Nice Observatory

Cannes

sand and water:
2 cohesionless materials!

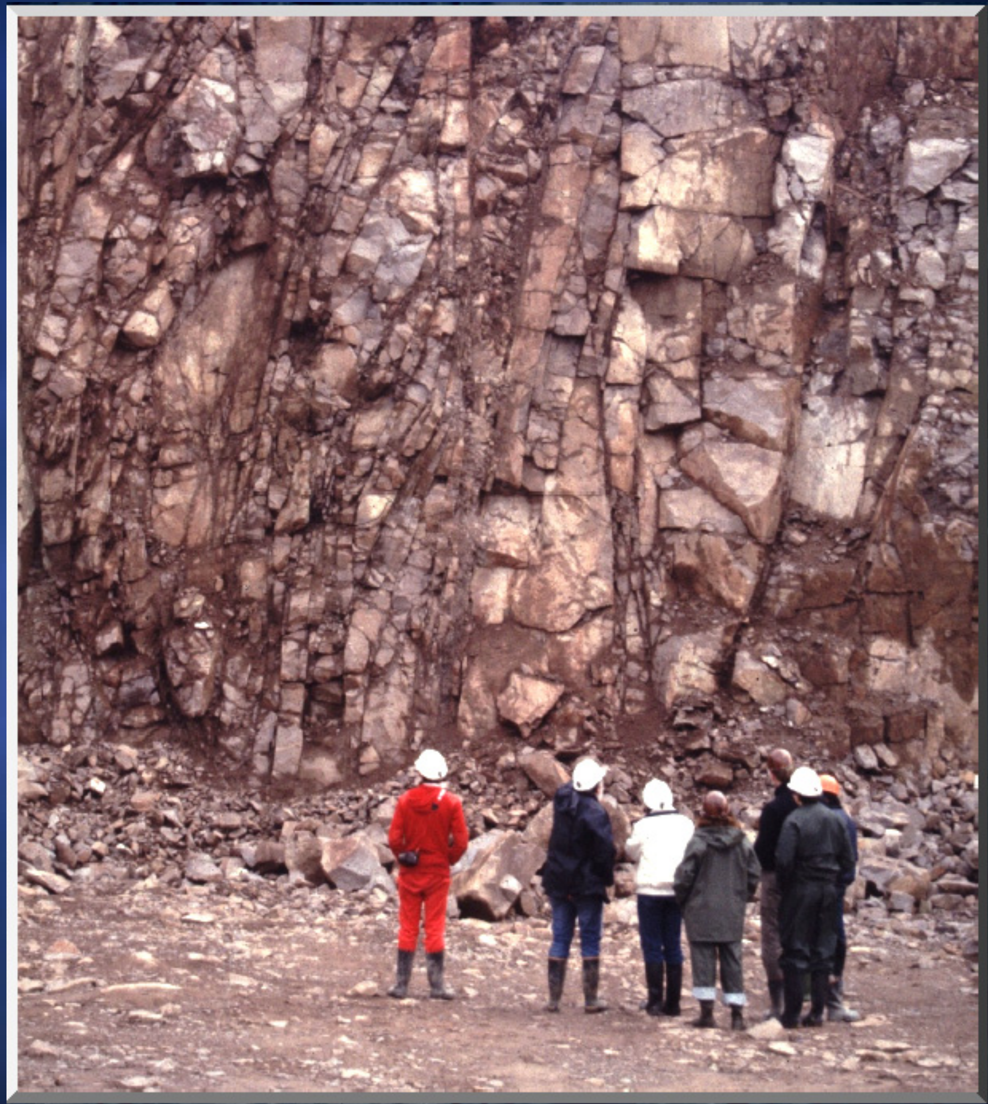
The Alpes
in the snow (another material)!

Both dynamical AND physical properties must be characterized

- To determine the global (collisional and dynamical) evolution of small body populations
- To determine the origin of observed properties (e.g. existence of binaries)
- To define efficient mitigation strategies

Rocks:

A Modeling Challenge



The modeling of material behavior is the biggest shortcoming in code calculations, and the primary reason for bad results..

What I won't talk about, but are important:

Eulerian v. Lagrangian codes

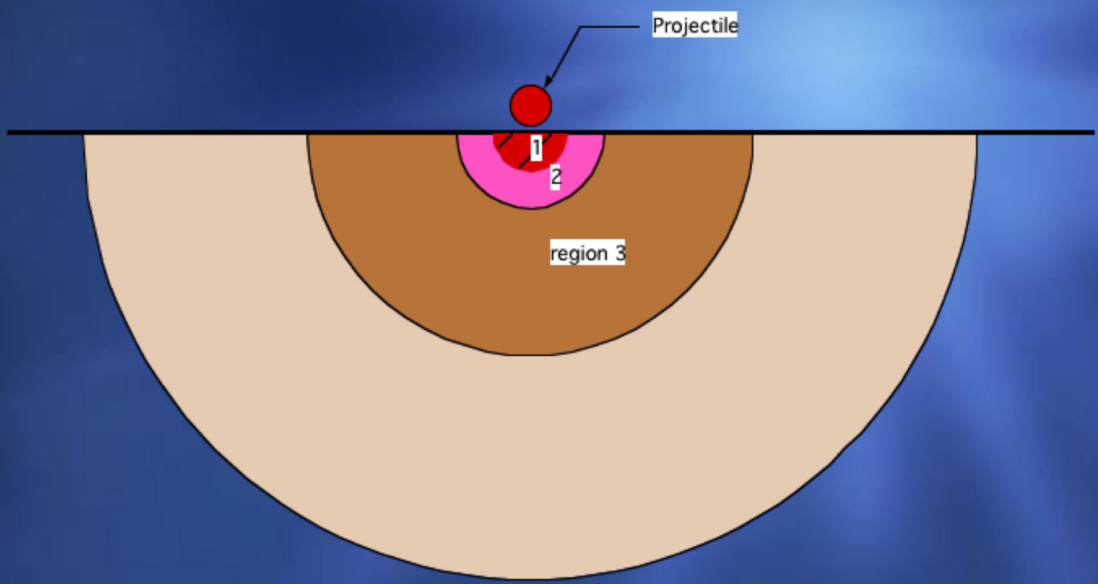
Handling Mixtures in Eulerian codes

Boundaries in Eulerian codes

Grid distortion in Lagrangian

Equations of States of rock materials

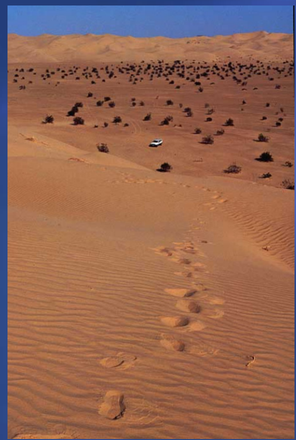
Understanding the process: Regions of Impact Process



1. ***r ~0-> a: Coupling of the energy and momentum of the impactor into the asteroid***
2. ***r ~ a -> 2a: Transition into point source solution, shock breakaway.***
3. ***r ~ 2a ->+∞ : Shock decays with distance, strength (& gravity) become important***

Strength v. Strength v. Strength

Rock Strength

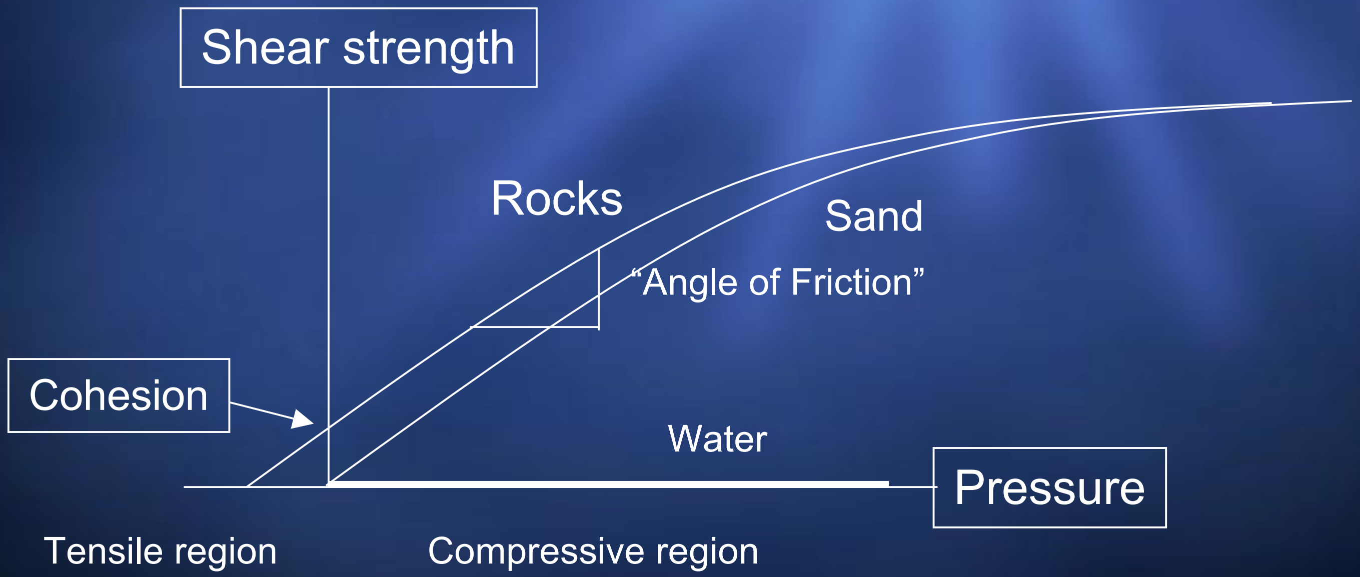


Sand Strength

Water Strength

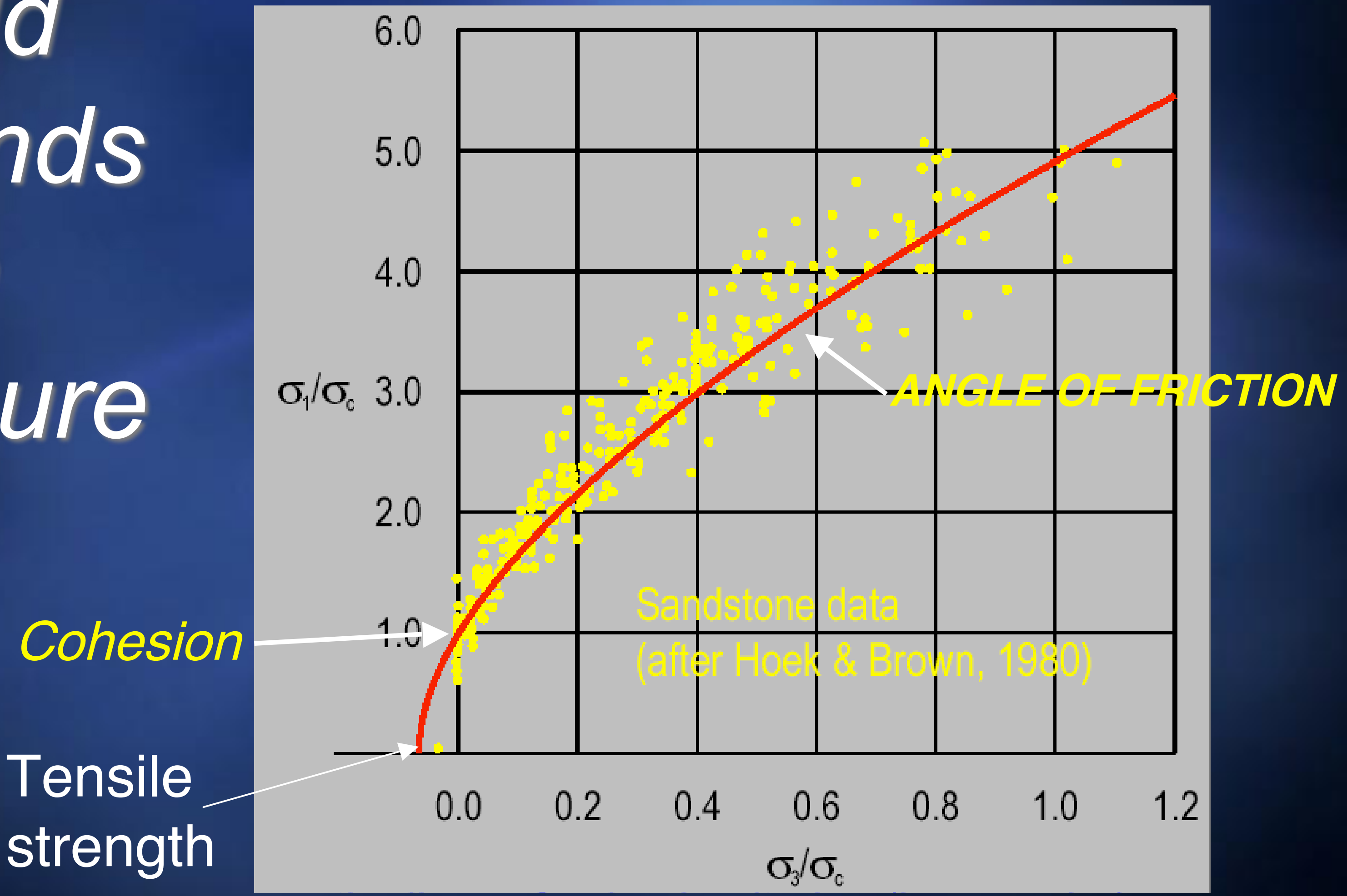
Strength:

- The Mohr-Coulomb (or Drucker-Prager) model:



Some real data

Yield depends on pressure

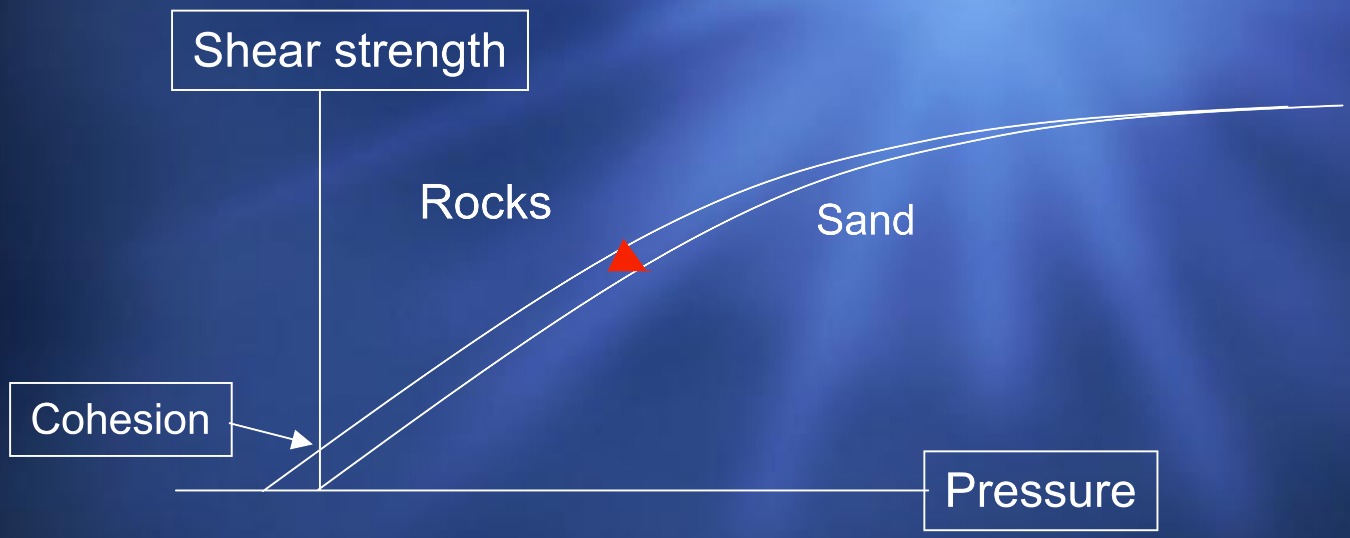


Strength

- A rock has each of:
  - Tensile strength
  - Shear strength (cohesion) ~same as tensile
  - Compressive strength ~5-7* tensile

- But at large pressure, the cohesion can be ignored…

And we have the model for large cohesion- less bodies or rubble piles:



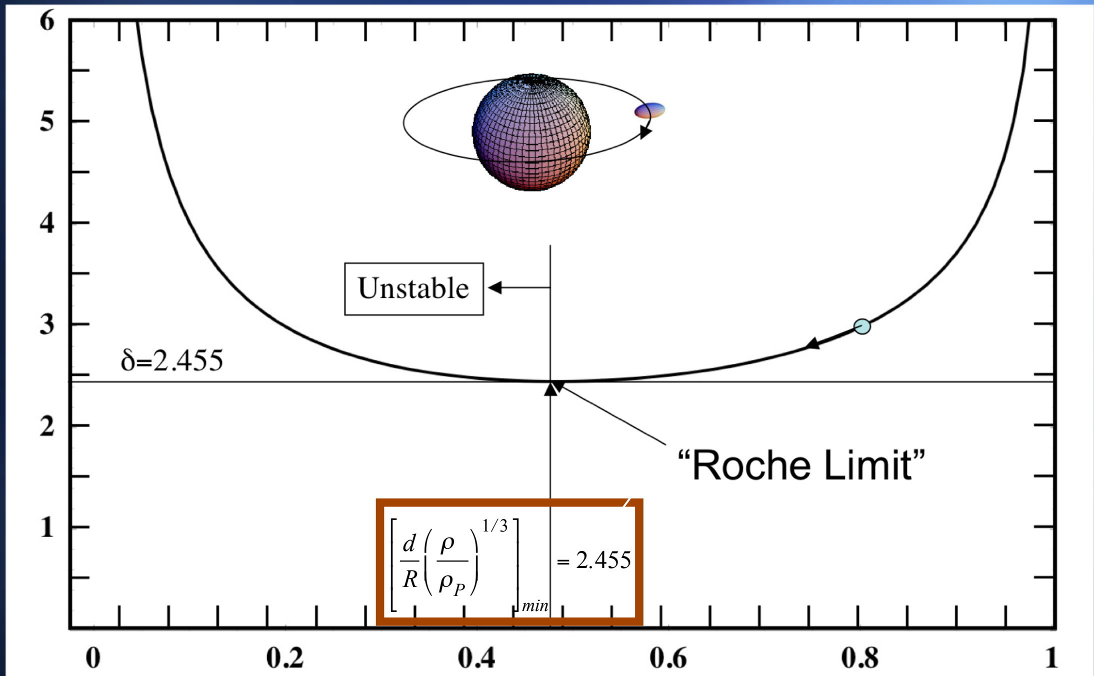
The ‘strength’ is due to the pressure, which is a result of self gravity holding the body together *(but it has no tensile strength)*

First application: Roche limit of cohesionless bodies

- THE ROCHE LIMIT IS A WELL KNOWN FEATURE FOR SMALL Orbiting (or passing) BODIES.

- But:
  - It assumes a fluid body
  - It requires an almost prolate shape with aspect ratios ~2.1:1

So here is the *fluid* tidal disruption problem



Aspect ratio α

Semi-major Axes: a, b, c

Aspect ratios:

$$\alpha = \frac{c}{a}$$

$$\beta = \frac{b}{a}$$

An Example (Phobos):

Does this look fluid to You??

Is it anywhere near the required shape for a fluid body?? (No: a=0.7, not 0.49)

A fluid model is not mandatory for any solid body, even when dominated by gravity (see further)

So:

"Satellites can orbit within their Roche limit because they have non-zero strength"

But, what is "strength"?

Here, we do not mean cohesion

BUT shear strength under pressure

The Problem Solved:

- Determine the tidal disruption limits for a geological material such as rock or sand.
  - Rubble Piles (Ignore cohesion)
  - Then what are the limit tidal disruption distances?

Ref: Holsapple and Michel, 2006, Icarus 183, 331.

Step 1: Determine the stress state

- Include spin, gravity, and tidal forces
- But there are different ways to do this:
  - Elastic Theory: Can determine state for “first yield” (but that depends on residual stresses, which cannot be known)
  - Plastic Limit Theory: Can determine states for “final failure” irrespective of past history.

The stresses at 'final failure' in an ellipsoidal body

$$\sigma_x = -\rho k_x a^2 \left[1 - (x/a)^2 - (y/b)^2 - (z/c)^2\right]$$

$$\sigma_y = -\rho k_y b^2 \left[1 - (x/a)^2 - (y/b)^2 - (z/c)^2\right]$$

$$\sigma_z = -\rho k_z c^2 \left[1 - (x/a)^2 - (y/b)^2 - (z/c)^2\right]$$

The magnitudes are determined by k_x, k_y and k_z which have specific components from each of gravity, spin and tidal forces. For the long x-axis is pointed toward the primary center:

$$k_x = \left(-2\pi\rho G A_x + \omega^2 + 2\frac{GM}{d^3}\right)x,$$

$$k_y = \left(-2\pi\rho G A_y + \omega^2 - \frac{GM}{d^3}\right)y,$$

$$k_z = \left(-2\pi\rho G A_z - \frac{GM}{d^3}\right)z$$

Ax=Ay=Az=2/3 for a sphere and are expressed in terms of elliptic integrals for an ellipsoid

ω= spin magnitude (about z)

M = primary's mass, ρ = body's density

d = distance (primary's and body's center)

Step 2: Solve the failure criterion for the tidal disruption limit distance

- The failure criterion is the Druker-Prager one (zero-cohesion):

$$\frac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_2)^2\right]=s^2\left[\sigma_1+\sigma_2+\sigma_3\right]^2$$

Define: $\Omega=\frac{\omega}{\sqrt{\pi\rho G}}$

$$s=\frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}$$

and solve for the dimensionless distance:

$$\delta=\left(\frac{\rho}{\rho_\mathrm{p}}\right)^{1/3}\frac{d}{R}=F\left[\alpha,\beta,p,\phi,\Omega\right]$$

p = m/M

ϕ = angle of friction

ρ_p, *R*: primary density and radius

α, β: aspect ratios

This corresponds to solve for “LIMIT” State where no further plastic re-adjustments are possible.

We have done that for many combinations of distance, spin, shape, and secondary size..

as a function of the angle of friction…
(so the fluid case with zero angle of friction is a special case)

Example: prolate bodies, spin-locked

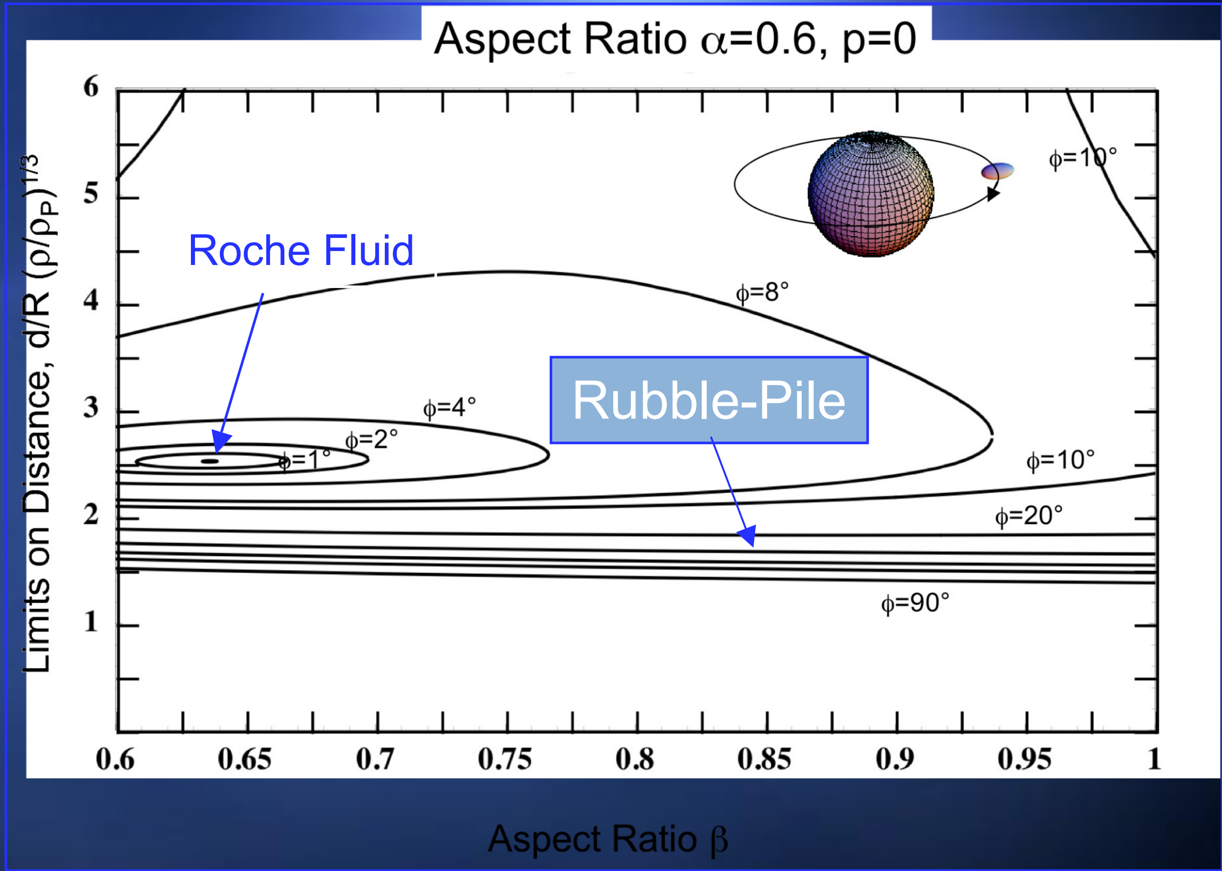
δ=d/R
(for $\rho=\rho_p$)

Semi-major Axes: a, b, c

Aspect ratios:

$$\alpha = \frac{c}{a}, \quad \text{(equal 0.6 here)}$$

$$\beta = \frac{b}{a}$$

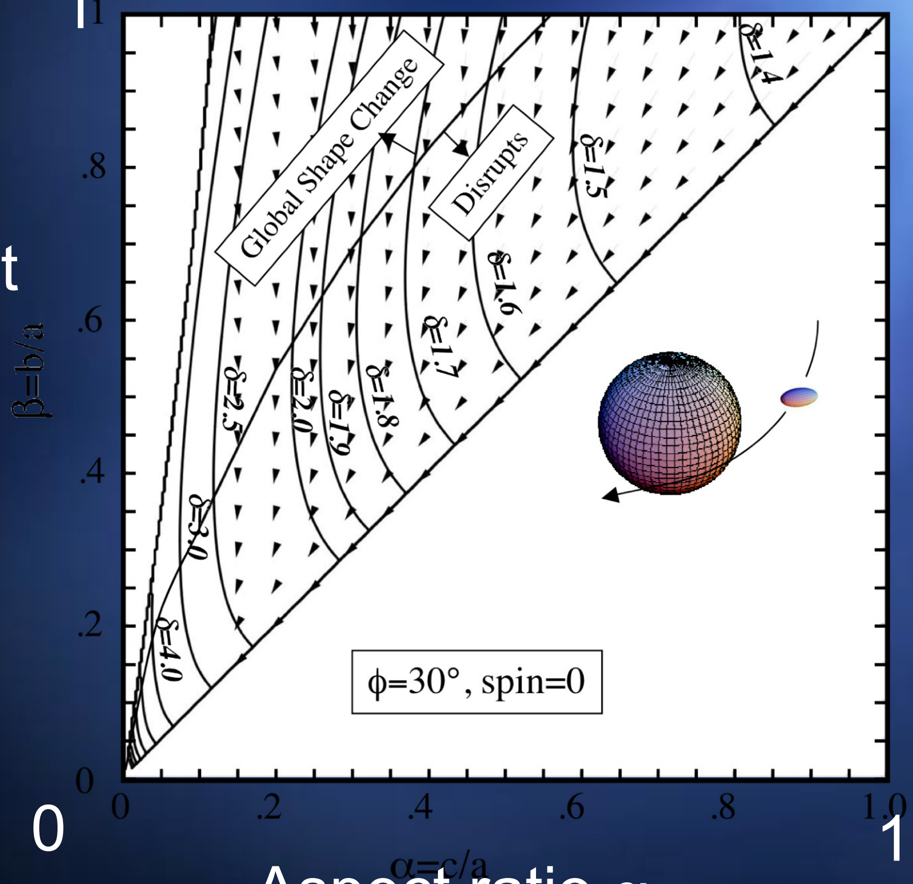


Aspect ratio β

And finally, what if it does have ‘final failure’?

Aspect ratio β

1



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Aspect ratio α

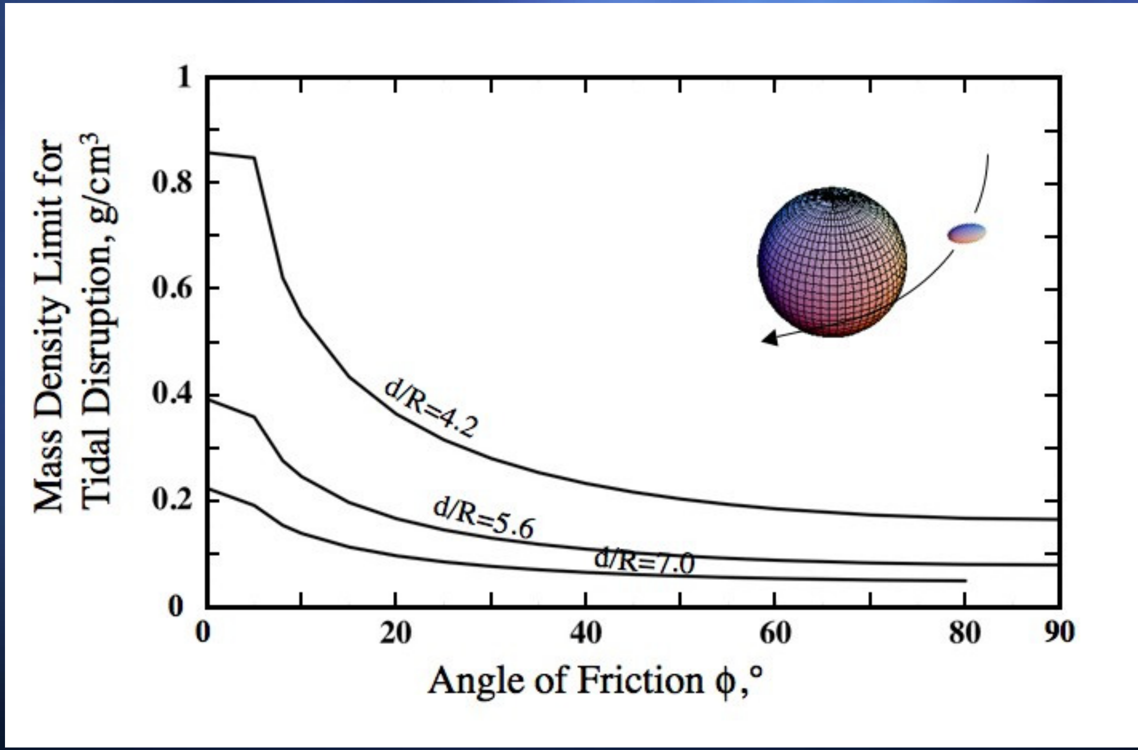
Prolate passing body,
Long axis ‘down’,
No spin,
30˚ friction angle

Application:
99942 **Apophis** *(2004 MN4)*

- In 2029: approach within 5.6 +/- 1.4 Earth's radii from Earth 's center.
- Ellipsoid with aspect ratios a=0.57, b=0.71 (Scheeres et al. 2005), rotation period=30 h.
- We can determine the bulk density for tidal disruption or reshaping vs. the angle of friction f

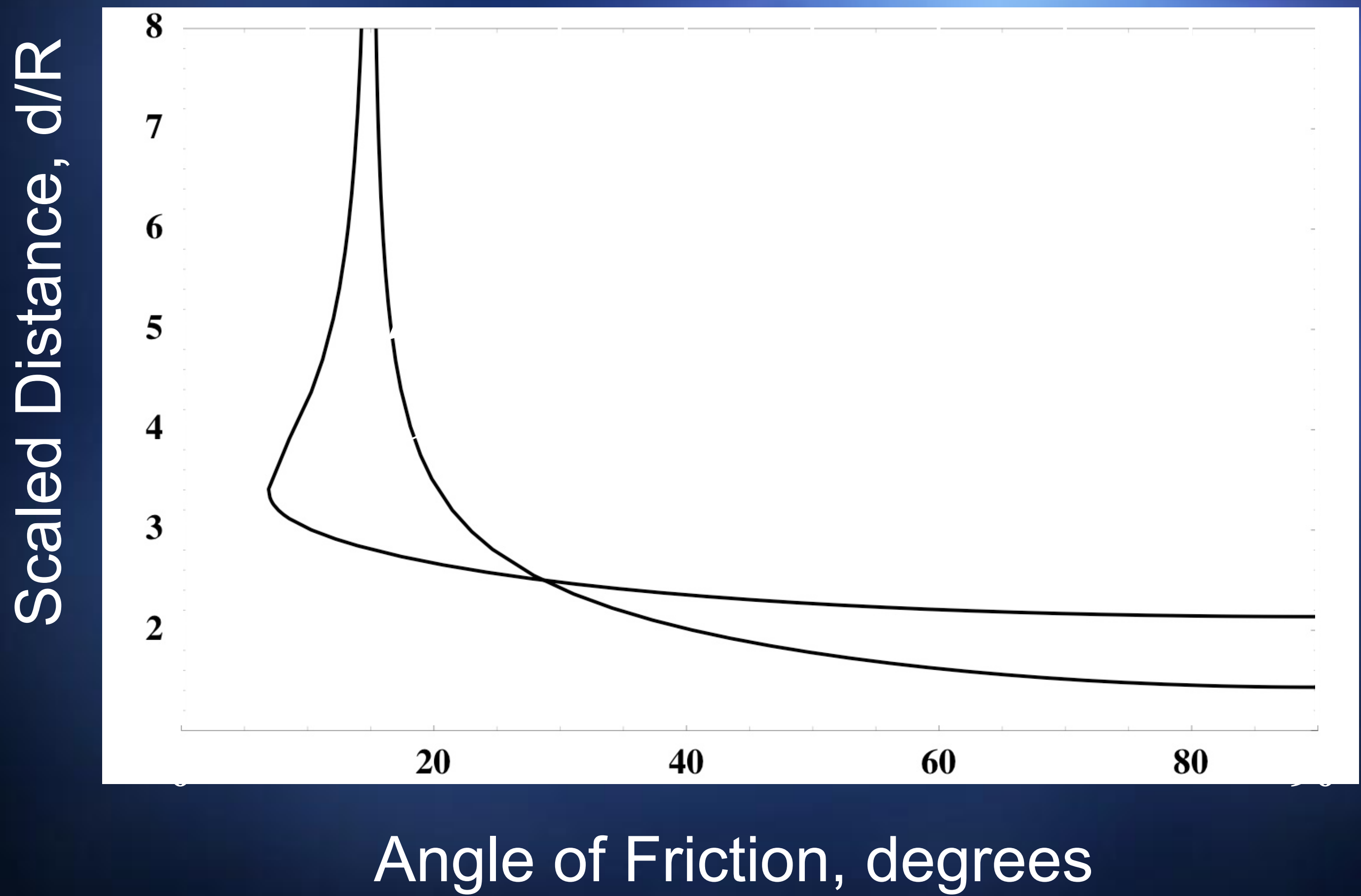
Application:
99942 **Apophis** (2004 MN4)

Minimum bulk density of Apophis for survival without tidal breakup during the passage by the Earth at d/R=5.6, 4.2, and 7.0, for various angles of friction (assumes the worst-case orientation of the longest axis pointed down)

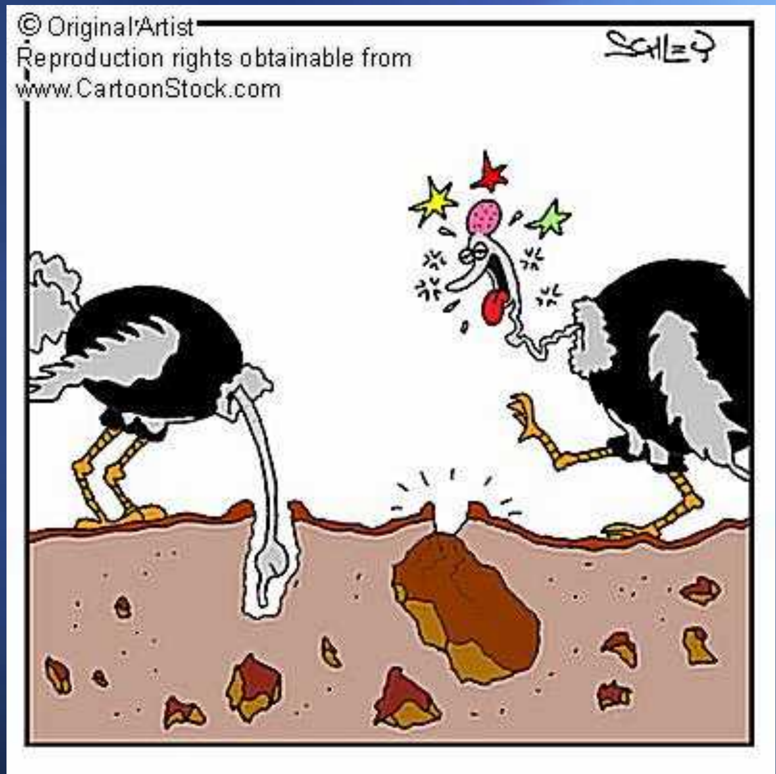


Application: (25143) Itokawa

Minimal distance to Earth for tidal effects



FUTURE STEP: ADD COHESION



« Ostriches trying to stick their heads in the sand »