EVOLUTIONS OF SMALL BODIES IN OUR SOLAR SYSTEM

Dynamics and collisional processes

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Plan

- Chapter I:

A few concepts on dynamics and transport mechanisms in the Solar System; application to the origin of Near-Earth Objects (NEOs)

- Chapter II:

On the strength of rocks and implication on the tidal and collisional disruption of small bodies

Preliminaries: orbital elements

a= semi major axis

e=eccentricity

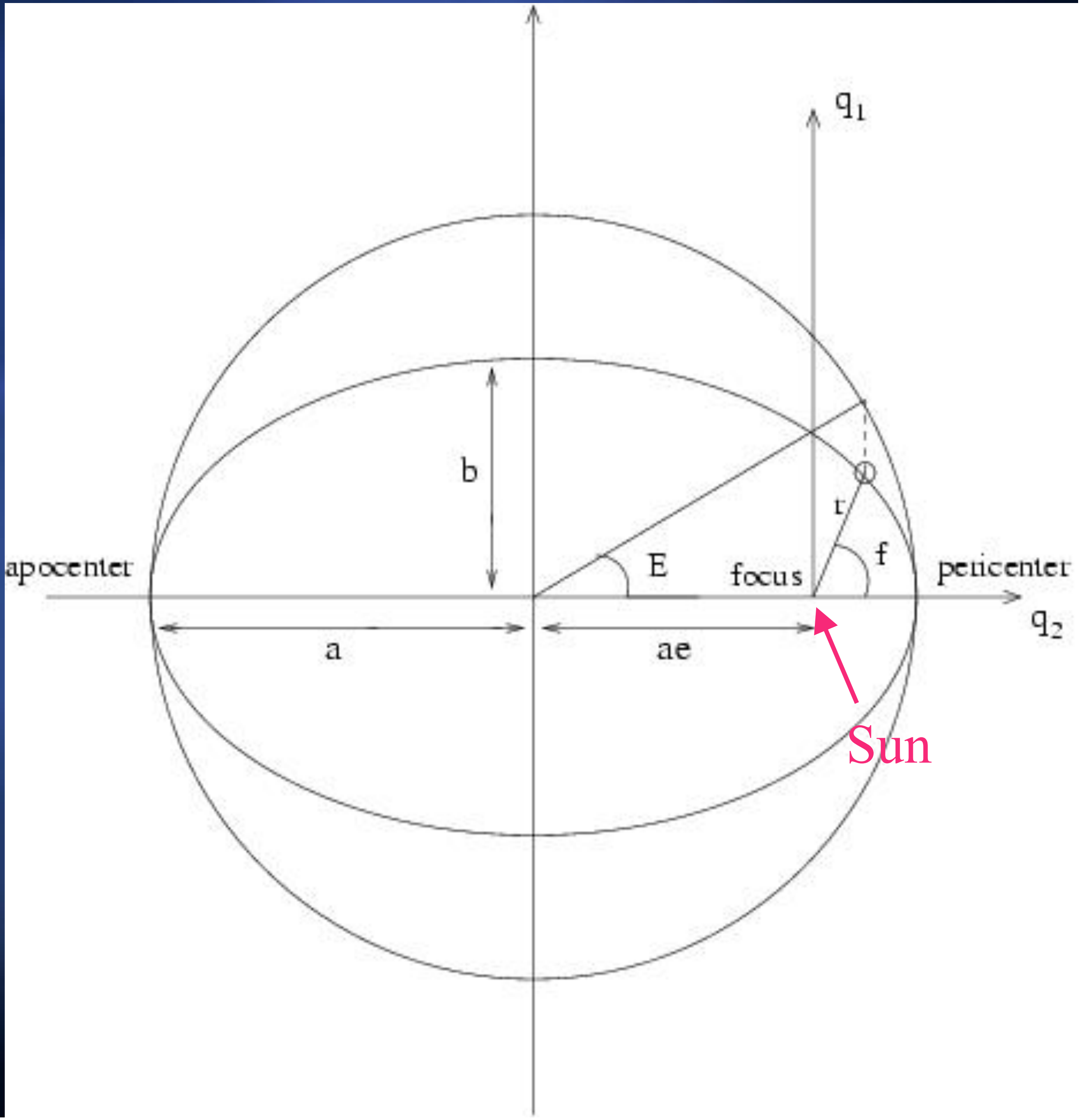
f=true anomaly

E=eccentric anomaly

Mean anomaly:

M=E-e sin E= n t

with $n=(GM_*)^{1/2}/a^{3/2}$ (orbital frequency)



Preliminaries: orbital elements

i = inclination

Ω = longitude of node

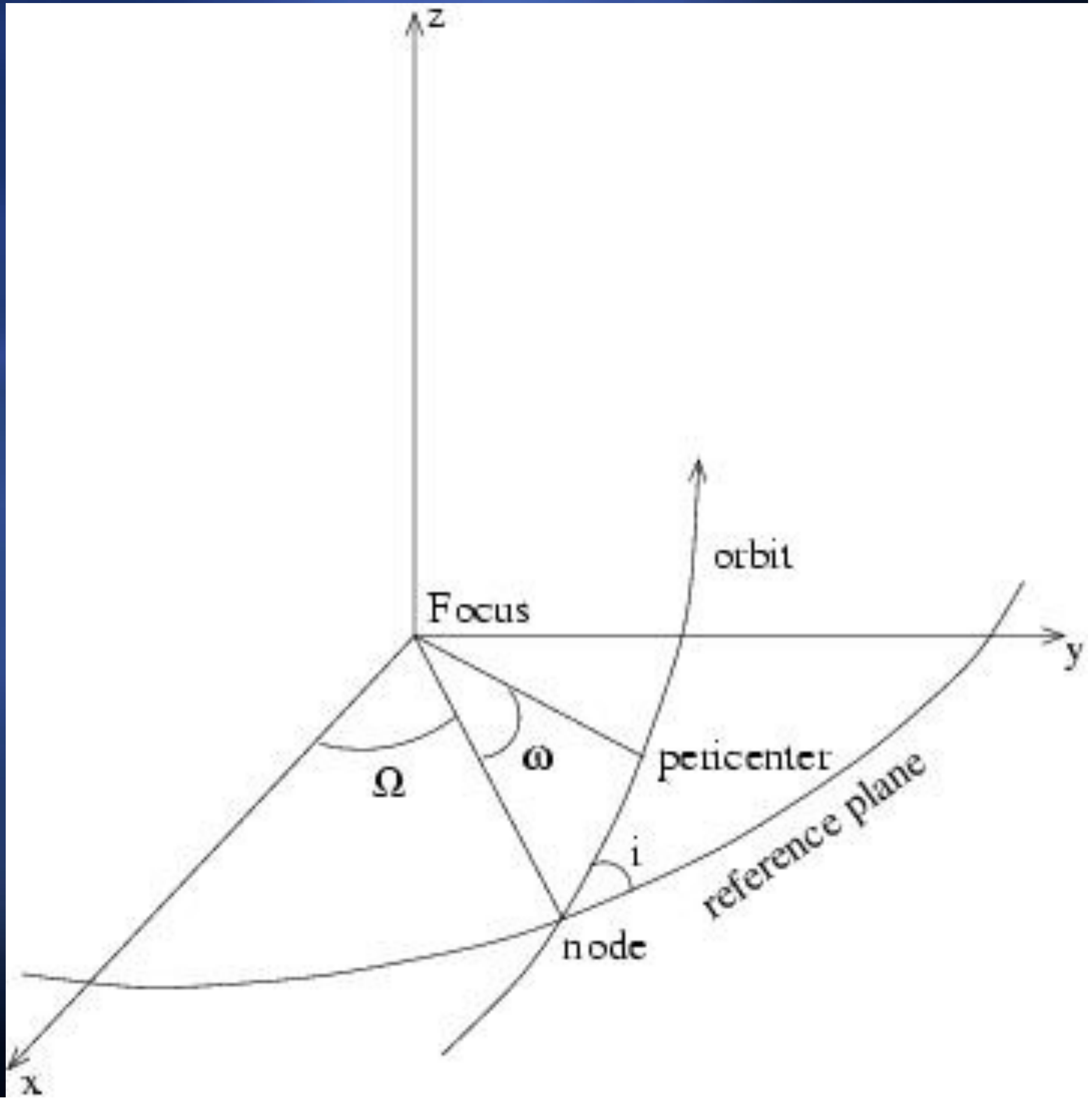
ω = argument of pericenter

Longitude of pericenter:

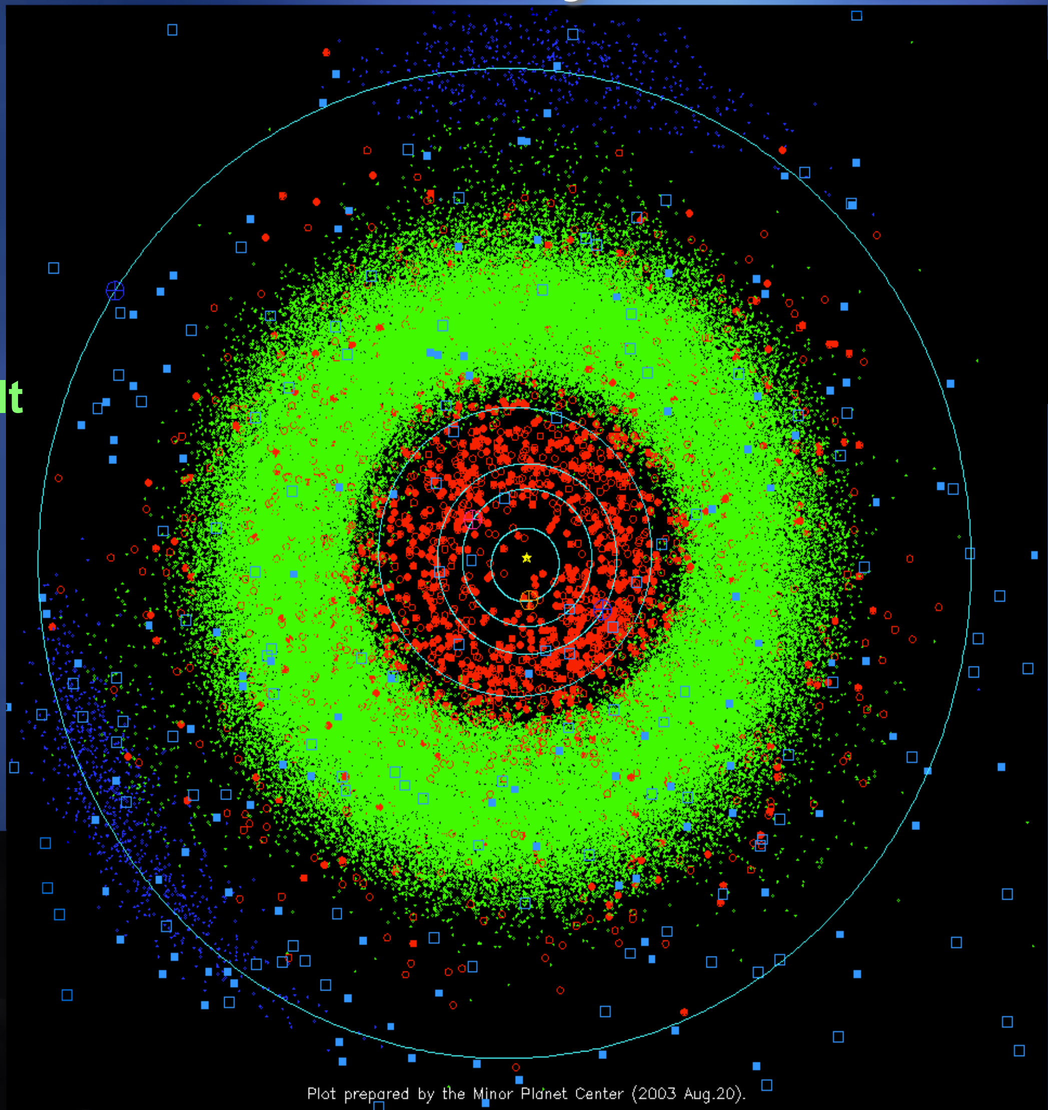
$\varpi = \omega + \Omega$

Mean longitude:

$\lambda = M + \varpi$



The small body populations in the Inner Solar System

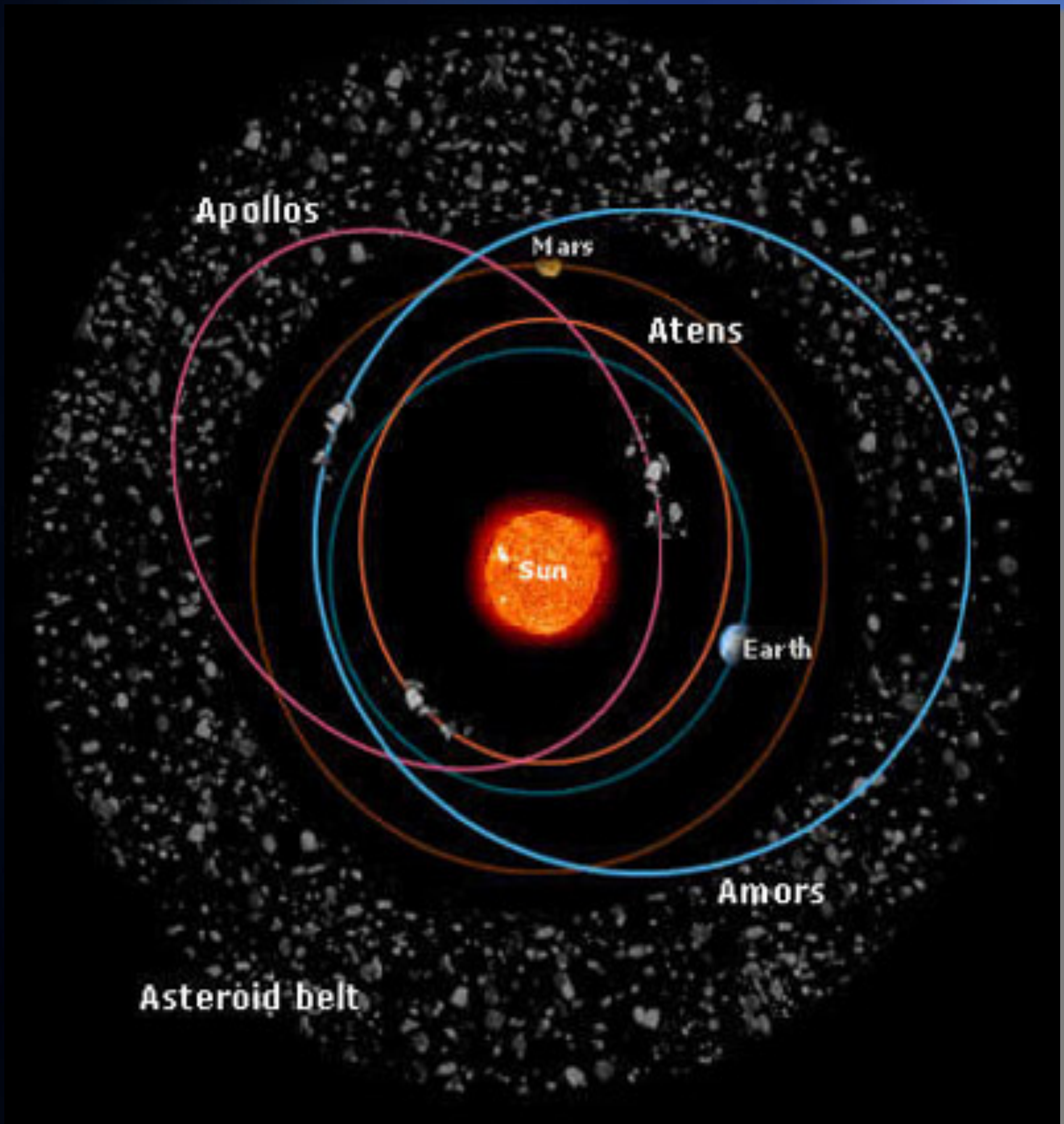


Green:Asteroid Main Belt

Blue squares:Comets

Red: objects with perihelion distance q<1.3 AU

The NEO population



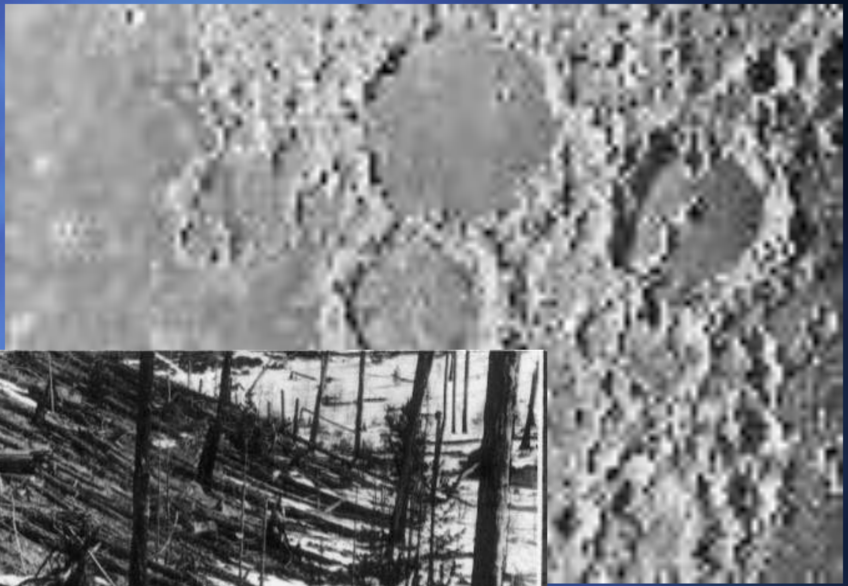
- **Amors**: a>1 AU
  1.017<q<1.3 AU
- **Apollos**: a>1 AU
  q<1.017 AU
- **Atens**: a<1 AU
  Q>0.987 AU
- **IEOs**: a<1 AU
  Q<0.987

1000 Objects
with D>1 km,
≈500 discovered

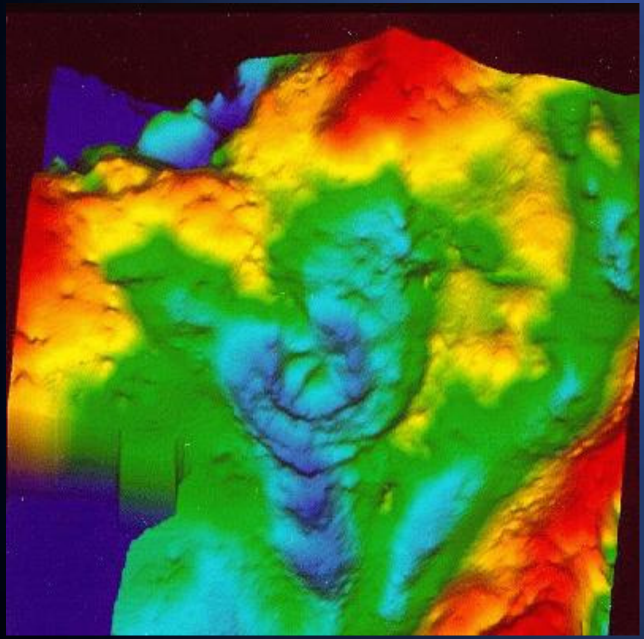
The NEO threat!

Impacts are real facts!

Moon



Venus



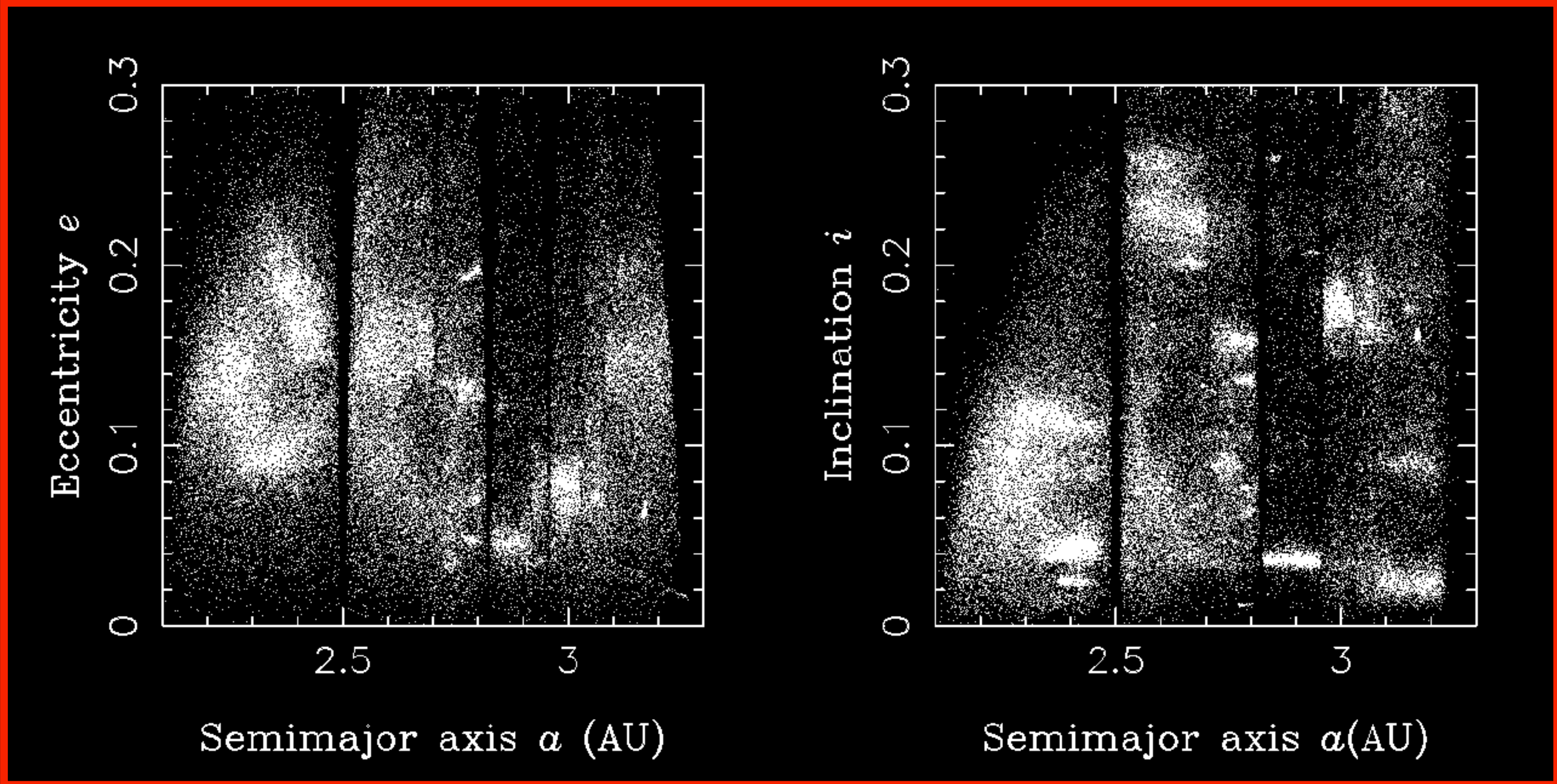
Earth:
Tunguska, 1908

The least likely natural disaster BUT the only that may be predicted and avoided!

Main transport mechanisms in the Solar System

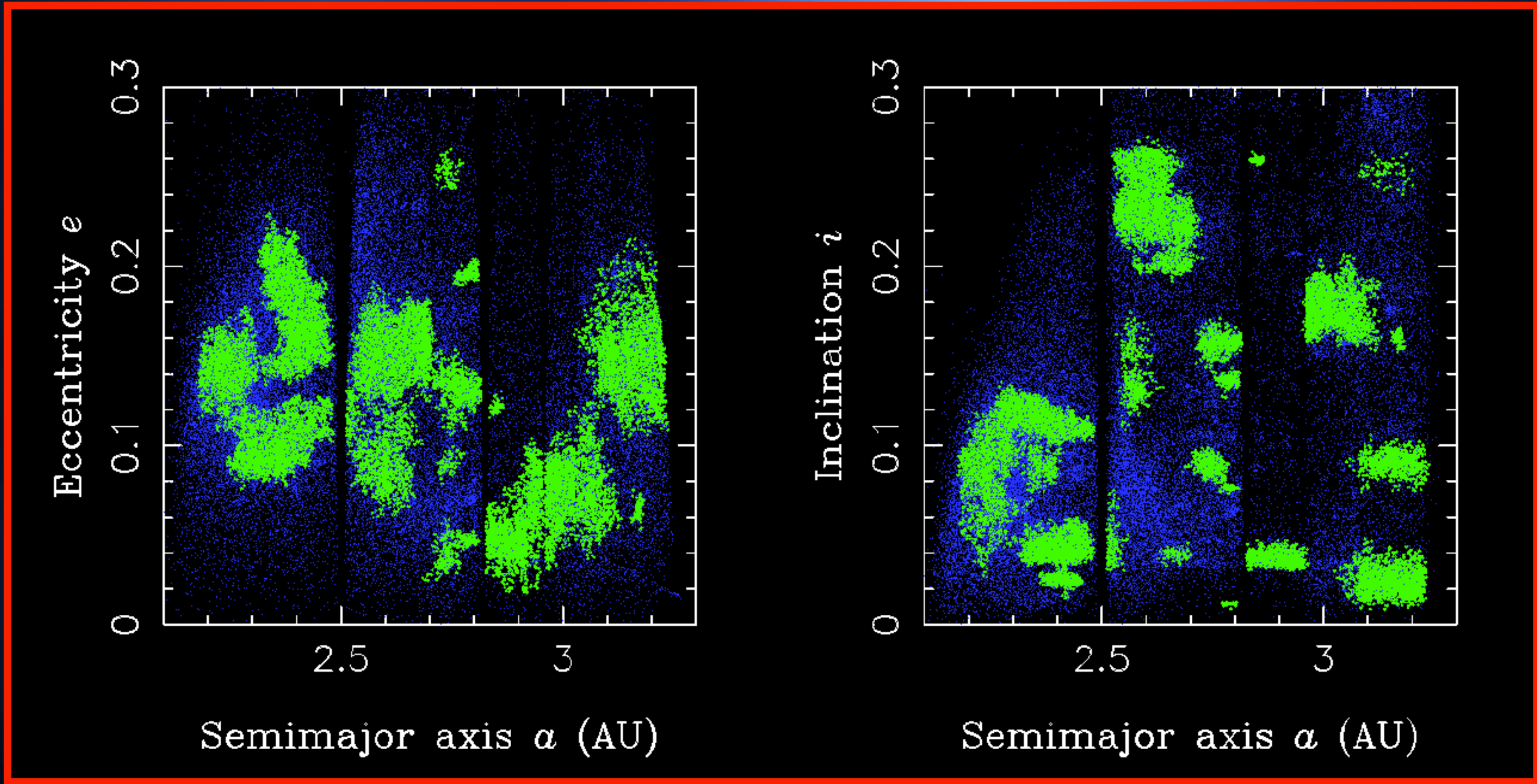
- Fast mechanisms:
  - Mean motion resonances with planets
  - First-order secular resonances with planets
- Slow diffusions (not described in this lecture):
  - Non-gravitational effects (Yarkovsky)
  - High-order and three-body resonances

The Kirkwood gaps in the asteroid Main Belt

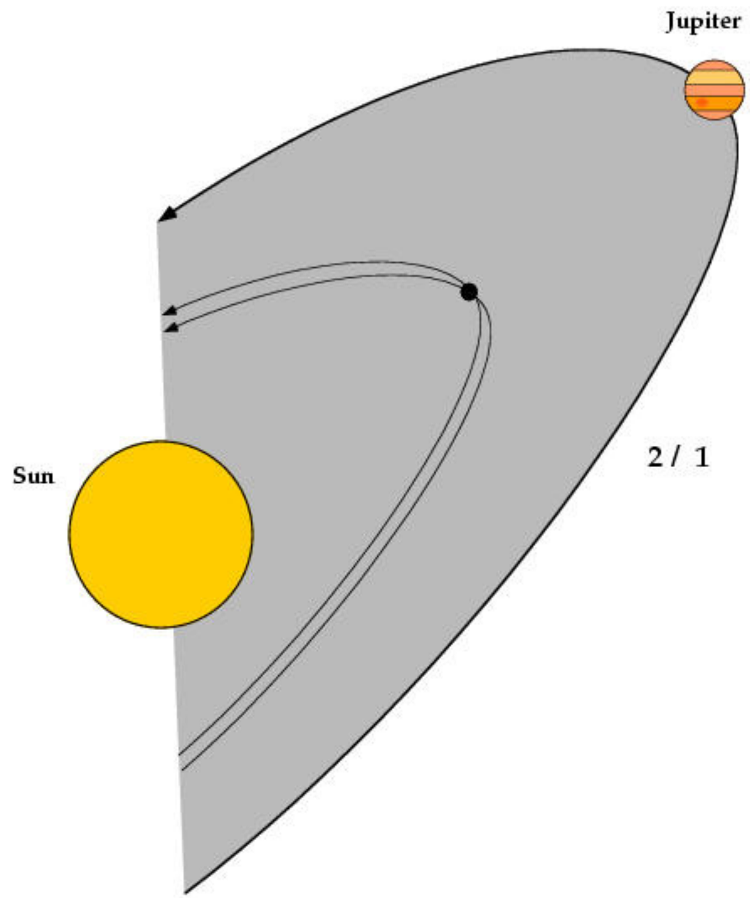


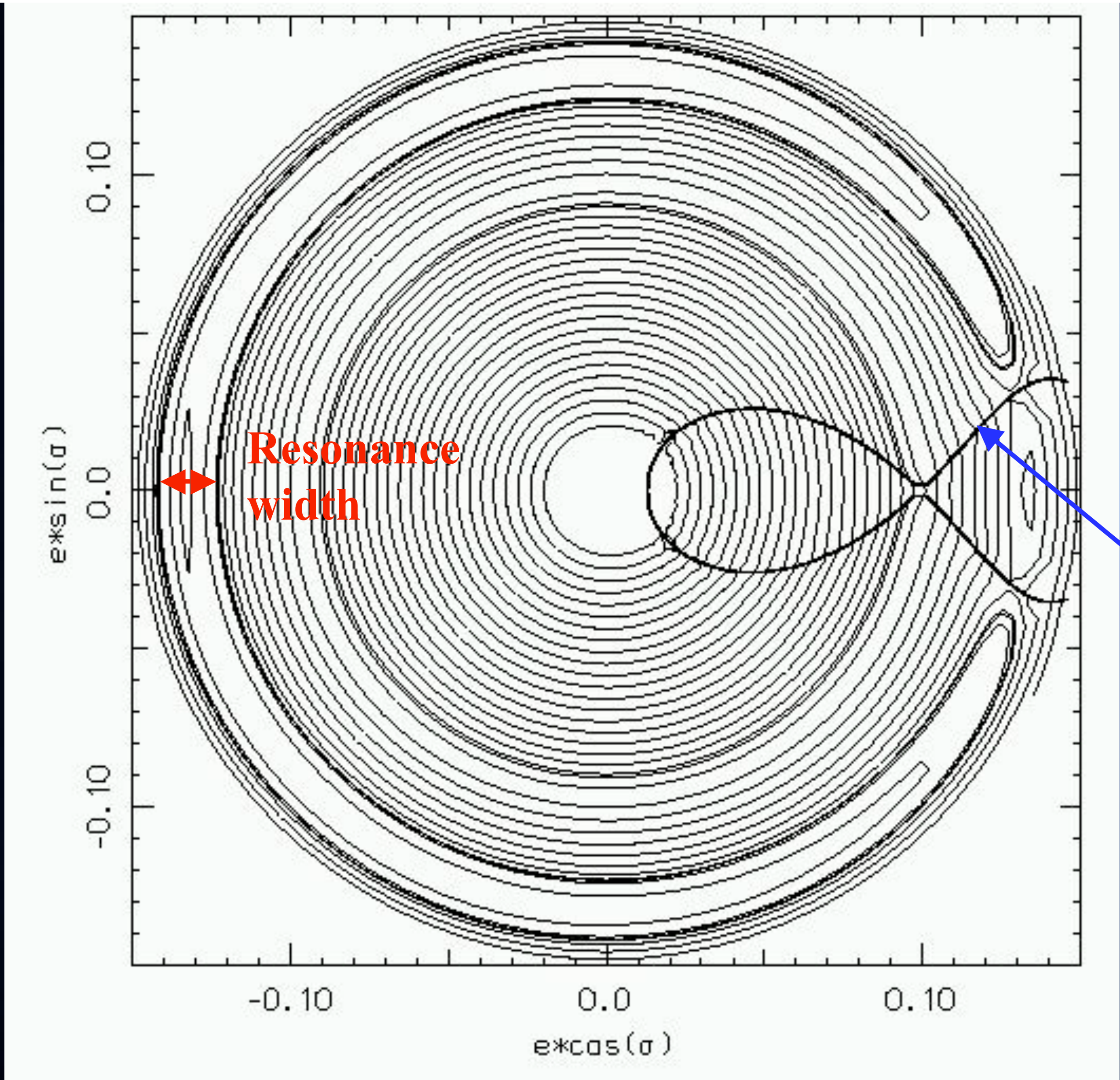
Collisions produce asteroid families!

This will be addressed in Chapter II



Mean Motion Resonances



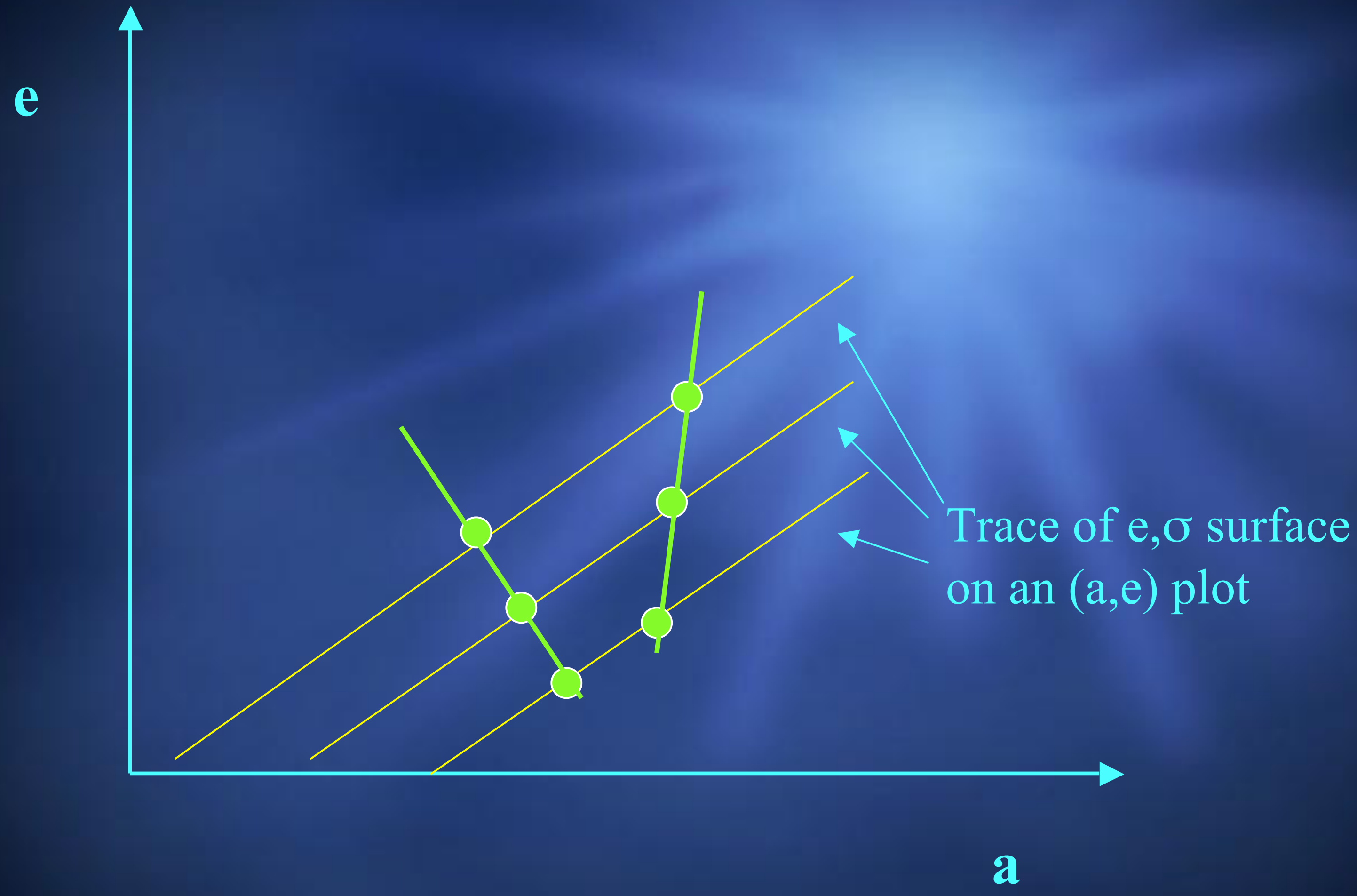


MM Resonance

(e,σ) surface plot

$\sigma_{i/j} = i\lambda_p - j\lambda - (i-j)\varpi$

Planet collision line

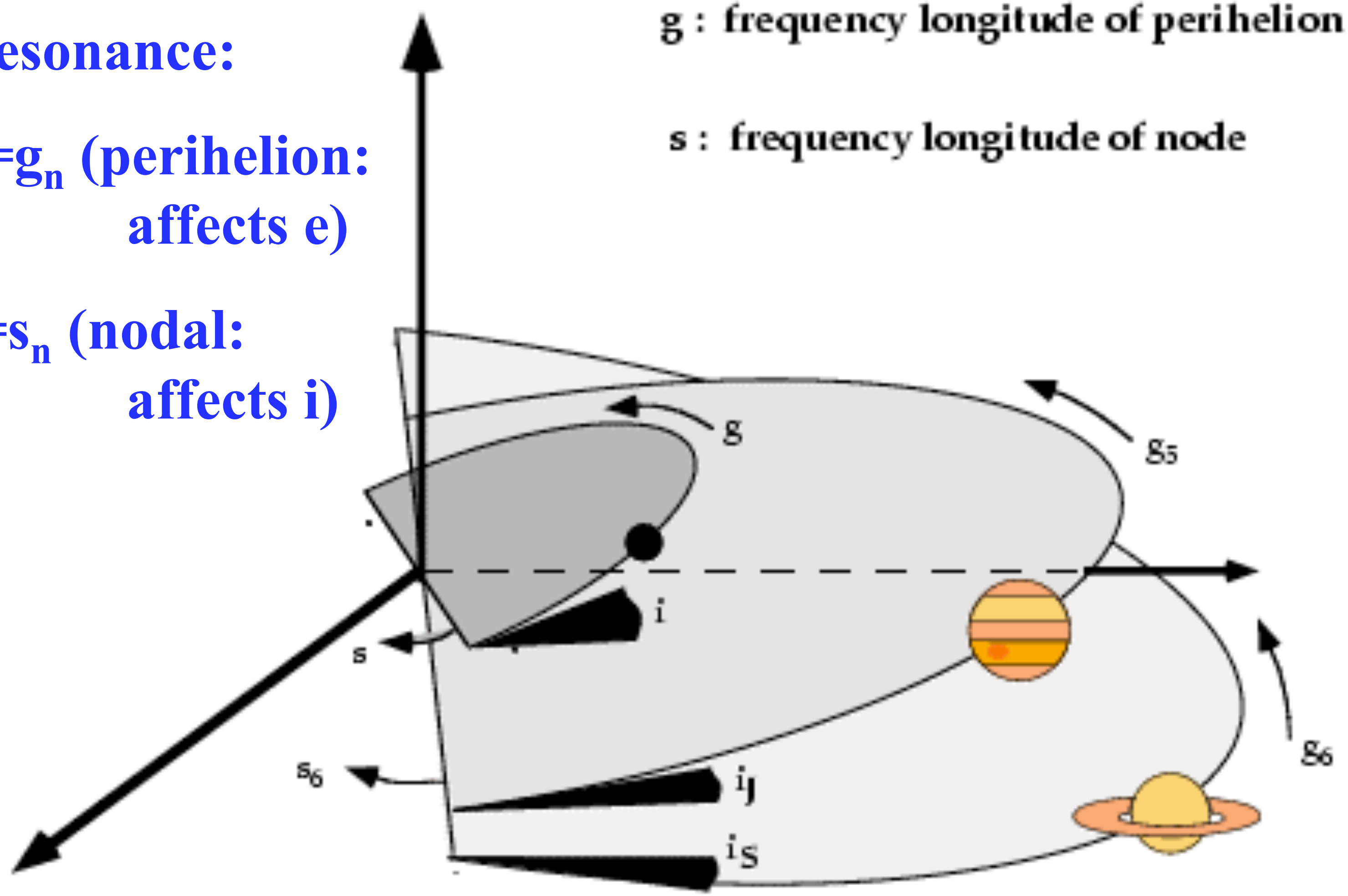

e
a
Trace of e,σ surface
on an (a,e) plot

SECULAR RESONANCES

Resonance:

$g=g_n$ (perihelion: affects e)

$s=s_n$ (nodal: affects i)



Main principle

$$H(\dot{\mathbf{r}},\dot{\mathbf{r}}_j;\mathbf{r},\mathbf{r}_j) = \frac{1}{2}\|\dot{\mathbf{r}}\|^2 - \frac{1}{\|\mathbf{r}\|} - \sum_{j=1}^{N_p} m_j\left[\frac{1}{\|\Delta_j\|} - \frac{\mathbf{r}_j \bullet \mathbf{r}}{\|\mathbf{r}_j\|^3}\right]$$

At first order in planetary mass (*j* = planet index), the hamiltonian of a massless body expresses as:

$$\mathrm{H}=-\frac{1}{2L^2} - \sum_{j=2}^{N_P} m_j P_j(L,G,H,L_j,G_j,H_j;l,g,h,l_j,g_j,h_j),$$

Keplerian part — Planetary perturbations

$$L = \sqrt{a} \qquad l = M$$

$$G = \sqrt{a(1-e^2)} \qquad g = \omega$$

$$H = \sqrt{a(1-e^2)}\cos i \qquad h = \Omega$$

Delaunay variables

Assumption: the small body is not in a mean motion resonance

- The Hamiltonian (at 1st order in planet masses) can be averaged over all mean anomalies *l* and l_j (fast angles)

$$\overline{\mathrm{H}} = -\frac{1}{2L^2} - \sum_{j=2}^{N_P} m_j \overline{P}_j(-,G,H,-,G_j,H_j;-,g,h,-,g_j,h_j)$$

L = cste, so we omitt the first term and expand the perturbation w.r.t. planetary eccentricities and inclinations:

$$\overline{\mathrm{H}} = -\sum_{j=2}^{N_P} m_j \left[K_0^j + (e_j,i_j)K_1^j + (e_j,i_j)^2 K_2^j + \ldots\right]$$

$(e_j,i_j)^r$ are terms prop. to $e_j^a i_j^b$, with $a+b = r$ and $a,b \geq 0$

Isolate the first term K_0

- It can be shown that:

$$K_0 = \sum_{\substack{\geq 0 \\ p,q \in \mathbb{N}}} c_{0,-v,v,0,p,q,0,0}\, e^{|2v|+2p} i^{|2v|+2q} \cos(2v(\underbrace{\varpi - \Omega}_{\omega}))$$

Thus, $K_0=f(\varpi-\Omega)=f(g)$

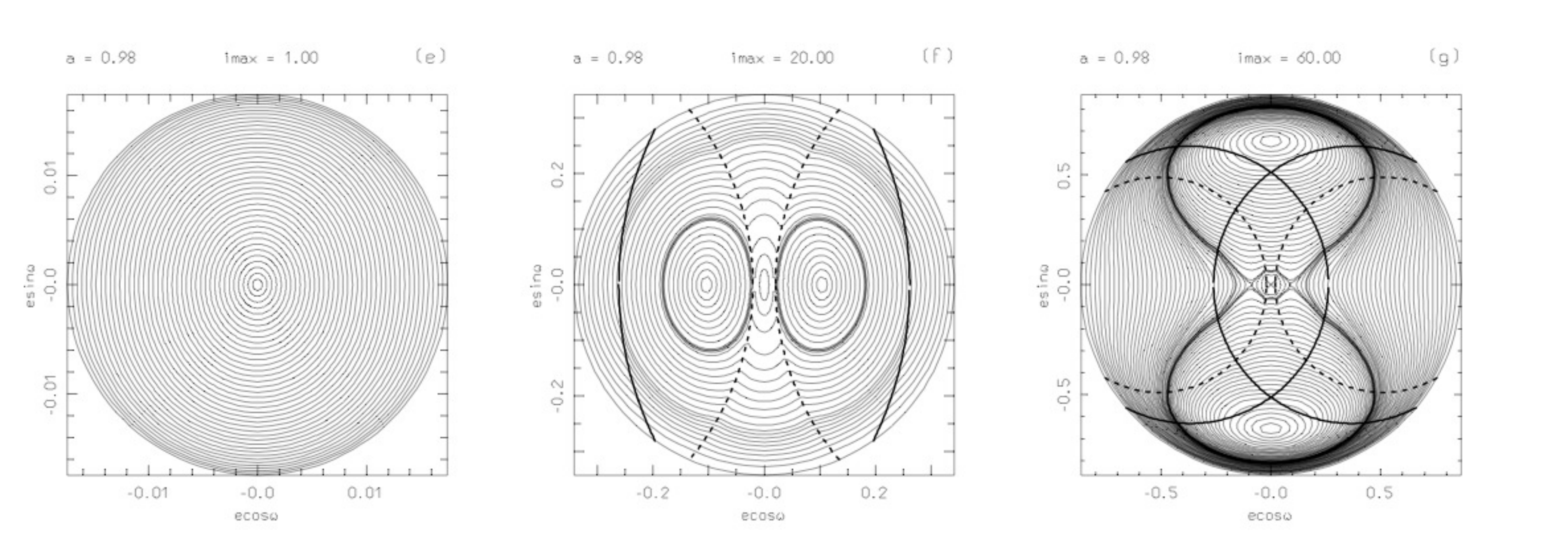
K_0 = 1 degree of freedom integrable hamiltonian in the variables *G,g* as it depends only on the angle *g* (= ω).

It is parametrized by the constants actions *L* and *H*.

Its highly non-linear dynamics can be studied in details (Kozai 1962) by drawing level curves in the (e, ω) plane on a surface *H* = constant.

Dynamics of K_0 at a=0.98 AU on 3 different surfaces of H=cst, each characterized by a value of i_{max} (1°, 20°, 60°)

- Polar diagram (e,ω)



From Michel & Thomas (1996, AA 307)

Location of secular resonances

- The free frequencies of ϖ and Ω of the asteroid's orbit in the (a,e,i) space are obtained by integrating wrt time:

$$\dot{G} = -\left(\frac{\partial K_0}{\partial g}\right),$$

$$\dot{g} = \left(\frac{\partial K_0}{\partial G}\right),$$

$$\dot{h} = \left(\frac{\partial K_0}{\partial H}\right)$$

Proper frequencies = average values over a complete cycle of the free oscillations with period T (from g=0 to g=g(T)=2π)

Secular resonance: (a,e,i) for which:

$$<\dot{\varpi}> = g_{planet}$$

or

$$<\dot{\Omega}> = s_{planet}$$

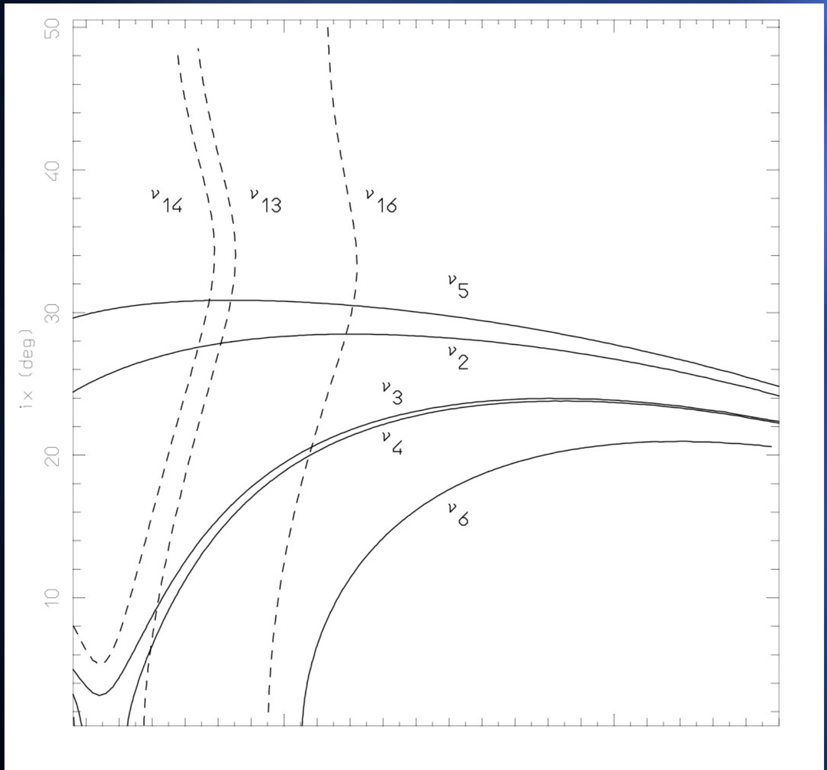
Ex: $\nu_6 \rightarrow g_6 \approx 28.22$ ''/yr

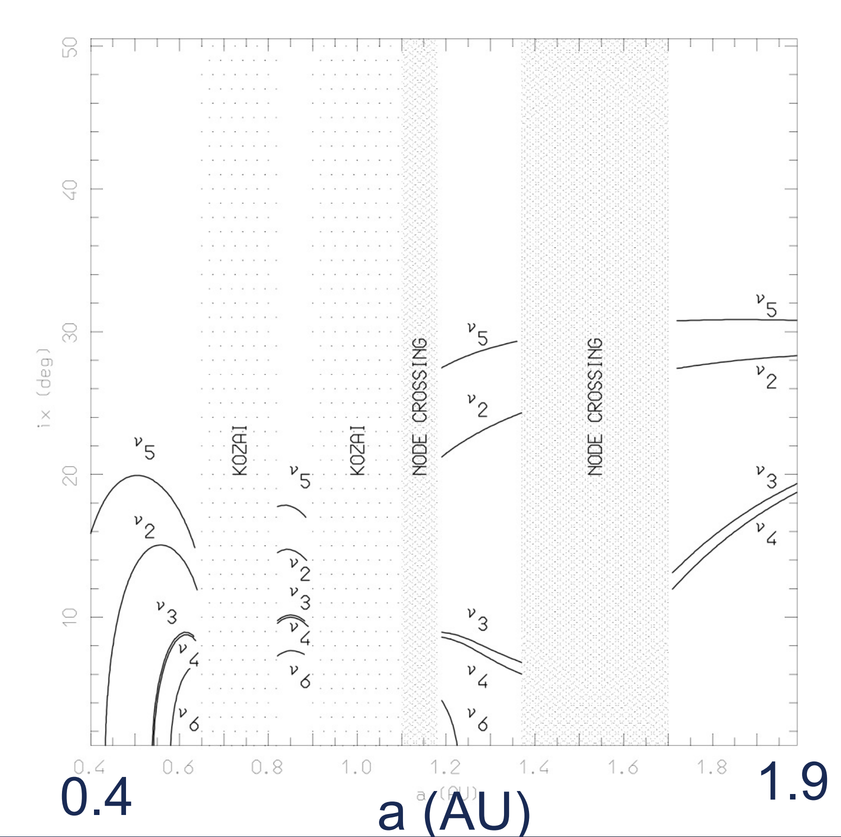
Some secular resonance locations (left: main belt, right: NEO region)

From Michel & Froeschlé (1997, Icarus 128)

50°

i°





1.5 a (AU) 3.5

e=0.1

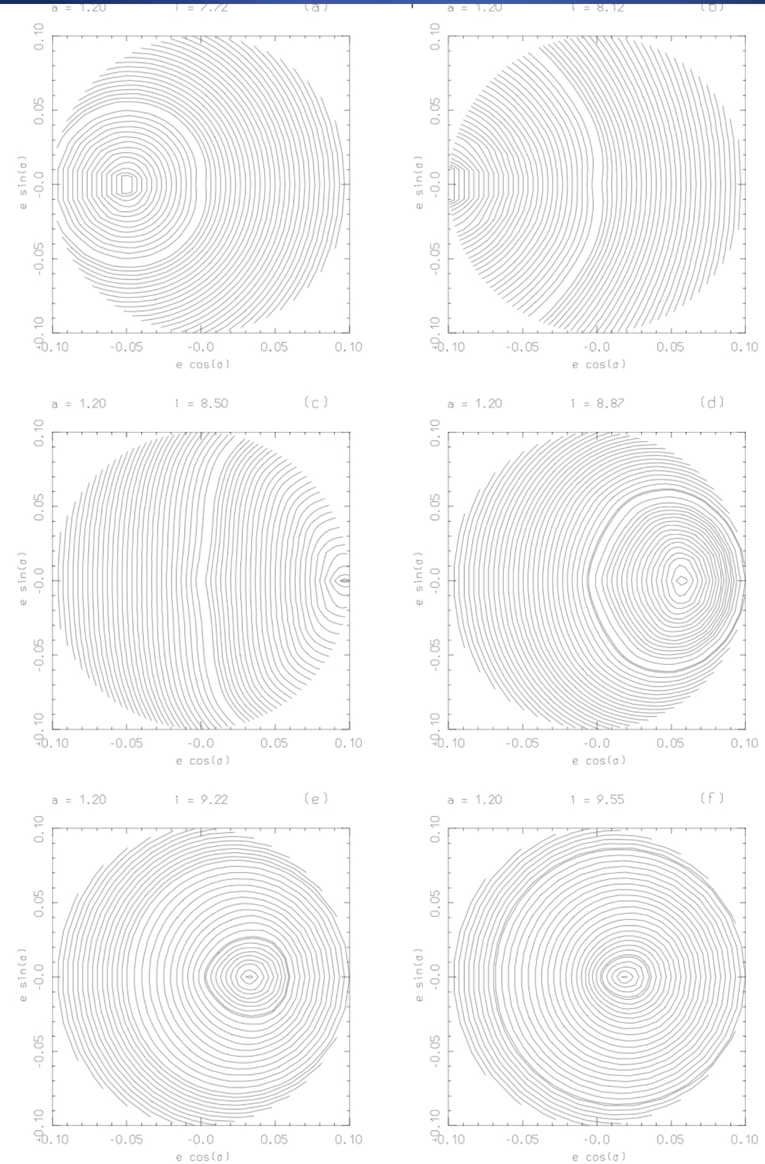
0.4 a (AU) 1.9

Effect of secular resonances

Needs to consider the first-order term in e_j and i_j of the Hamiltonian (K_1)

Ex: effect on eccentricity of ν_4 at a=1.2 AU

0.1

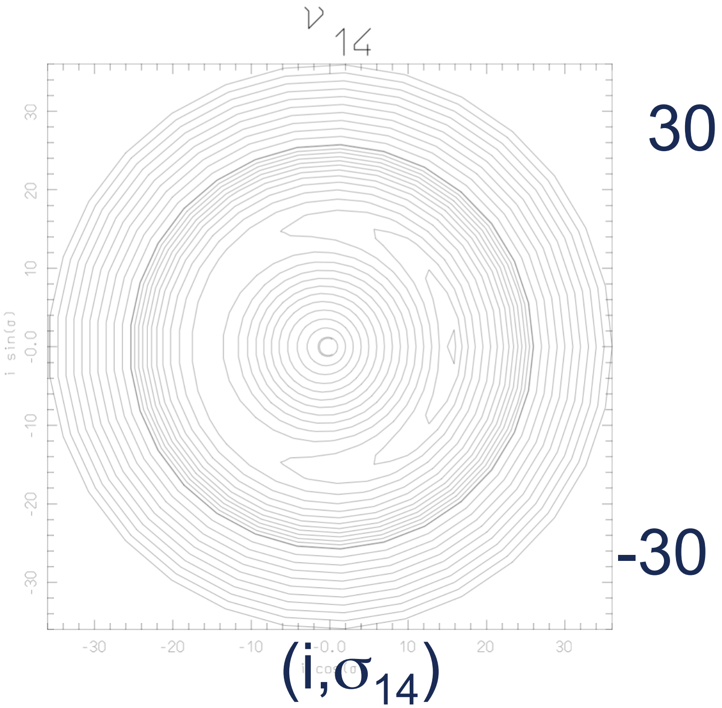


(e, σ_4)

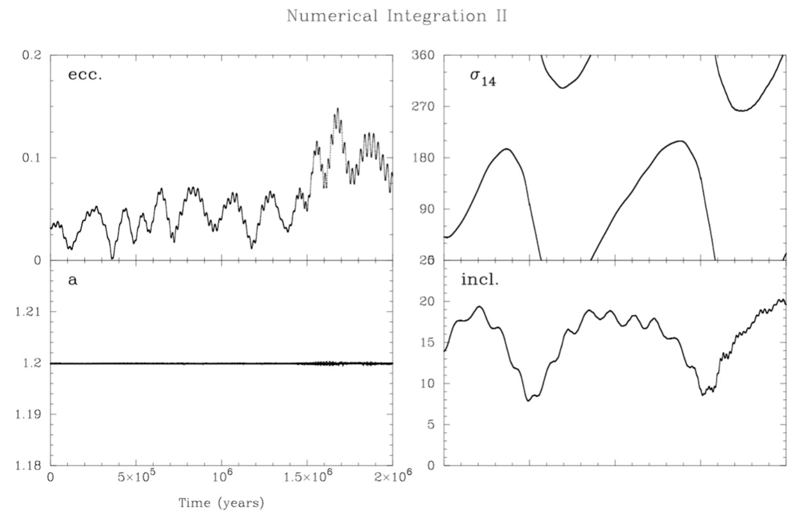
Effect of secular resonances (II)

Semi-analytical theory

a = 1.2 AU



Numerical integration



Effect of resonance overlapping

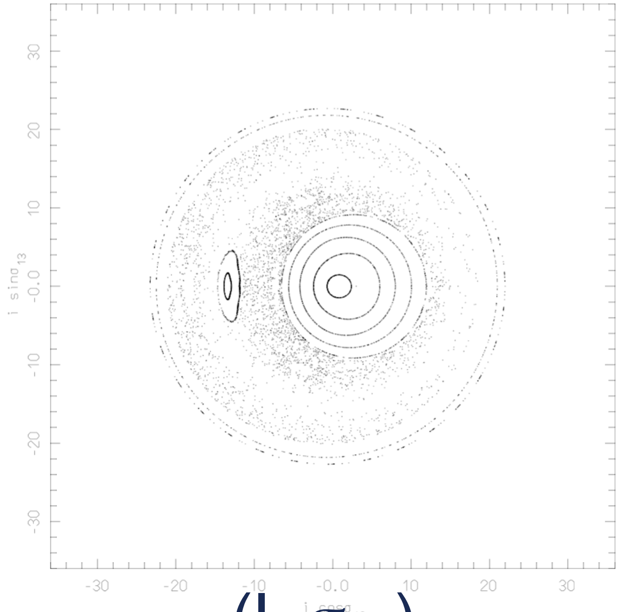
Overlapping of
ν_{13} and ν_{14}

Surface of section
at $\sigma_{14} = \pi$

a = 1.2 AU

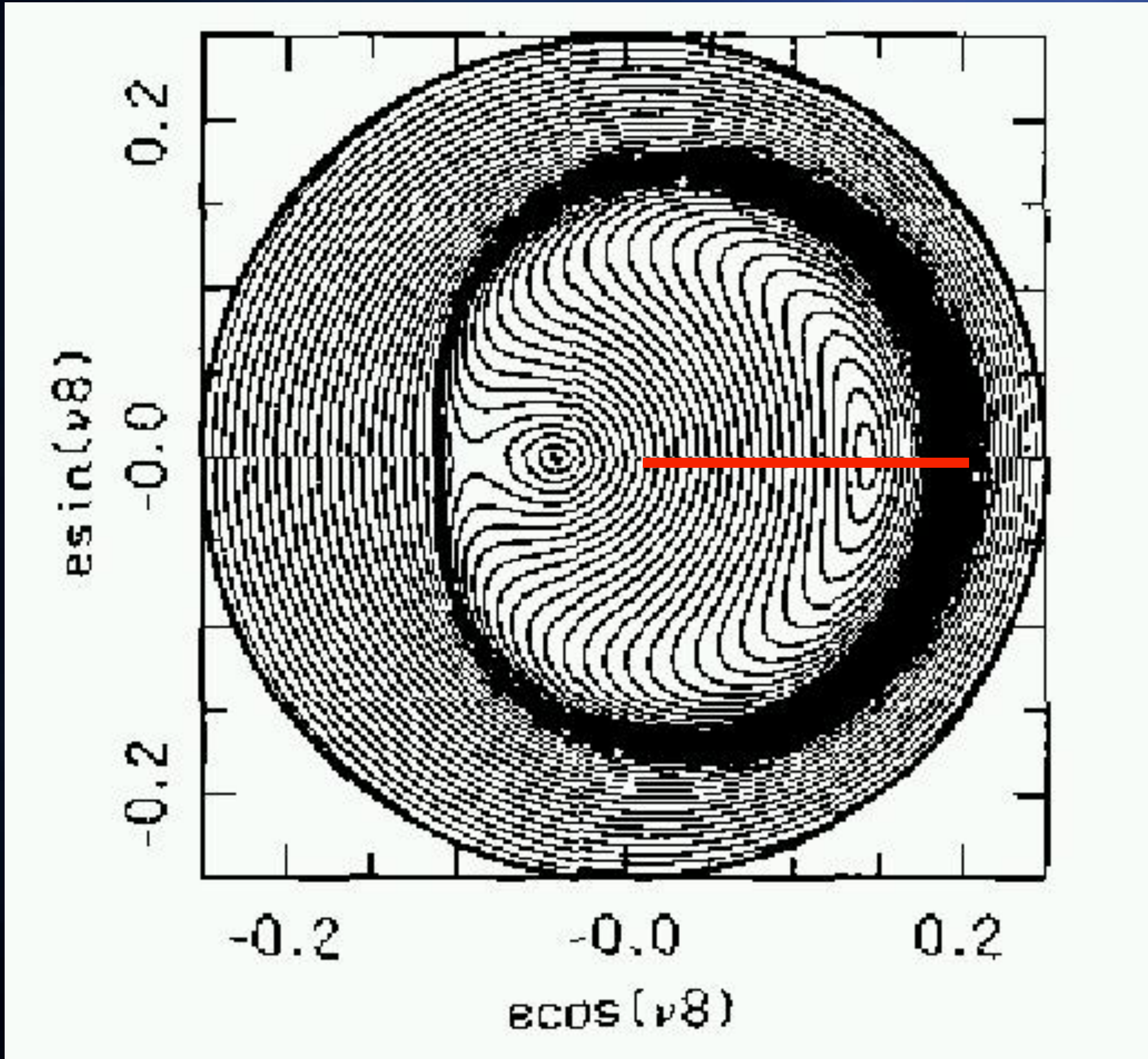
30

-30



(I, σ_{13})

Example: Dynamics of the g=g8 resonance at 41 AU



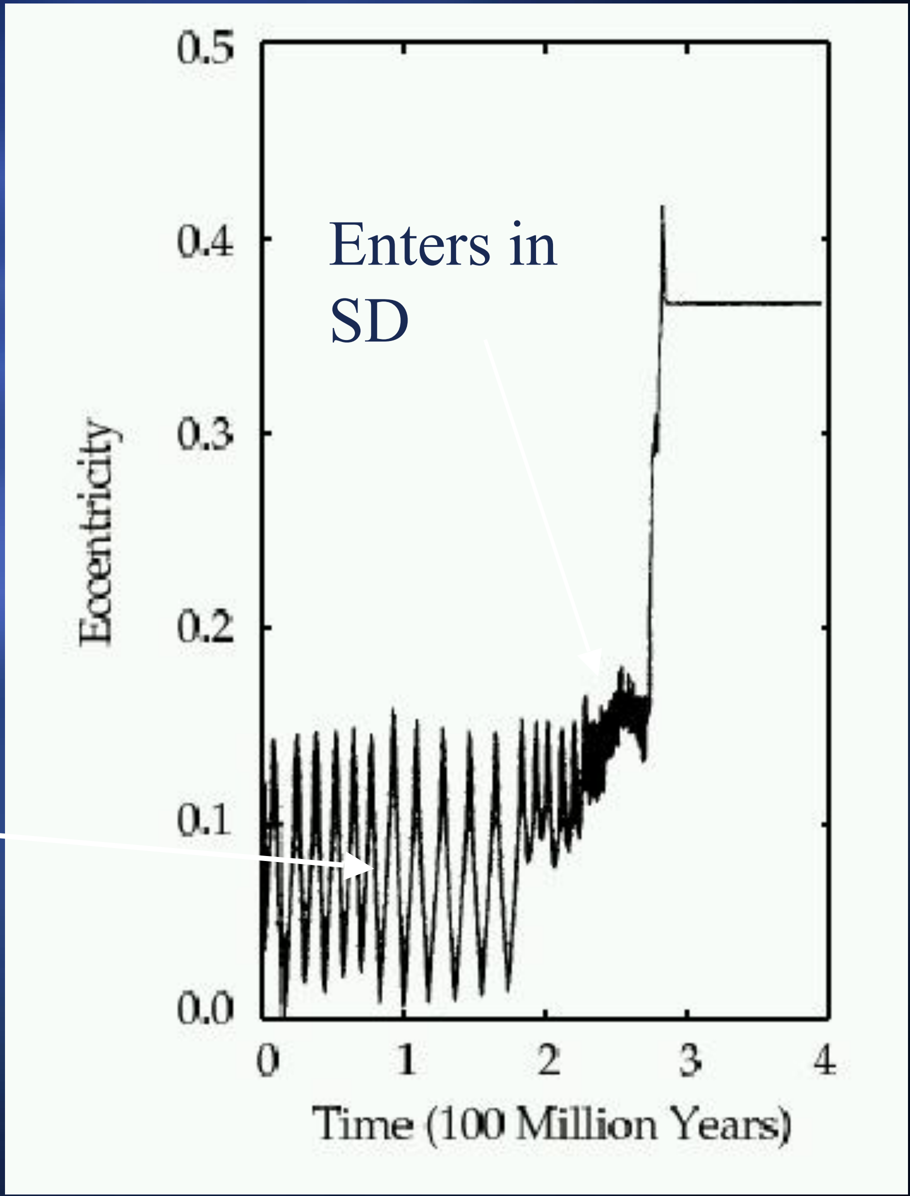
ν_8

$\sigma_8 = \varpi - \varpi_N$

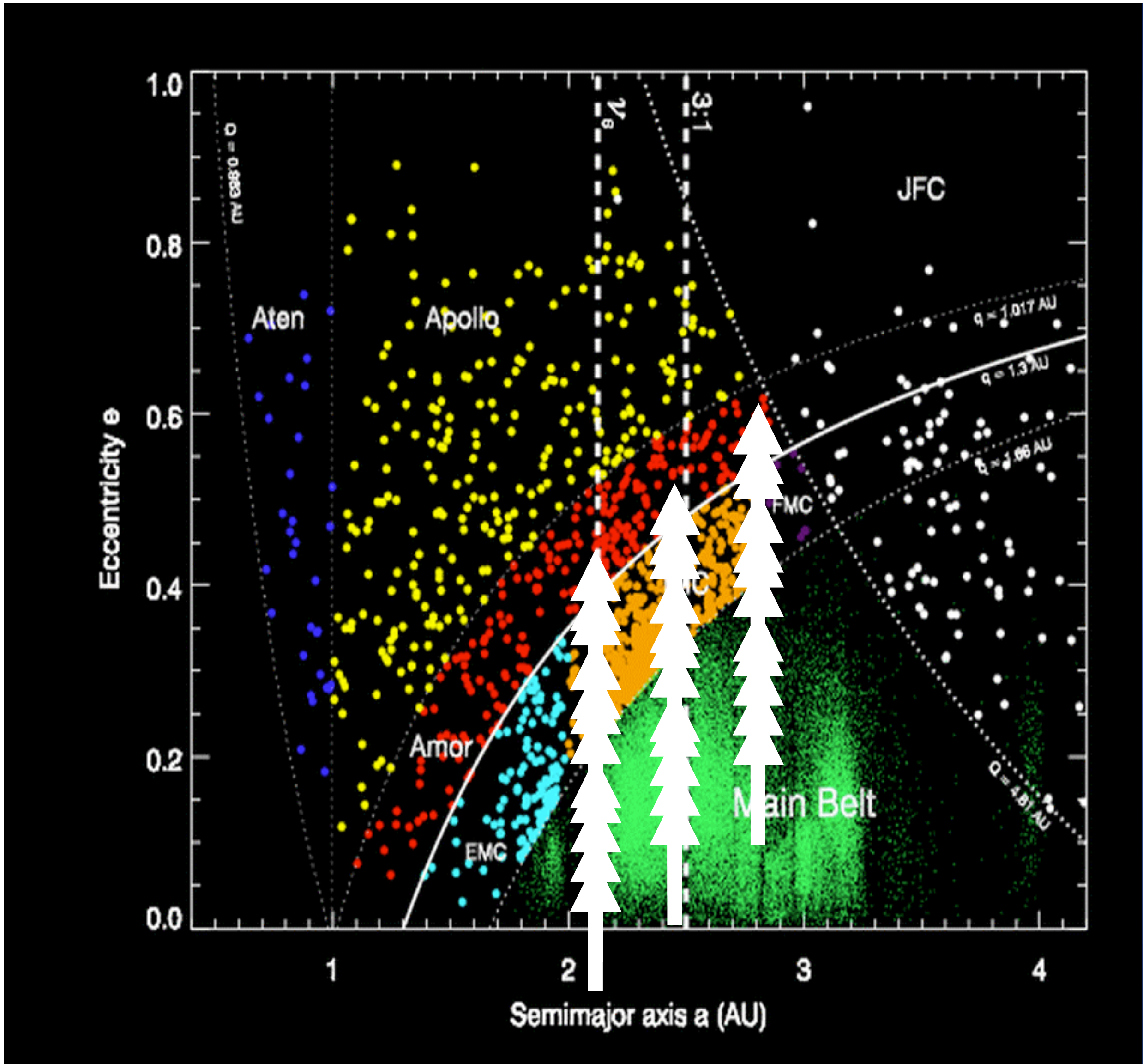
Morbidelli et al., 1995

Simulation of the evolution of a body in the g=g8 resonance by Holman and Wisdom, 1993

Secular resonance driven slow oscillations



Origin of NEOs



Asteroids from different regions of the Main Belt (MB) are injected into resonances which transport them on Earth-crossing orbits

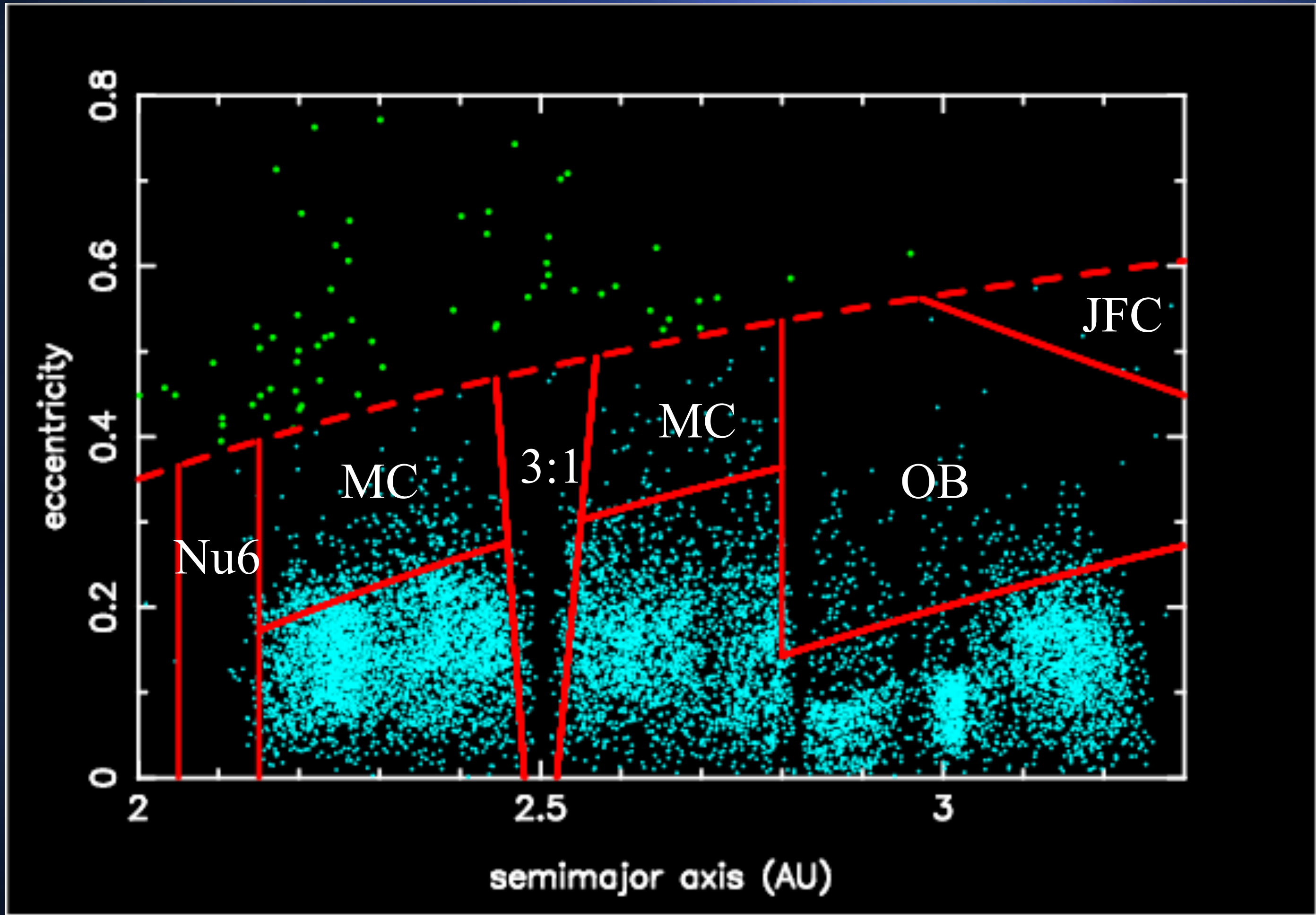
Hamiltonian describing the evolution of a massless body:

$$H(\dot{\mathbf{r}},\dot{\mathbf{r}}_j;\mathbf{r},\mathbf{r}_j)=\frac{1}{2}\|\dot{\mathbf{r}}\|^2-\frac{1}{\|\mathbf{r}\|}-\sum_{j=1}^{N_p} m_j\left[\frac{1}{\|\Delta_j\|}-\frac{\mathbf{r}_j\bullet\mathbf{r}}{\|\mathbf{r}_j\|^3}\right]$$

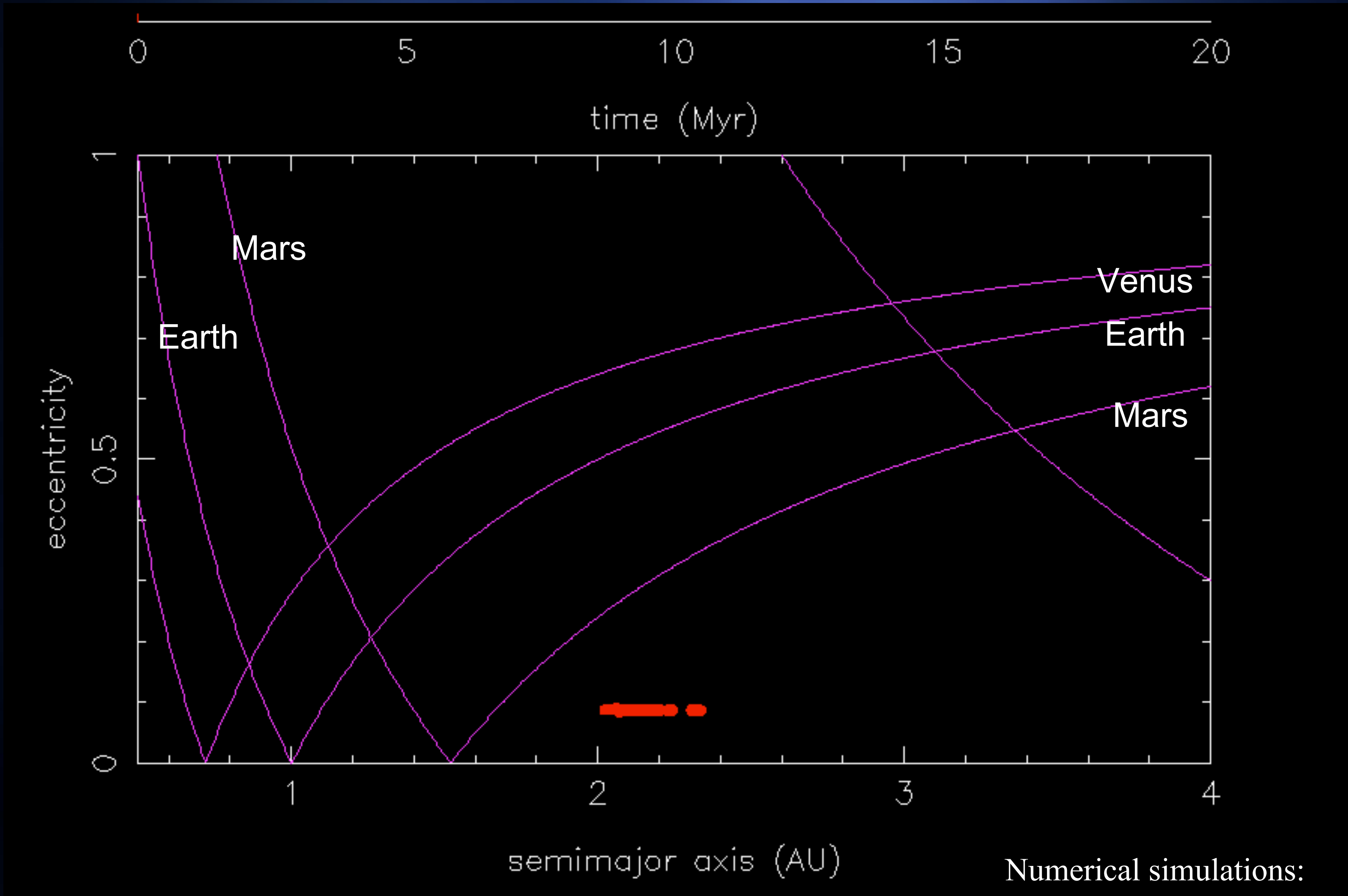
(heliocentric frame)

$\Delta_j = \mathbf{r}_j - \mathbf{r}$ $\quad G = M_{sol} = 1$

SPECIFIC SOURCES OF NEOs:

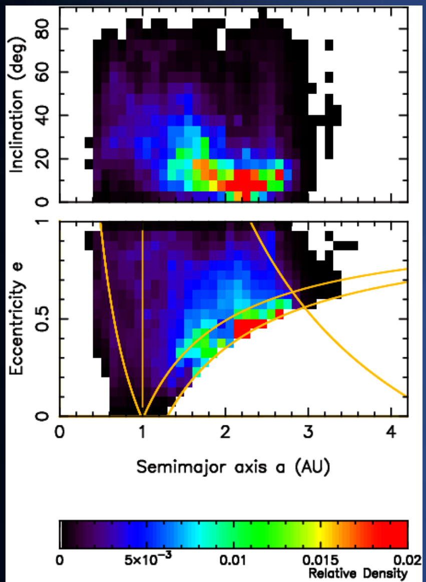


Fast resonances: Main Belt Asteroids become rapidly NEOs by dynamical transport from a source region (in a few million years)

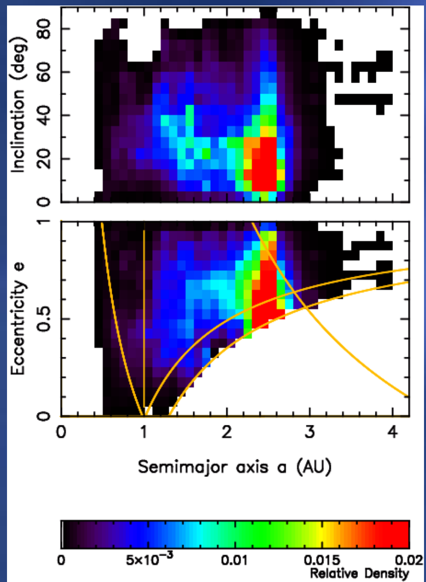


Numerical simulations:
Several 1000 particles

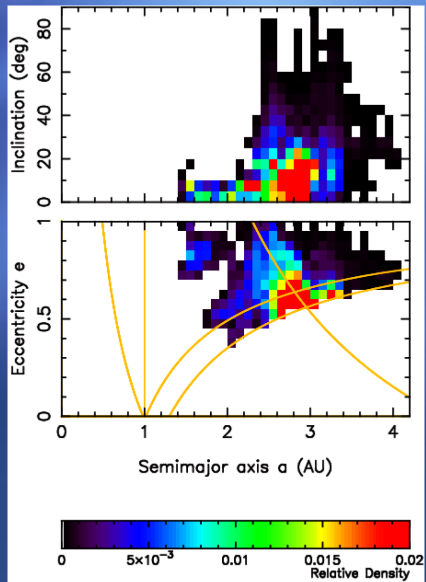
Combine the sources of NEOs so that applying observational biases on the total distribution reproduces the observed distribution



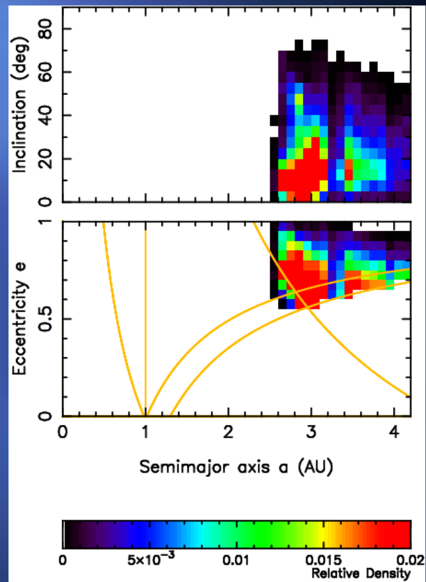
Mars Crossers



3:1



External Belt

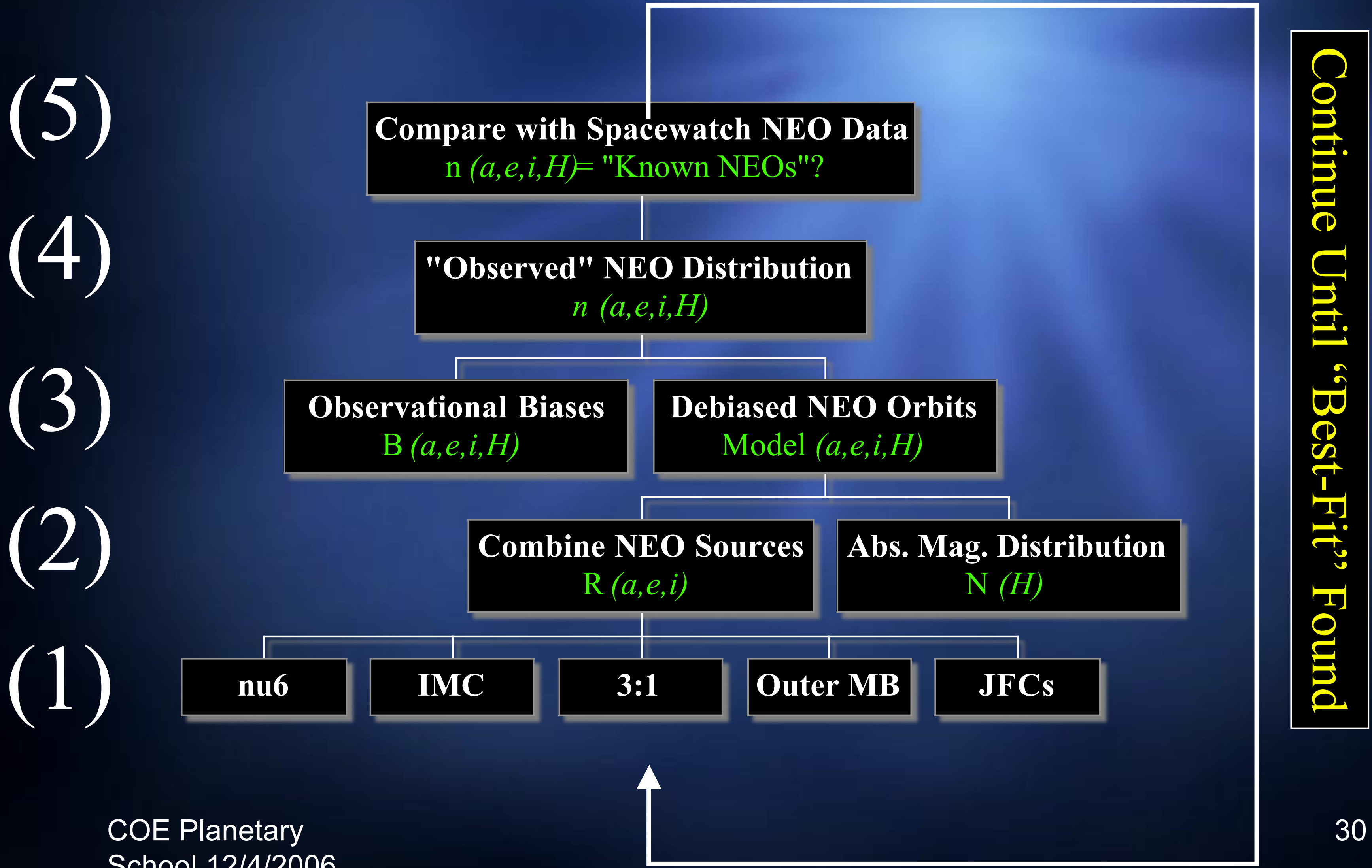


Comets (Jup.)

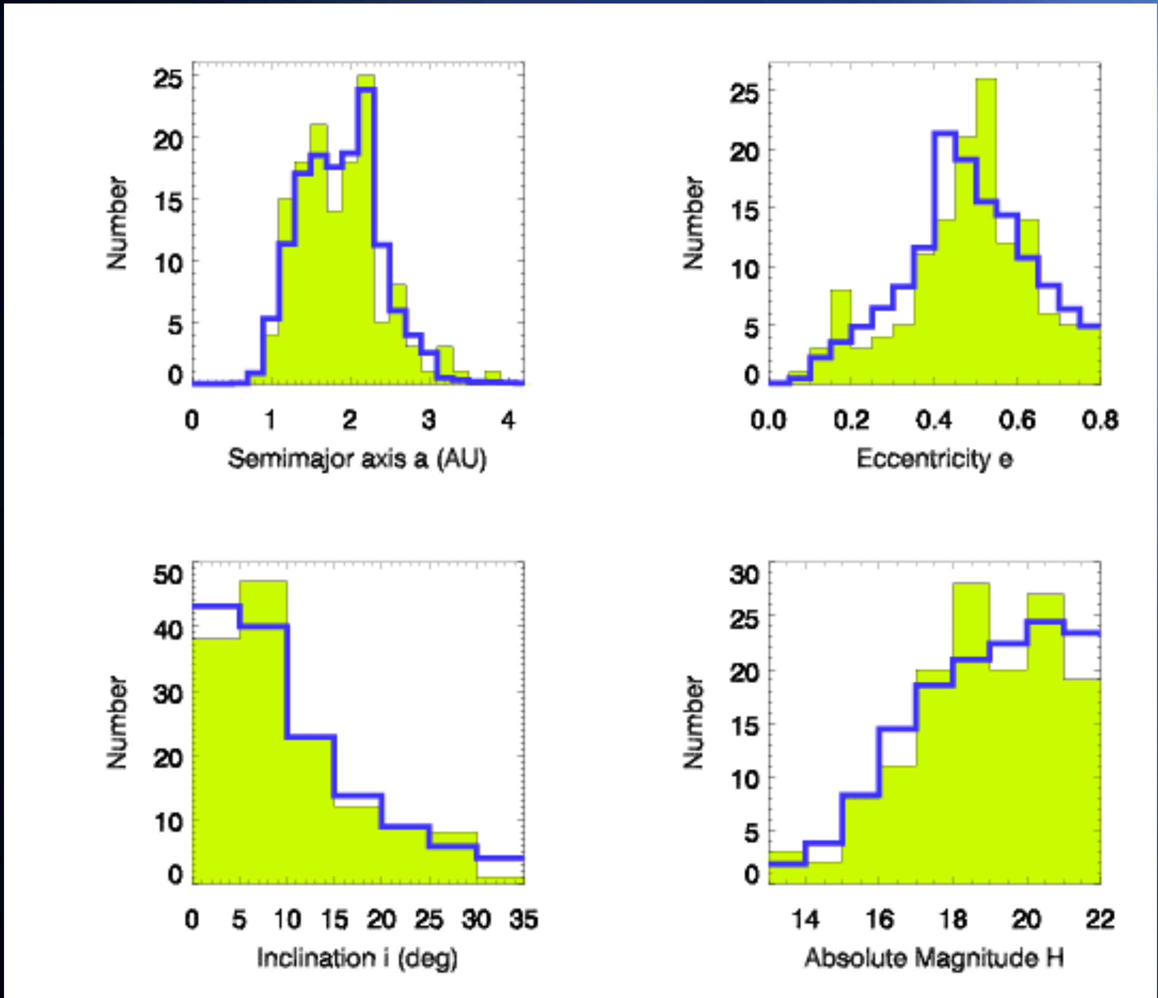
Combine NEO Sources
R *(a,e,i)*

nu6 | **IMC** | **3:1** | **Outer MB** | **JFCs**

Comparison between the *biased* model of NEOs and real data



Comparison Between Discovered NEOs and Best-Fit Model



Weighting factors

ν_6	0.36 ± 0.09
IMC	0.29 ± 0.03
3:1	0.22 ± 0.09
Outer MB	0.06 ± 0.01
JFC	0.07 ± 0.05

Model fit to 138 Spacewatch NEOs with $H < 22$

Our model of real orbital and absolute magnitude distributions of *Near Earth Objects*

~1000 NEOs with H<18 and *a*<7.4 AU

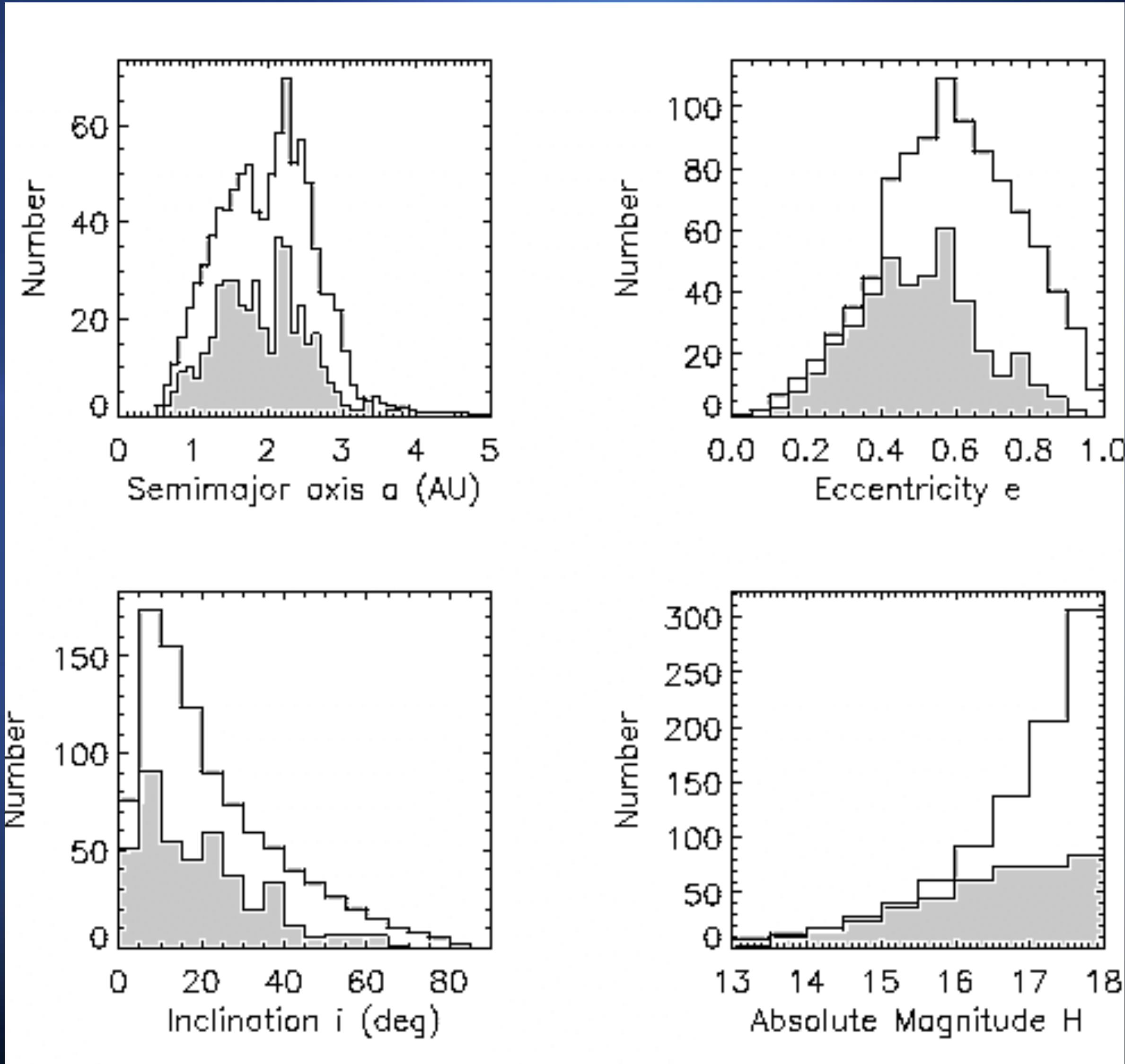
32% Amors

61% Apollos

6% Atens

94% of asteroidal origin

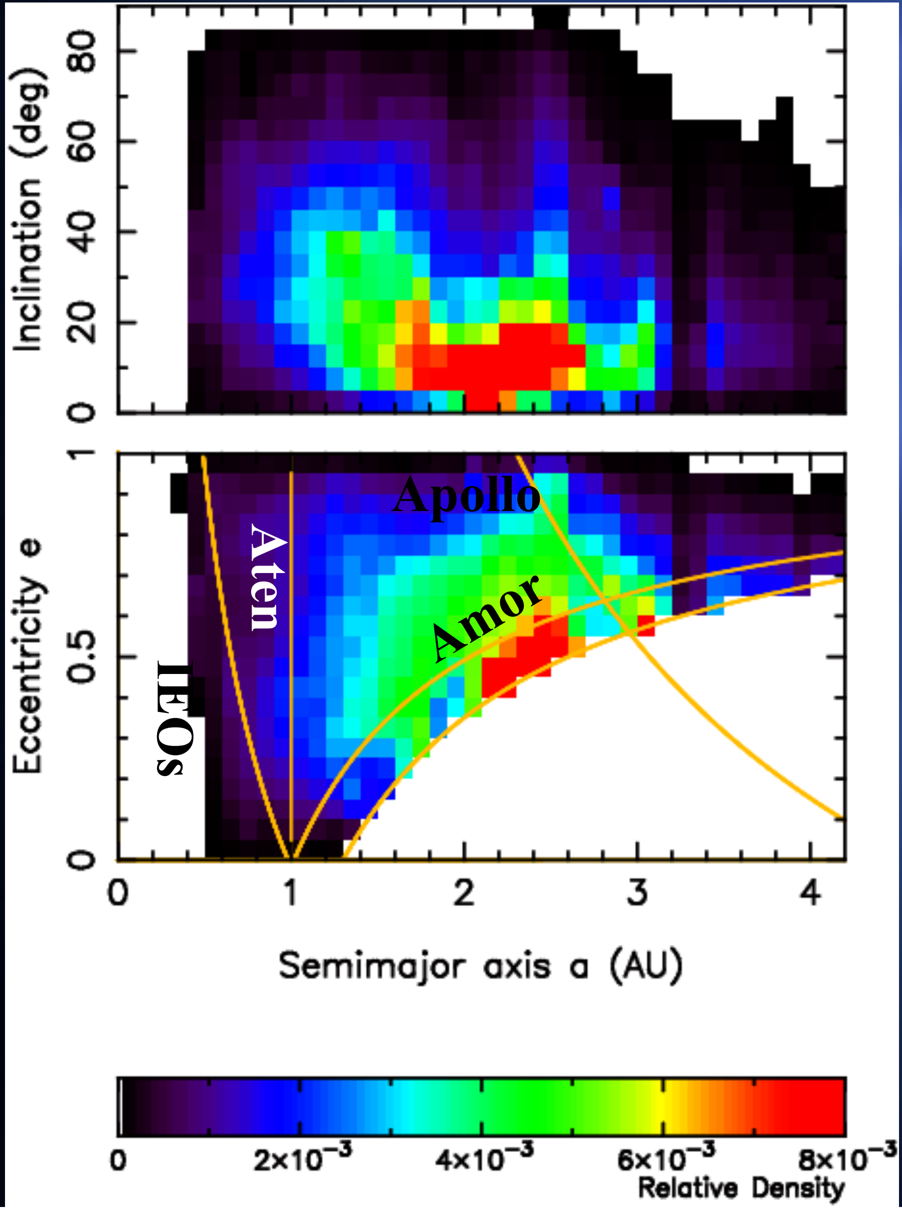
6% dormant comets (Jupiter family)



(Bottke et al., 2000, 2002)

White = model; Gray = observations

Debiased NEO Orbital Distribution



- The NEO population having *H* < 22 and *a* < 7.4 AU consists of:
  - → 32% Amors.
  - → 61% Apollos.
  - → 6% Atens.
- 2% are IEOs (Inside Earth's Orbit).

Bottke et al. 2002

Estimate of 1 impact with energy > 1,000MT per 64,000 years

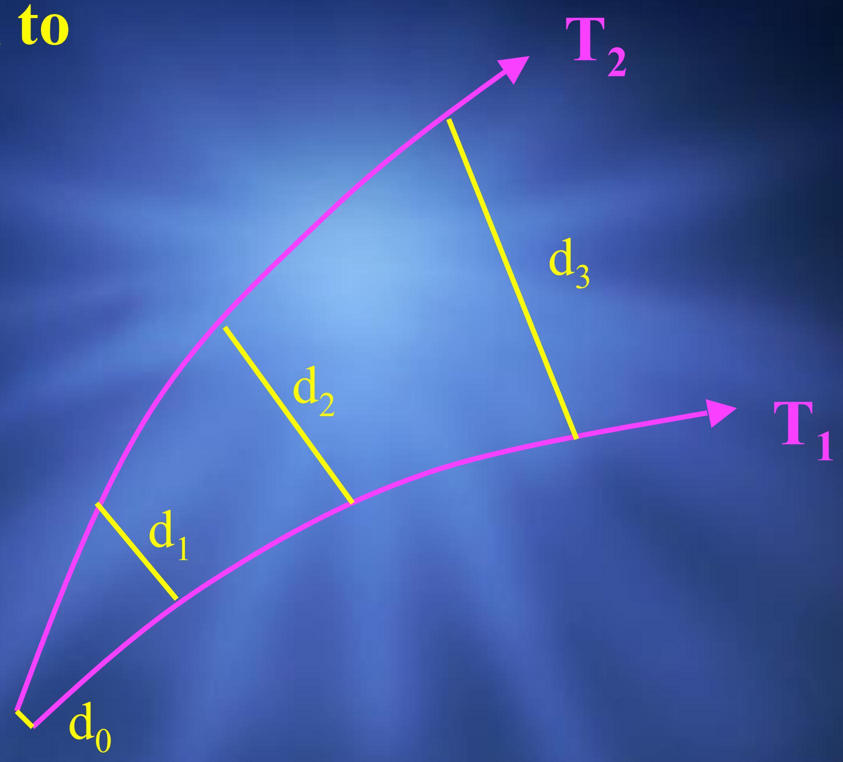
Known NEOs carry only 18% of this total collision probability (*H*<20.5)

Morbidelli et al. 2002

Impact Energy	Mean Frequency (years)	Mean projectile's size	Completness
1,000 MT	63,000	277 m (H=20.5)	16%
10,000 MT	241,000	597 m (H=18.9)	35%
100,000 MT	935,000	1,287 m (H=17.5)	50%
1,000,000MT	3,850,000	2,774 m (H=15.6)	70%

The Lyapunov exponent: a tool to characterize the chaotic nature of an evolution

$$L = \lim_{t \to \infty} \mathrm{Log}(d_t)/t$$



NEOs have positive Lyapunov exponent indicating chaotic evolutions

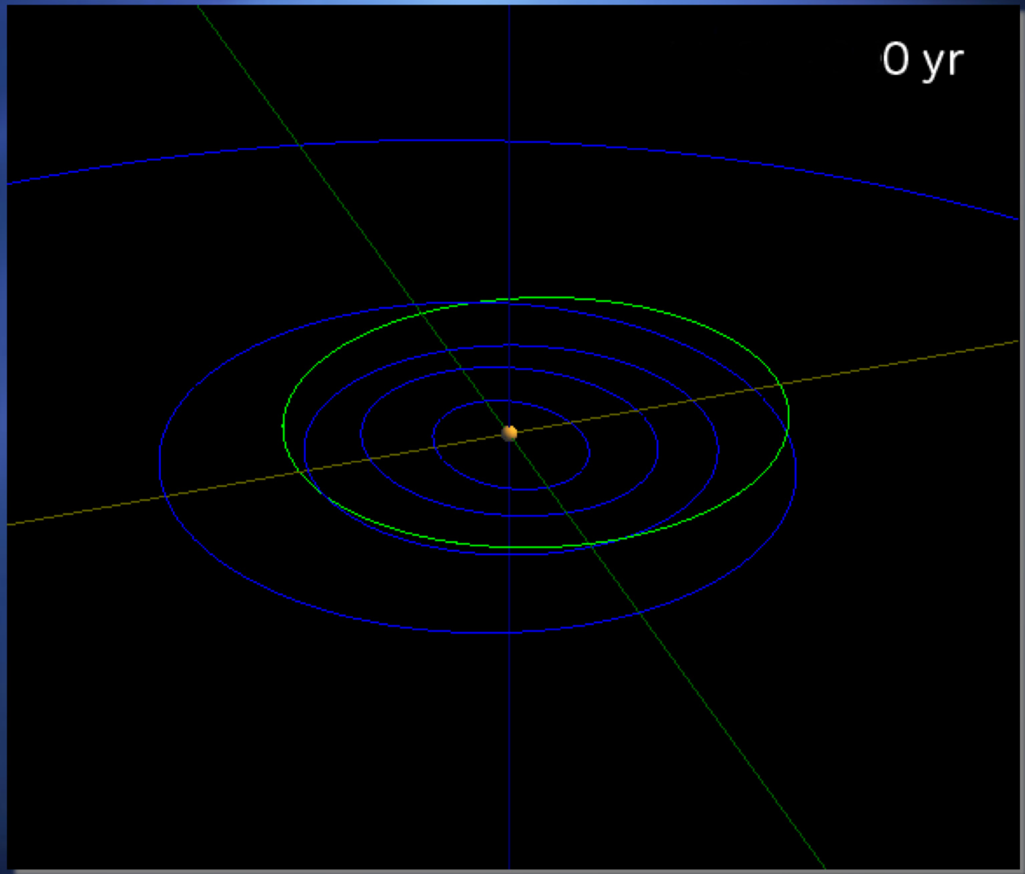
⇒impossible to make long term predictions of individual trajectories

NEOs have chaotic evolutions Example of Itokawa

Computation of the evolutions of **100 initially very close orbits**

Expected timescale for a Collision of Itokawa with the Earth: 1 Myr

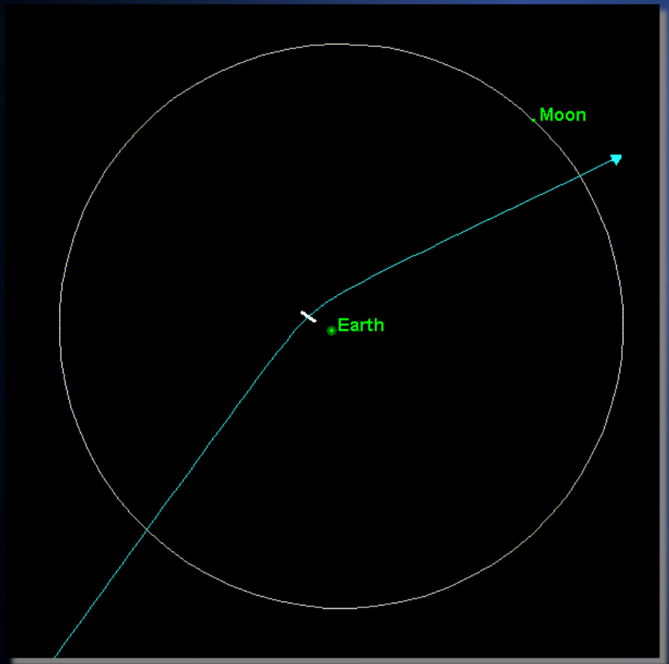
P. Michel & M. Yoshikawa, 2005, Icarus 179, 291-296.

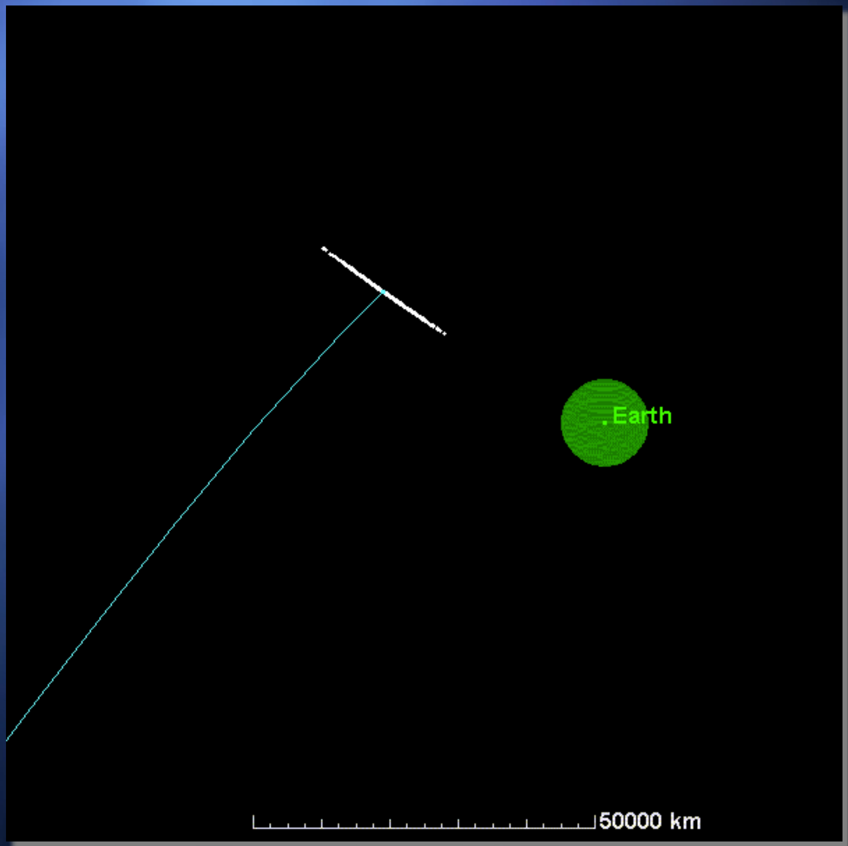


On a shorter term: the threatening object Apophis (size: 300 m)

Trajectory uncertainty: 600 m within which a solution leads to a collision in 2036

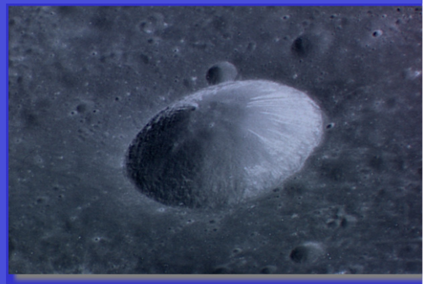
In 2029: approach within 32,000 km!!



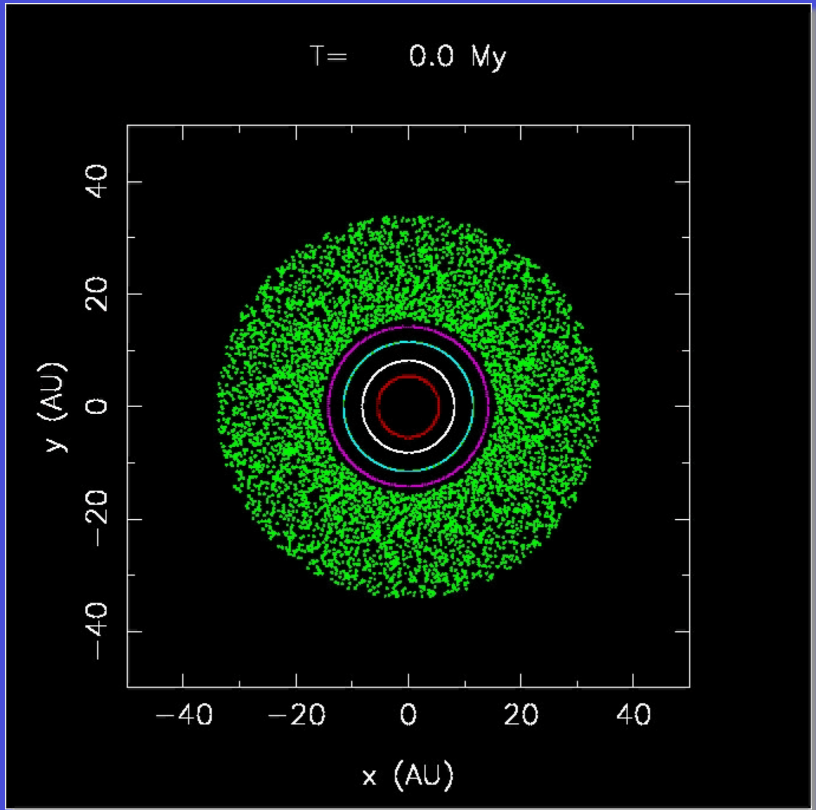


Origin of the Late Heavy Bombardment (3.9 Byr ago)

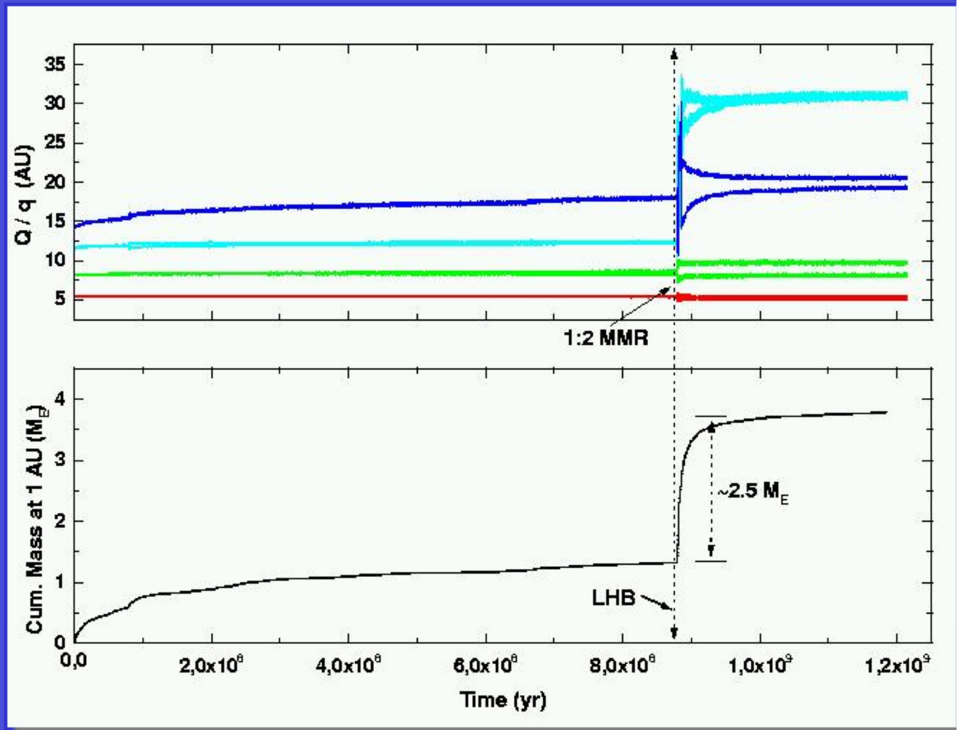
Lunar craters



3 articles published in Nature (Vol. 435, 2005)



External Solar System (in red: Jupiter)
In green: disk of planetesimals



1st scenario which simulatneously explains: giant planet excentricities, origin of Trojans, LHB, and structure of the Kuiper Belt !

Conclusion I

- Mean motion and secular resonances = efficient transport mechanisms by increasing eccentricities or inclinations
- Most NEOs come from the main belt through resonance channels
- LHB can be explained by passage of Jupiter and Saturn in the ½ MM resonance