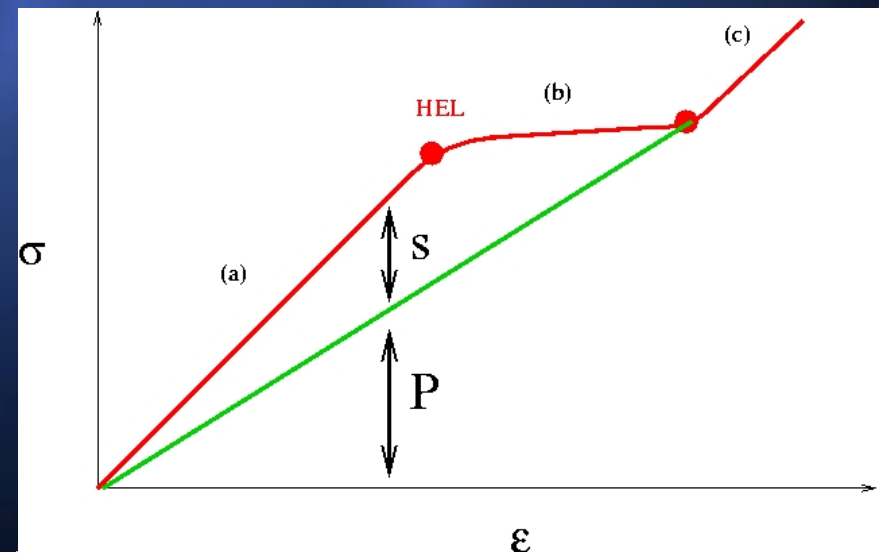
$$\sigma_{\alpha\beta} = -P \delta_{\alpha\beta} + S_{\alpha\beta}$$

$$S_{\alpha\beta} = \mu(\epsilon_{\alpha\beta} - 1/3 \epsilon_{\gamma\gamma} \delta_{\alpha\beta})$$

Yielding criterion:  
 $S_{\alpha\beta} \rightarrow f S_{\alpha\beta}$

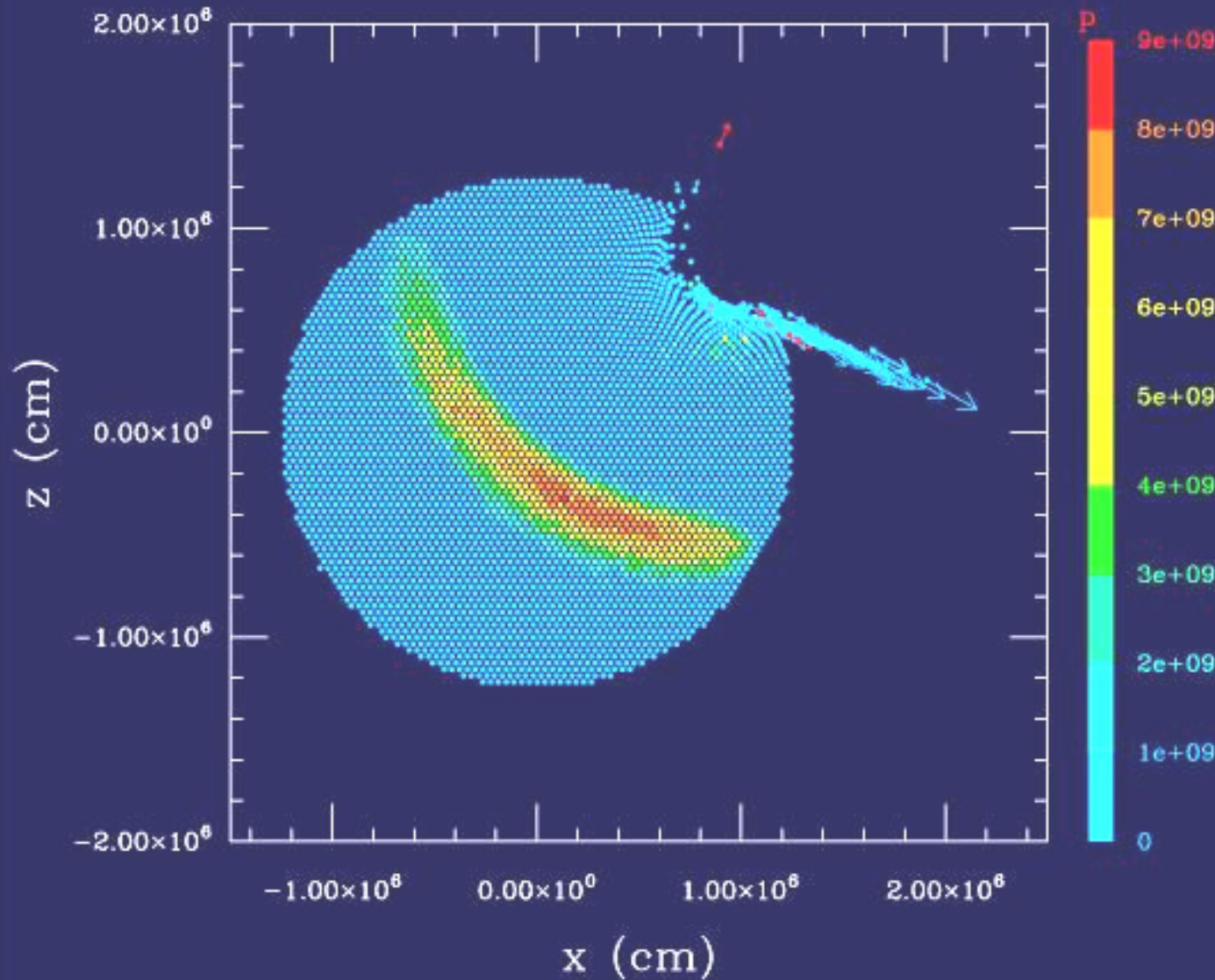
Conservation equations

SPH techniques





impact phase  $t = 2.50132$  s



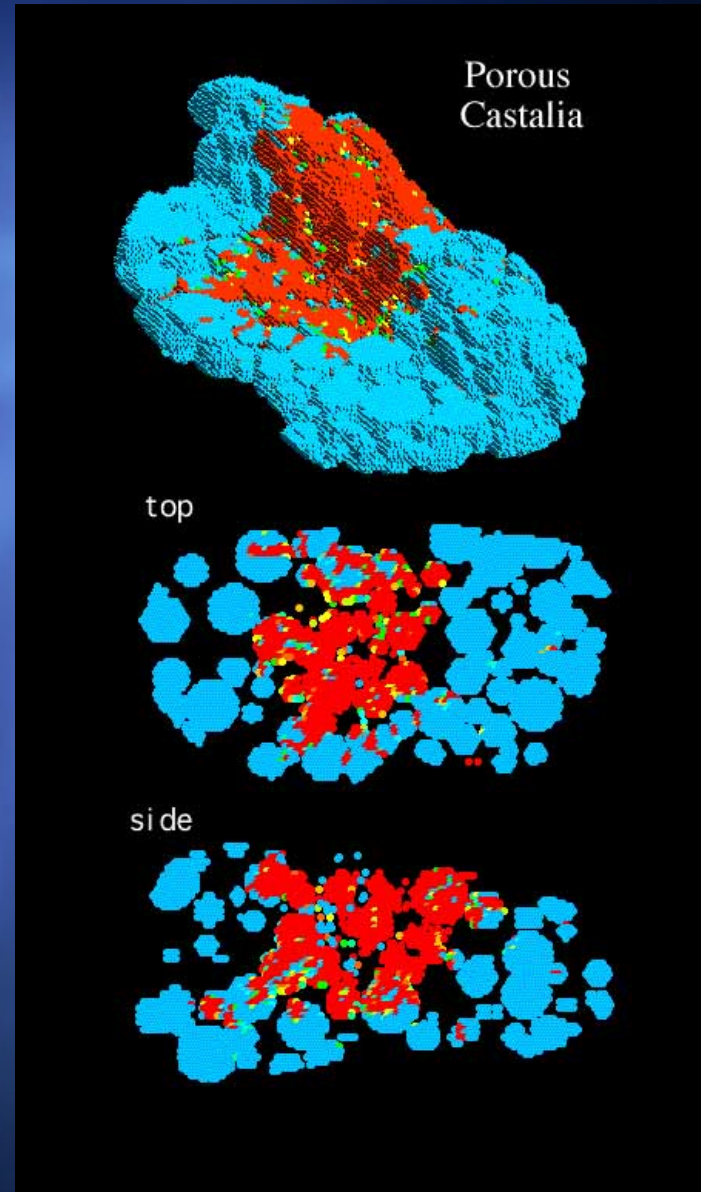
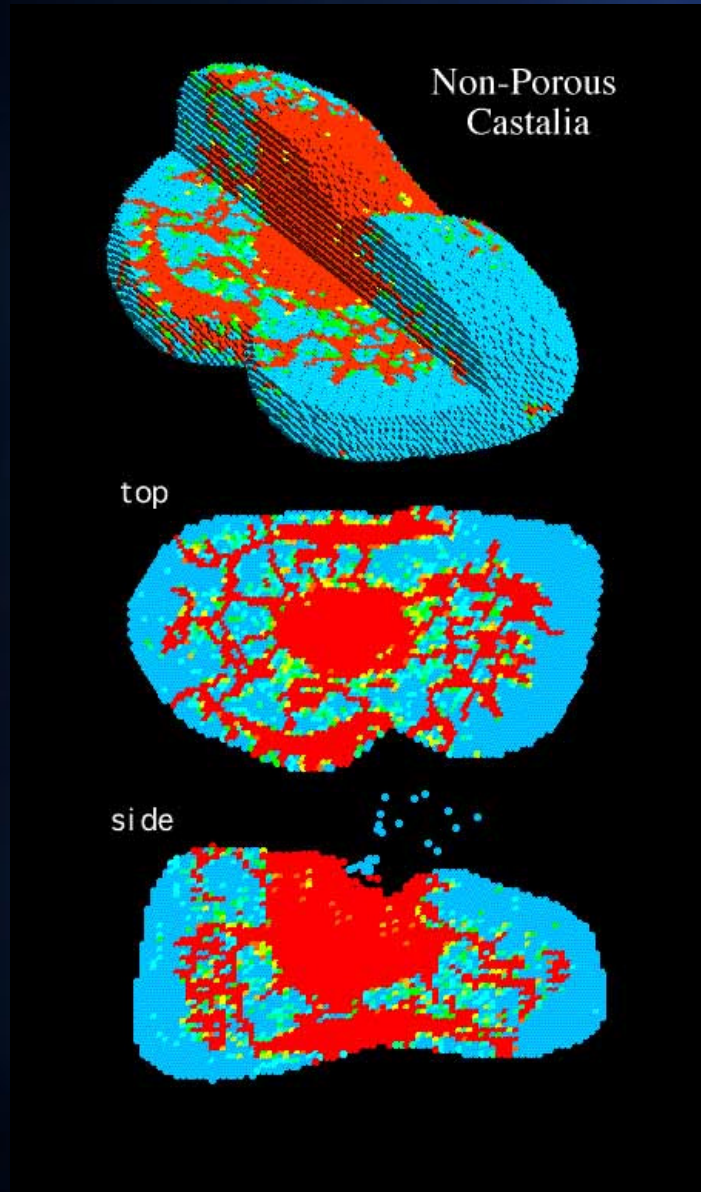
Fragmentation  
Phase

Shock wave  
Propagation

Impact  
velocity:  
5 km/s

Impact angle:  
 $45^\circ$

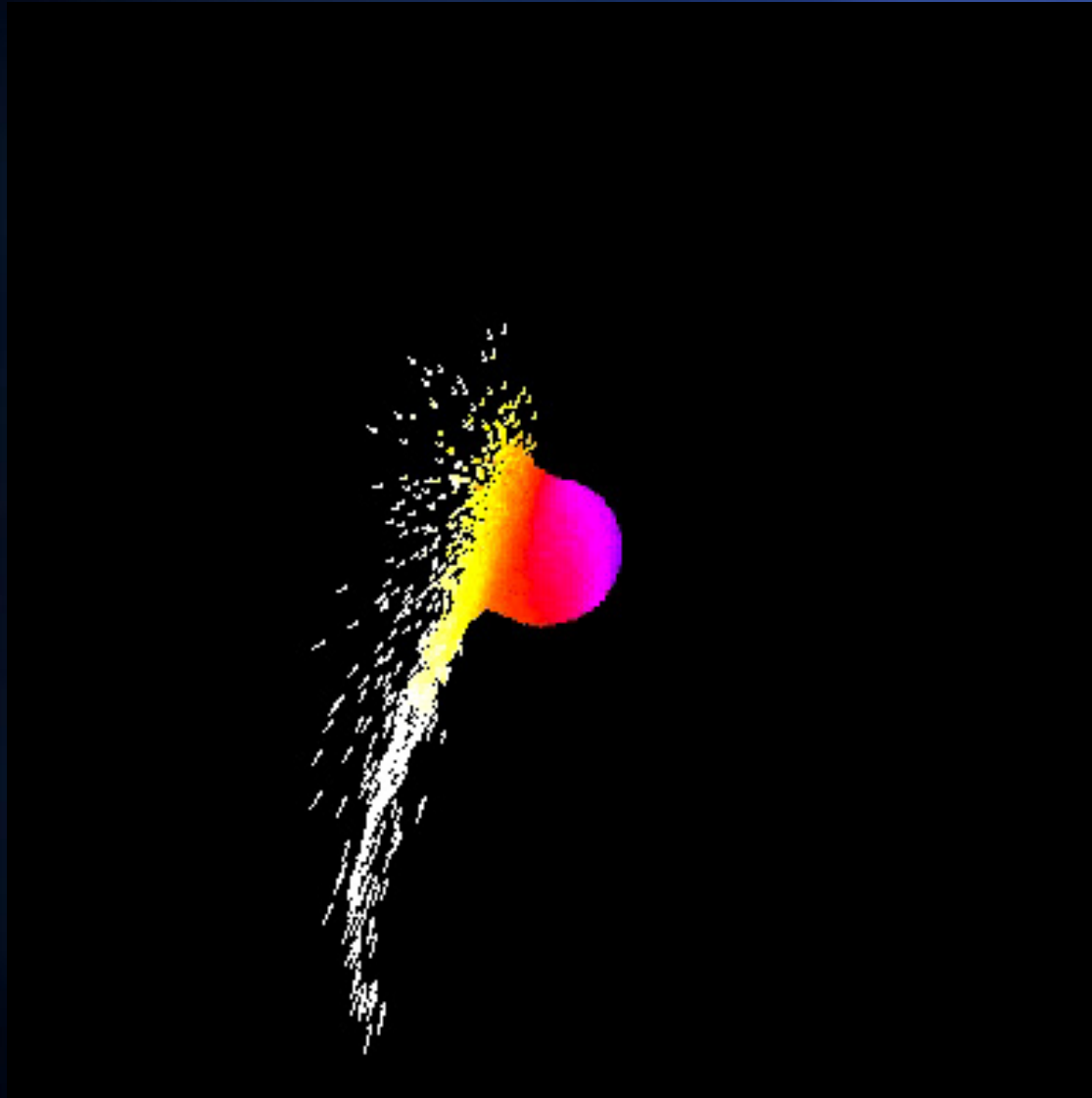
P. Michel & W. Benz



From Asphaug et al. 1998, Nature **393**.

Impact angle:  $66^\circ$ ,  $V = 5 \text{ km/s}$

$D=164 \text{ km}$



Velocity  
distribution  
At the end of  
the  
fragmentation  
phase

Colors from  
**Yellow** to **Blue**  
indicate  
velocities from  
**large** to **small**

*Intermediate  
impact regime*

# *Gravitational Phase: parallel N-Body simulations*

- ⊕ Several hundreds of thousands km-size fragments can be generated by the fragmentation phase

➔ Impossible to compute their gravitational interaction by classical methods:

The CPU time required to compute  $N$  interactions between  $N$  particles is of  $O(N \times N)$  !!



Using the so-called hierarchical tree method (tree code):  
CPU Time =  $O(N \log N)$

# *Gravitational Phase: parallel N-Body simulations*

⊕ Parallel N-Body code: *pkdgrav* (Parallel K-D tree GRAVity code); developed at UW by T. Quinn, J. Stadel, D.C. Richardson

- Detects and handles collisions between massive particles. Several options:

1. Systematic particle merging
2. Merging/Bouncing of particles depending on impact speed and spins.

Particle shape: spherical

# *Simulations of Collisions in the Gravity Regime*

- ⊕ SPH hydrocode → crack propagation through the target
- ⊕ Nbody code → gravitational interaction between intact fragments



**Simulation of target shattering + fragment dispersion and/or reaccumulation**

Michel et al. (2001), Science Vol. 294, pp 1696-1700.

# Results!

Simulations of asteroid disruptions **have**  
1. **successfully reproduced asteroid families**  
2. **suggest that most kilometer-sized objects**  
**are gravitational aggregates**



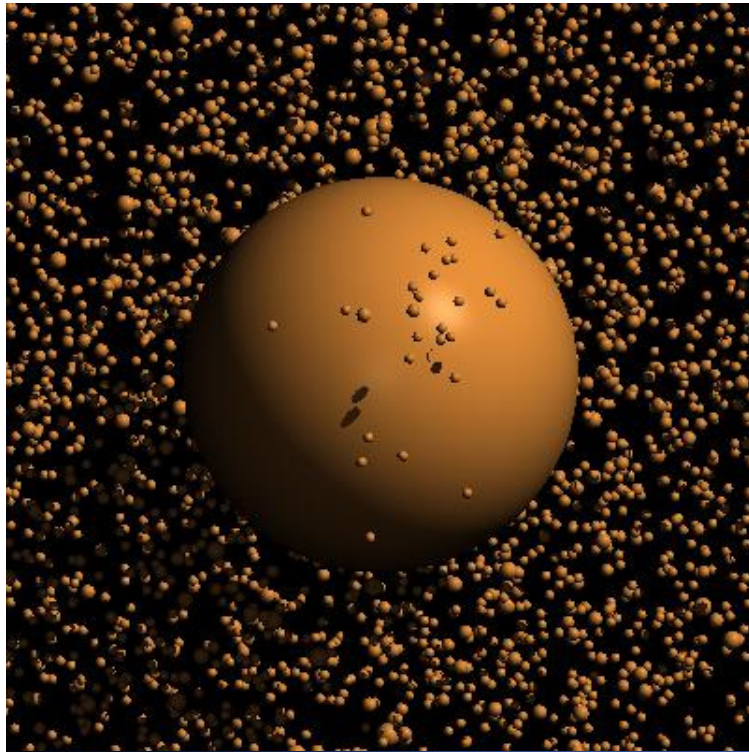
Michel et al., *Science* 294 (2001)

COE Planetary  
School 12/4/2006

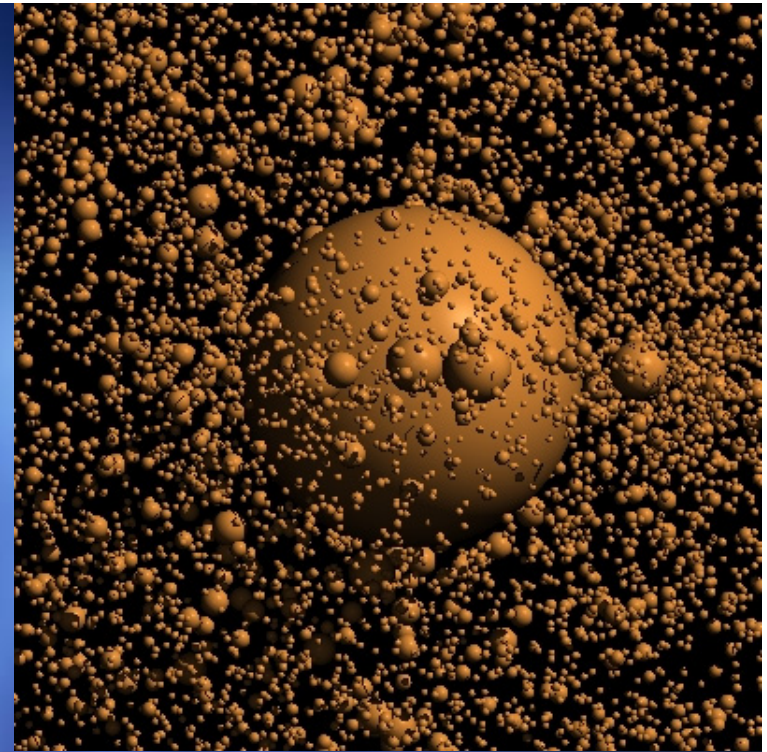
Impact energies and  
collisional outcomes **depend**  
**highly on the internal structure**  
**of the parent body**



Michel et al., *Nature* 421 (2003)



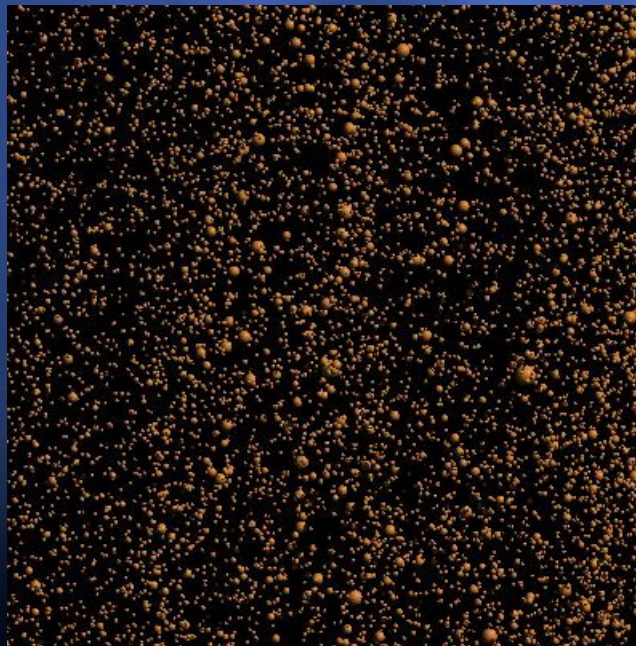
Michel, Benz, Tanga,  
Richardson, *Icarus*,  
160, 2002.



T=84 minutes

Different phases of  
the reaccumulation  
process

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T=2 minutes

T=2 seconds



Implication: **most asteroids** originating from the disruption of a larger one - such as most NEOs - should be **rubble piles**

**The Japanese mission Hayabusa brought us some evidence** in this direction: where are the craters?? why so many debris ?? What about the small bulk density ( $< 2 \text{ g/cm}^3$ )

Release 051101-3 ISAS/JAXA



Release 051101-4 ISAS/JAXA



# From Velocities to Orbital Elements

**Gauss Formulae:** transformation velocities to orbital elements

$$\frac{\partial a}{a} = \frac{2}{na\sqrt{1-e^2}} [(1+e\cos f)V_t + e\sin f V_r]$$

$$\partial e = \frac{\sqrt{1-e^2}}{na} \left[ \frac{e+2\cos f + e\cos^2 f}{1+e\cos f} V_t + \sin f V_r \right]$$

$$\partial i = \frac{\sqrt{1-e^2}}{na} \left[ \frac{\cos(\omega + f)}{1+e\cos f} V_w \right]$$

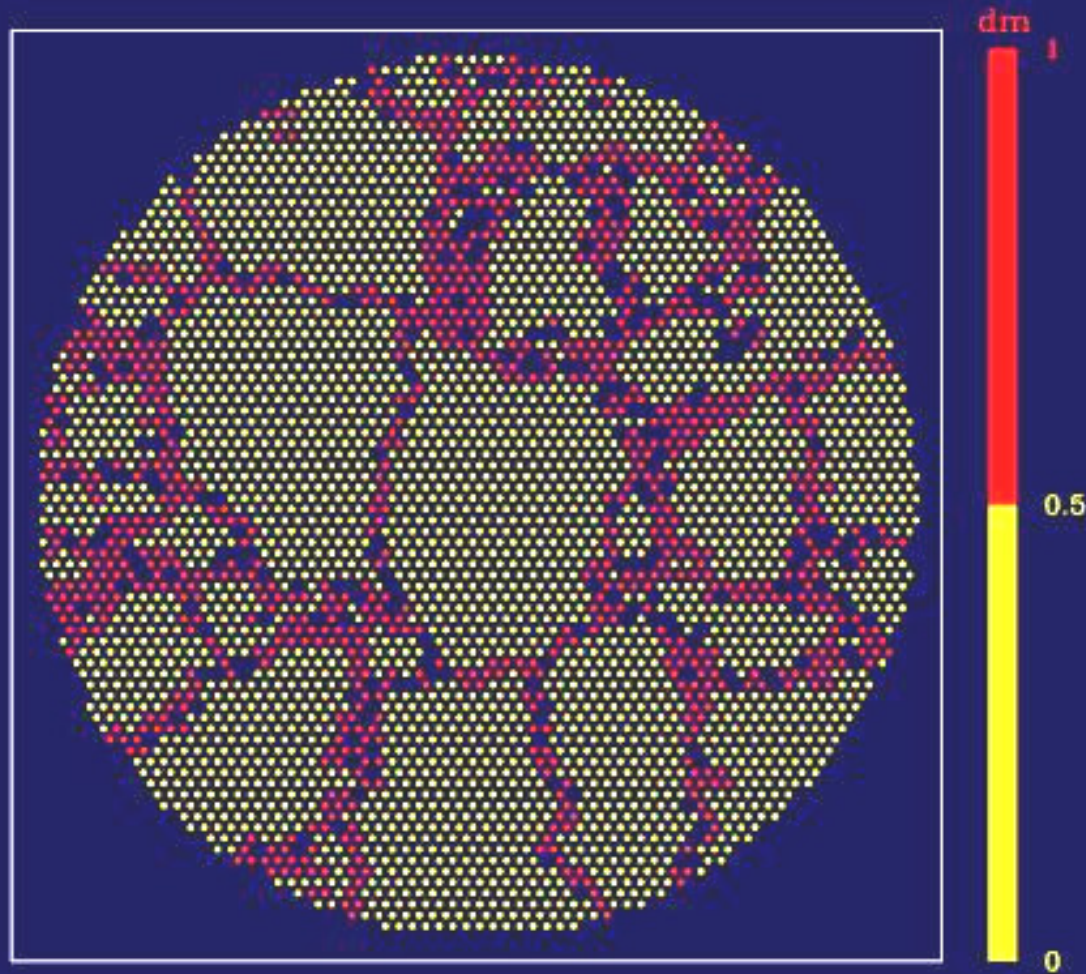
(a, e, i, w, f, n) = orbital elements of Parent body (family barycenter)

**Requires to assume *a priori*  $\omega$  and  $f$  of the parent body at the impact instant**

# *Effect of the Parent Body's Internal Structure*

- ⊕ Previous simulations assumed monolithic parent bodies
- ⊕ Large asteroids are likely to undergo shattering events before disruptive ones
- ⊕ **What is the outcome of the disruption of a pre-shattered parent body?**

Pre-shattered parent-body



Yellow zones=  
fragments

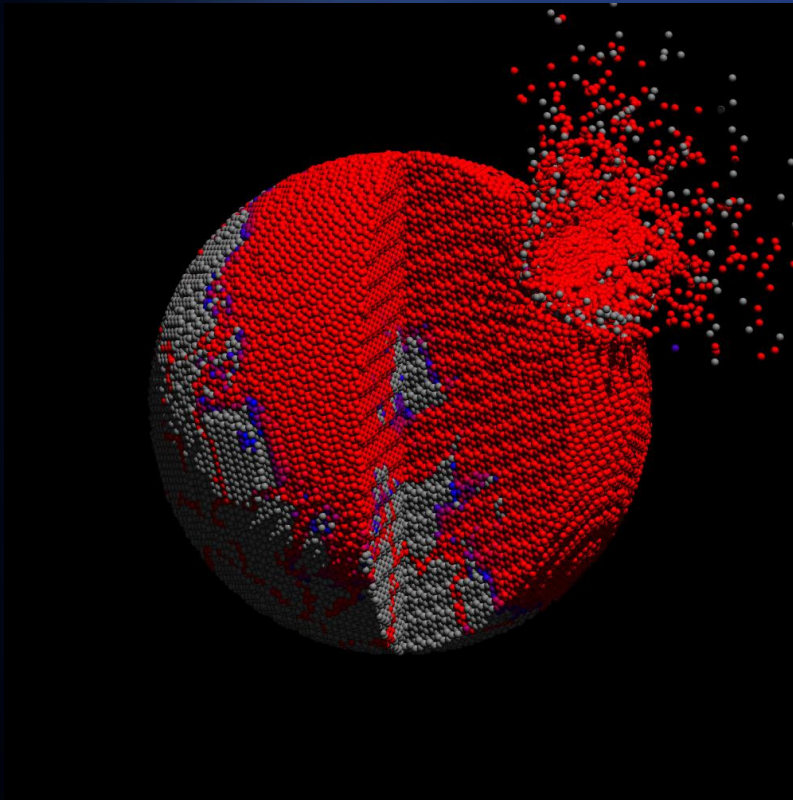
Red zones=  
dammage  
(separation  
between  
fragments)

Black points=  
void

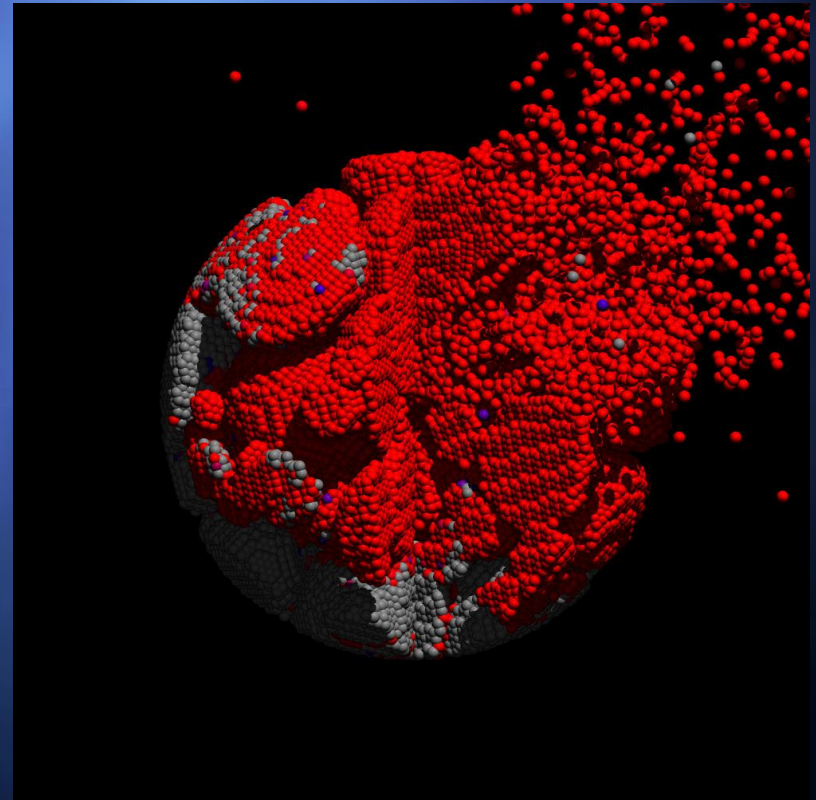
W. Benz & P. Michel

# *Two types of pre-shattered internal structures*

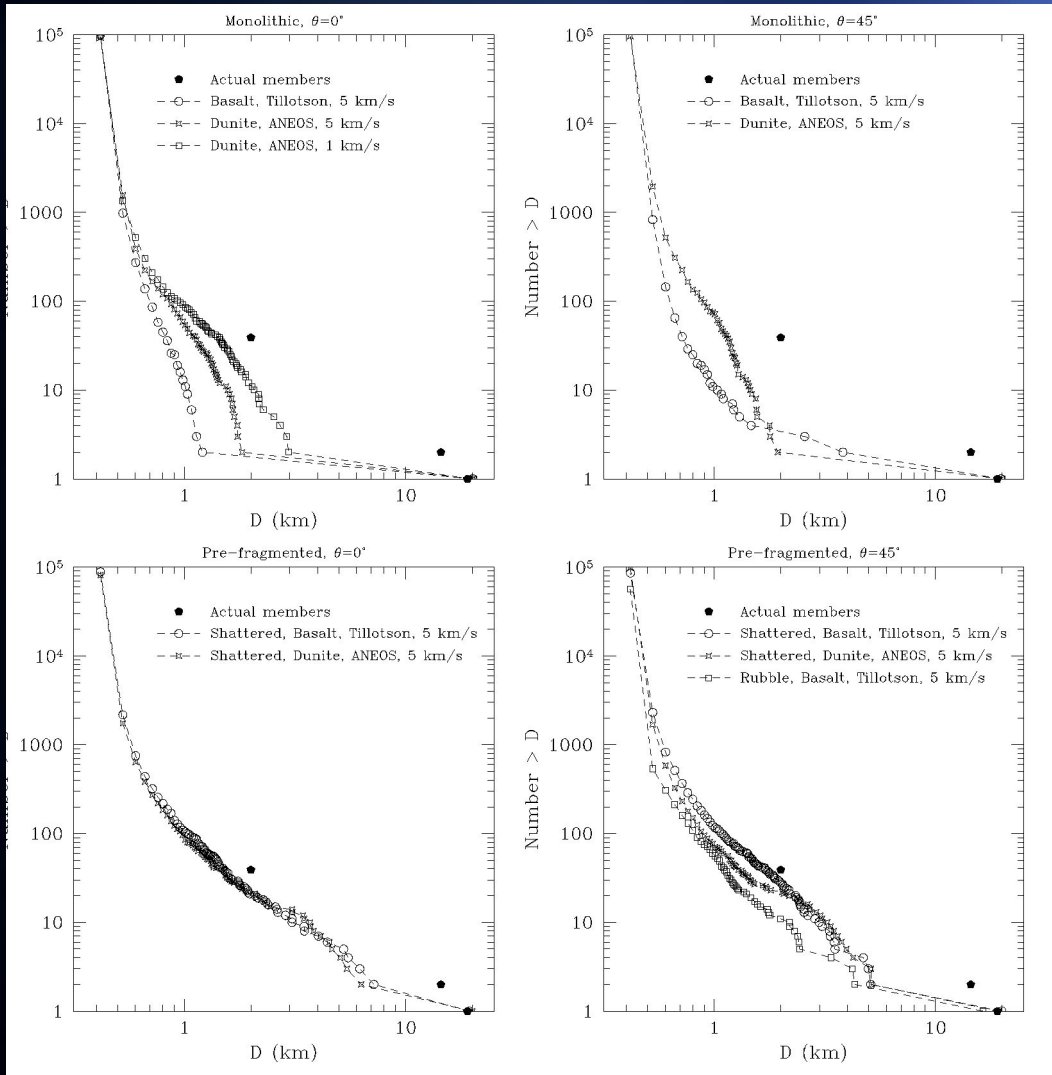
Presence of damage zones



Presence of damage zones + voids



# Monolithic/Pre-shattered Parent Body



Monolithic Parent Body

$N > D$  vs  $D$  (km)

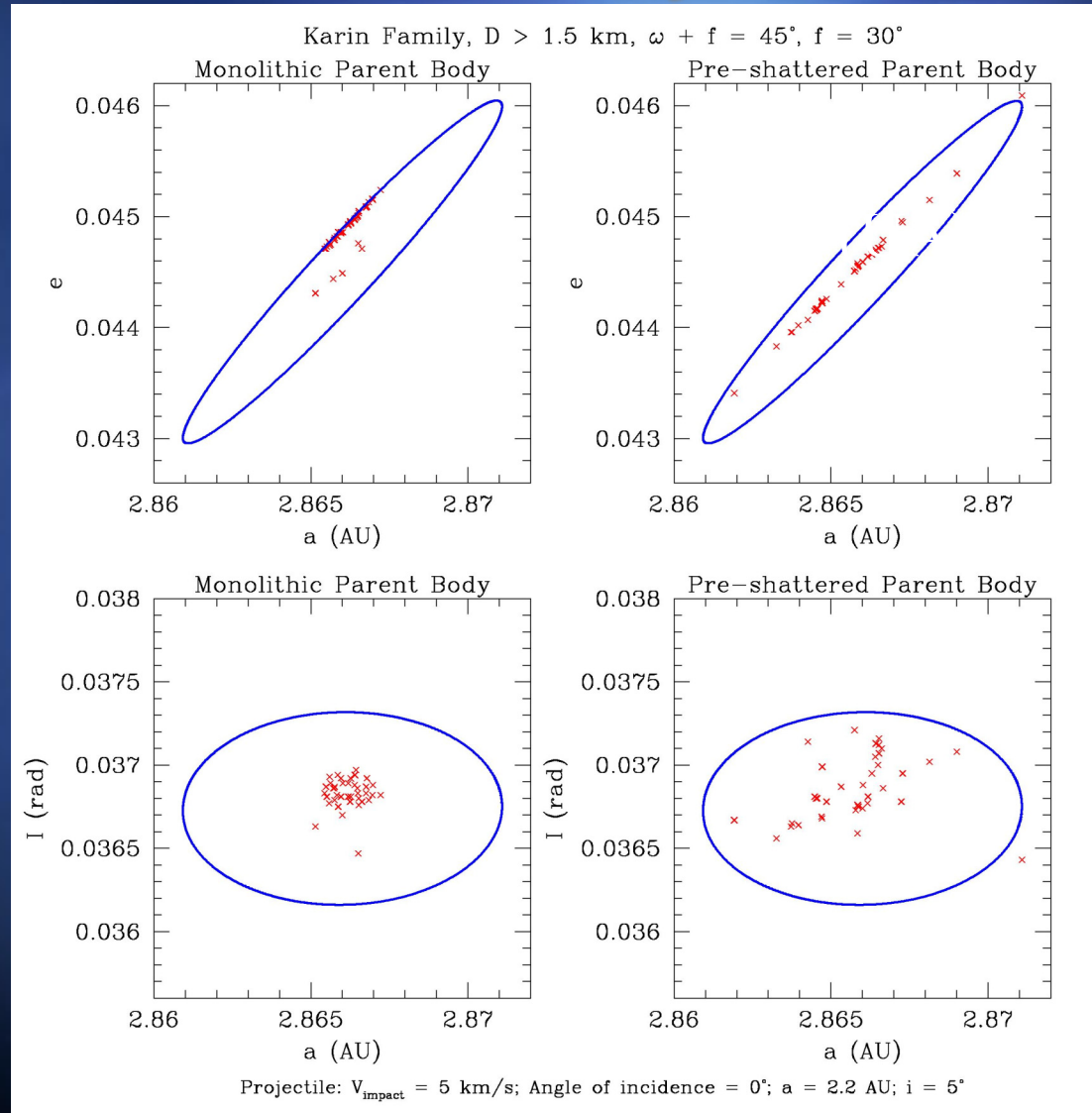
Pre-shattered Parent Body

# Monolithic/Pre-shattered Parent Body

Ellipses =  
spreading of  
the real family

Crosses =  
simulation

I (rad) vs a (UA)



# *So how can we improve the models?*

## ⊕ Compare, Compare, Compare

### ⊕ to real experiments

- ⊕ Large explosive field tests
- ⊕ Carefully controlled lab tests

### ⊕ to impact craters

- ⊕ (but what was the impactor?)

## ⊕ Test, Test, Test

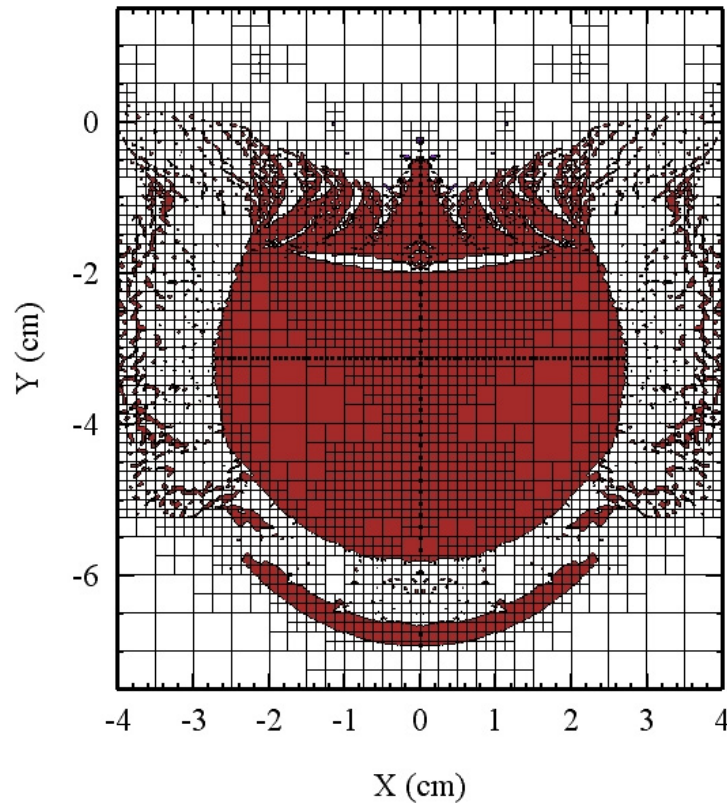
### ⊕ real materials

- ⊕ Crushability
- ⊕ Strength in different states



# *Experiments = first and crucial step for code validation*

test2: N&F Disrupt GSI=100 HSRG, Materials at 1.00e-03 seconds



Example:

Simulation by an Hydrocode of the Impact experiment on basalt of Nakamura & Fujiwara in 1992

The core fragment is successfully reproduced

# *“Some” Current Shortcomings:*

- Most strength models do not address all types of “strength”
- Codes often have “hidden features”
- Equations of state of some materials are still uncertain
- We do not often enough make comparisons to any experiments

# *Some more specific shortcomings*

- We cannot model well enough to distinguish details for a particular crater
- *We cannot handle mixtures well*
- *Mixing rocks and atmospheres, and porosity makes for very difficult code calculations*
- *We don't do chemistry*

# *However, on the positive side*

- ⊕ **In the gravity regime:** we were able to reproduce qualitatively the main properties of asteroid families → reaccumulation processes may dominate and « accurate » modeling of fragmentation may not be so crucial (needs to be checked) for qualitative studies
- ⊕ **In the strength regime:** the SPH hydrocode including a model of brittle failure has at least reproduced successfully some experiments on basalt targets
- ⊕ **Future challenge:** characterizing the behavior of porous materials and differentiated objects, first in the strength regime (with confrontation to experiments) and then in the gravity regime (formation of C-type asteroid families, impact response of comets, KBOs ...)

*Arigato Gozai-Masu*  
*Thank you for your attention*  
*Merci beaucoup ...*